GEOPHYSICS

1940

A. I. M. E.

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TRANSACTIONS

OF THE

AMERICAN INSTITUTE OF MINING AND METALLURGICAL ENGINEERS

(INCORPORATED)

and Petroleum

Volume 138

GEOPHYSICS
1940

PAPERS AND DISCUSSIONS PRESENTED AT MEETINGS HELD AT NEW YORK, FEB. 18–21, 1935; FEB. 17–21, 1936; FEB. 15–19, 1937; FEB. 14–17, 1938; FEB. 13–16, 1939; FEB. 12–15, 1940.

PUBLISHED BY THE INSTITUTE
AT THE OFFICE OF THE SECRETARY
29 WEST 39TH STREET
NEW YORK, N. Y.

NOTICE

This volume is the fourth of a series devoted to papers and discussions on Geophysics. The four volumes are as follows:

1929, Geophysical Prospecting (Vol. 81, TRANSACTIONS A.I.M.E.)

1932, Geophysical Prospecting (Vol. 97, TRANSACTIONS A.I.M.E.)

1934, TRANSACTIONS A.I.M.E., Vol. 110, Geophysical Prospecting

1940, TRANSACTIONS A.I.M.E., Vol. 138, Geophysics

Volume 74 of the TRANSACTIONS contains one paper on electrical prospecting.

PRINTED IN THE UNITED STATES OF AMERICA

THE MAPLE PRESS COMPANY, YORK, PA.

PREFACE

The evolution in the activities of the Institute with respect to Geophysics (physics of the earth) is suggested by the changes in the name of the Committee. In 1934, when the Committee on Geophysical Prospecting sponsored Volume 110 of the TRANSACTIONS, the latter bore the same title as the designation of the Committee—namely, Geophysical Prospecting. A couple of years later the name of the committee was changed to Geophysical Methods of Exploration. This was regarded as a somewhat broader and more accurate term. This year both the Committee and the volume bear the single word, Geophysics. The contents of the present volume reflect a broadening of the subjects dealt with by the Committee. Some may question the desirability of this trend. However, this much can be said: The problem of finding and delineating mineral deposits in the crust of the earth will continue to become more and more difficult and costly as those lying nearer the surface are exhausted. Efficient and economical methods of prospecting and exploration will be at a premium. The successful searcher—whether he is called a geologist, a geophysicist, or something else—will have to know all he possibly can about "earth physics." It will be a function of the A.I.M.E. to record for its members progress in this art in so far as it is applicable to the effective search for mineral deposits of all kinds.

It is only fair to say, and the other officers listed on pages 9 and 10 will agree, that the man who worked most diligently, and who is largely responsible for the present volume, is Sherwin F. Kelly, who served as vice-chairman of the Committee in 1935 and 1936; and as chairman of the Committee during each of the three following years.

A. B. PARSONS,
Secretary A.I.M.E.

NEW YORK, N. Y.
November 1, 1940.

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INTRODUCTION

Six years have elapsed since the appearance of the last A.I.M.E. Transactions volume devoted to geophysical prospecting. The intervening years have seen a shifting emphasis, as some old methods have declined in importance and new possibilities loomed; but clearly discernible throughout these changes is the trend toward a broadening of the field and a strengthening of the analytical processes. A brief comparison between the last Geophysical Prospecting volume, No. 110, issued in 1934, and the present one, will suggest some of these changes as well as point to a few of the still unsolved questions.

The major problem that we have always with us is that of solving the multilayer case in electrical resistivity work. At least five papers dealing with the theoretical aspects of this troublesome question appeared in the 1934 volume, and now, six years later, we have three articles in the present volume testifying to the unremitting research that this problem entails. The definitive paper has not yet been written, if indeed it ever will be.

In the earlier volume was published the first comprehensive description of a method that in the ensuing years has established itself as an accepted and integral part of the oil exploration program; that is, electrical coring, or electrical well logging, now so widely applied in most of the world's oil fields. No paper on this subject appears in the 1940 volume, possibly a testimony to its taken-for-granted acceptance. Nevertheless, continuous improvements have been made in the technique, and new offshoots have developed therefrom. One of the latter is gamma-ray well logging, which permits the taking of records in cased holes. Although no article on this particular radioactive technique is included in the present volume, there is one that indicates that in the future radioactive measurements may play a larger role in the correlation of geological horizons. It is interesting that this is the first paper on radioactivity to find a place in the Transactions. Possibly it foreshadows a growing usefulness for the phenomena, as they become better understood.

No paper on gravitational methods appears in the 1934 volume, which therefore gave no hint of the procedure that has grown so enormously since then and now dominates the gravitational techniques. One lonely gravimeter, early in 1934, was the sole foreshadower of this new method. In the 1940 volume the papers on gravitational methods are virtually preempted by the discussion of gravimeters in all their variety. The

presence of a paper on the pendulum emphasizes the tenacious hold of the instrument in that fundamentally important field of geophysics, the accurate determination of gravity and of the shape of the earth, and the formulation of geological theories concerning the natures of the continents and the ocean basins of the world. Possibly the inclusion of such a paper will help to break down the artificial distinctions between the applications of geophysics commonly denoted as "geophysical prospecting" and that broader field of geophysics, which may be termed the "physics of earth phenomena."

In the seismic techniques too, there has been a pronounced shift in emphasis. Six years ago the reflection methods of seismic prospecting made their first appearance in the A.I.M.E. volumes, accounting for three out of five papers on seismic procedures. Now, in 1940, there are eight papers dealing only with the reflection techniques. Five of these are included in a symposium intended to enlighten a reader who is not a seismic geophysicist, on the fundamental principles and general procedures involved. The other three papers consist of studies of specific problems.

That the refraction method may yet have other fields of application is emphasized by the papers herein devoted to the utility of the technique for determining overburden depths. An article on this same subject appeared in the 1934 volume, which might be considered almost prophetic in view of the subsequent wide adoption of the method by the United States Army Engineers for the study of proposed dam sites. A particularly interesting application of geophysics is found in the discussions of seismic studies of earth dams, and of soil dynamics and house vibrations. The field of applied geophysics is of ever-increasing importance to civil engineers.

In contrast with the discussions under the headings of seismic, gravitational and electrical methods, the articles on the magnetic techniques give rather the impression of quiet consideration of "case histories." A few specific problems come up for attention and simplified methods of interpretation are offered, but in the main we find the space devoted to descriptions of field surveys and the correlation of the results with geological structure.

An outstanding innovation in this volume is the inclusion of papers dealing with the educational preparation that should be given to students envisaging a career in geophysics. Comments on this important feature will be deferred until after some further attention has been paid to progress in geophysics not reflected so directly in the papers included herein.

Use of Geophysical Methods by Government Surveys

One of the most impressive trends in recent years has been that toward the increasing utilization of geophysical methods by government

geological surveys. The geophysical techniques are rapidly becoming recognized as an integral part of geological field work, and more and more are we seeing a breakdown of the artificial mental barrier that has all too often existed between the concepts of geology and geophysics. Probably the first governmental recognition of geophysics in North America was given by the Ontario Bureau of Mines in 1926. The Honorable Charles McCrea, then Ontario Minister of Mines, specially commissioned the writer in that year to make geophysical studies in the Cobalt-South Lorrain and Red Lake areas of Ontario. The first sustained support, however, came from the United States Bureau of Mines which, a little later, adopted the techniques and used them in the examination of mining districts. In 1936 the United States Government's geophysical activities were transferred to the U. S. Geological Survey. This permitted a broadening of scope so as to include other than mining problems. As early as 1933 the U. S. Geological Survey was exhibiting its interest in geophysics by cooperating with the States of Illinois and Kentucky in the electrical study of fault patterns in the fluorspar fields of those states. This work started under the auspices of the Public Works Administration and was carried on for several years; it formed an integral part of the areal geological study. Under similar arrangements, investigations were made on Alabama limonite ores. With the official formation of its Geophysics Section, the U. S. Geological Survey undertook such problems as the investigation of water supplies in the western United States and Hawaii, and recently was projecting a survey for ground water in Puerto Rico. It began to make geophysical studies of dam and reservoir sites to determine their suitability for the projected structures. Buried channels are being investigated by magnetic and electrical resistivity methods. Geological problems in various states are attacked through geophysical techniques, such as studies of stratigraphy in Michigan and a search for possible laccolith feeders in the Highwood Mountains of Montana. Along with the study of these varied geological problems, the Survey continues the investigation of mining districts and mineral resources by geophysical methods. To facilitate the more efficient carrying out of field and laboratory work, a geophysical laboratory has been equipped for the U. S. Geological Survey at the University of Maryland.

Working on a more restricted area than the Government Survey, the Geological Survey of Missouri has conducted a very thorough areal geophysical investigation of that state during the last decade. This has involved a magnetic study, which, by the end of 1939, covered approximately 31,000 square miles, and it is anticipated that the entire state eventually will be covered. This magnetic investigation has been of great value in giving the trends of subsurface formations, and the results are already in considerable demand because of the oil and gas possi-

bilities in Missouri. Electrical resistivity determinations of the depth of overburden have been used to map preglacial drainage channels, and spontaneous polarization studies have been made to find pyritic deposits. Another State Geological Survey to undertake geophysical work was Alabama, which, in 1935, was making a magnetometer survey of the state. Wisconsin has used electrical studies in excavation classification work, marsh soundings, quarry investigations, and ground-water search. Minnesota also has adopted geophysical methods in the search for road materials.

In Canada, the Ontario Geological Survey has incorporated magnetic studies in some of its geological field work, and New Brunswick has followed a similar plan. During the summer of 1939 the Newfoundland Geological Survey employed a consulting geophysical company to cooperate in its geological study of copper-mining and gold-mining districts. Several years previously the Government of Chile followed a similar program and Peru was planning extensive geophysical prospecting in its oil-bearing areas, while the Government petroleum-producing organization in Argentina has its geophysical commission.

The most comprehensive program of geophysical work by governmental agencies in South America has been in Brazil. The Mineral Production Survey of Brazil boasts a completely equipped geophysical department with apparatus for carrying out electrical, seismic, gravitational and magnetic surveys. This work is directed toward the examining of potential mineral-producing areas for the purpose of encouraging, or discouraging, further effort on the part of private capital. The results of some of these surveys appear in the present volume.

Some readers may recall that in 1931 the Imperial Experimental Geophysical Survey of Australia published an extensive report on the work carried out in that country with the cooperation of the British Government. This early work was followed by the establishment in 1934 of an Aerial, Geological and Geophysical Survey of Northern Australia, which ever since then has been periodically issuing printed reports of its work. These reports cover the geological and geophysical studies of mining regions throughout the Commonwealth, as the various states have collaborated in this program. The cooperation of consulting geophysical companies has also been secured.

Not far away, in New Zealand, the Government, through its Department of Scientific and Industrial Research, plans geophysical investigations of various oil and mining regions, and to aid in the selection of dam sites for hydroelectric power stations.

New Zealand and Australia are not the only countries on the far side of the Pacific that are evincing interest in geophysical methods, for in 1938 it was reported that China had organized a geophysical survey under the Commission of Natural Resources, and that the Japanese were

actively pushing geophysical work in Manchukuo, and investigating the possibilities for utilizing these technics in exploring for Japanese mineral resources.

Traveling westward, some of the governments of India and Africa are contemplating incorporating geophysical procedures in their geological surveys, or have already done so. In South Africa, the Witwatersrand area has been so thoroughly investigated by commercial geophysicists that the Geological Survey of the Union is concentrating its geological work in other areas, and plans to investigate underground-water problems, geological structures, and deposits of base minerals away from the Rand. The private geophysical studies on the Reef area resulted in outlining an 18-mile westward extension of the ore-bearing structures, through magnetic tracing of shale beds in known stratigraphic relationship to the ore conglomerates. Some shafts have already reached the gold-bearing horizons in this new zone, others are being sunk, and an impressive increase has been made in the extent of the gold-producing area of the Rand.

The geophysical attack on vital water-supply problems has been successfully prosecuted in Palestine and in Spain. In Palestine, electrical measurements were used to obtain structural data bearing on the existence and location of adequate aquifers. The Spanish Geological Survey has used seismic and electrical methods to determine the continuity of water-bearing horizons. As a matter of fact, for a number of years geophysical techniques have been an integral part of that Government's geological and mining investigations, including an effort to determine the geological nature of the rocks underlying the Straits of Gibraltar, by taking seismic observations across the Straits.

In Poland, aside from private commercial search for oil, the Polish Geological Survey, for several years prior to the Germanic invasion, had been cooperating in the exploration of various parts of the country. Some of this work was particularly concentrated in the Forelands of the Polish Carpathian Mountains, where seismic, magnetic, electrical and gravitational methods were employed. The magnetic and gravitational work was also extended over large areas in the Polish lowlands. Mineral zones in various mountain formations have also been explored by magnetic and gravitational methods.

Russia has devoted a great deal of official attention to geophysics, and its government geophysicists have been active in the search for oil and metals.

Sweden, the first Government to give active support to geophysical work, over two decades ago, through its encouragement of some of the early experiments in electrical prospecting, is still continuing its active interest in these methods. As a result, several new copper and lead-zinc bodies have been discovered recently in that country. In addition, the

Government-sponsored program includes gravimetric and seismic methods for structural surveys, and electrical techniques for locating faults.

Germany has given many workers to the field of geophysics, and is devoting considerable governmental attention to these methods. As would be expected, most of the geophysical work in that country is Government-controlled, and the little commercial work is "coordinated" to the official General Geophysical Survey. The governmental work is carried out in cooperation with several geophysical institutions at various universities, and embraces all fields of geophysics.

One naturally wonders whether or not the German type of government monopoly over geophysics can result in a progress as remarkable as that which has taken place in recent years in the United States, where competition is the father of invention. Advances are made at a rapidly mounting pace, and the changes come so quick and fast that only a brief survey can be made of progress achieved since 1934. The most spectacular of these changes have occurred in the fields of gravitational and seismic geophysics.

Gravity Methods

The first discovery of an oil field by pendulum methods was that of the Cleveland pool in 1934, and the Tom O'Connor pool, checked the same year, was the first one credited to the gravimeter. Since then, the pendulum has practically disappeared as a prospecting instrument, but the gravimeter has grown in importance and well-nigh replaced the torsion balance, as well as the pendulum. Yet the torsion balance retains its preeminence for detailed work where gradient and curvature values are required, and speed is not so essential. The gravimeter is still undergoing development; doubtless improvements will be made so as to decrease drift and make the machines lighter and stronger, and at least two instruments have been designed recently for use under water. Gravitational surveying seems to be on the increase, and more money is being spent on this technique than ever before. One significant development in gravimetric work is the recent production in Sweden of a torsion balance and of a gravimeter adapted to mining exploration. A paper on that gravimeter appears in this volume.

In spite of its ousting from the field of prospecting, the pendulum nevertheless has its own field, and an important one. The pendulum remains an essential instrument for studies of gravity in connection with geodetic work and the investigation of the earth's crustal structure. Gravity has been studied at sea, with pendulums in submarines, and major anomalies have been found to parallel some of the island arcs, notably in the Caribbean Sea. The U. S. Coast and Geodetic Survey has established profiles of pendulum stations in the Atlantic States, which have shown regional gravity anticlines and synclines paralleling the

Appalachian trends. Companion work in Newfoundland and Canada by the Dominion Observatory staff shows these trends to continue into the Acadian region. Thickening of the crust in the pre-Cambrian is offered as an explanation of the marked negative anomalies observed there. However, if an effective gravity map of this continent is to be formulated, faster equipment will have to be used to make second-order traverses to supplement these first-order ones. The gravitational work in the Atlantic coastal plain has been supplemented by magnetic as well as by seismic profiles.

Prior to the invasion of The Netherlands, a semiofficial Committee for Geophysical Research had planned an extension of the net of gravity stations already established in that country.

Seismic Methods

The oceanic deeps and continental platforms have also been subjected to study by seismic refraction, as well as by gravitational techniques. The U. S. Coast and Geodetic Survey, Woods Hole Oceanographic Institution, the Geological Society of America, the Geophysical Research Corporation, and the American Geophysical Union have been cooperating in a study extending from the foot of the Piedmont Plateau in Virginia to the edge of the continental shelf, 80 to 140 miles off shore, to determine the thickness of the consolidated sediments. These sediments overlie the basement igneous complex with thicknesses from nothing at the foot of the Piedmont Plateau to ½ mile at the coast line, and 2 miles at the continental shelf. The evidence has been interpreted to indicate a downwarping, rather than faulting, as the type of subsidence probably responsible for the foundering of the ancient land mass of Appalachia.

A similar study in Great Britain has shown comparable slopes of the basement complex and thicknesses of the sedimentary mantle along the continental shelf, southwest of England.

The recent establishment of a seismic station in Bermuda will probably give an answer to the question as to whether or not the speed of seismic waves is the same beneath ocean basins as beneath the continental areas. Thus, eventually light may be thrown on the geological constitution of the great suboceanic areas of our globe, and a suggestion by Robert Mallet to this effect, made in the middle of the nineteenth century, will at last be brought to fruition.

An interesting adaption of seismic techniques is found in the studies instituted by Harvard University and the Massachusetts Institute of Technology, whereby large quarry blasts have been recorded on portable seismographs. Usable records have been obtained over a distance of 200 miles, and the results have been used to formulate deductions as to the continental structure in the area, to a depth of nearly 15 miles.

As fixed seismic stations become more numerous, and the net of gravity stations assumes a closer mesh, and magnetic and electrical methods are more and more brought to bear on them, many of the baffling problems of crustal geology will yield to the geophysical attack. Increasing knowledge will be obtained of the interrelationships of regional and local structures, and their influence on economic deposits. To this end more seismic stations should be established, and a widespread system of bench marks set up for gravity and magnetic studies. The adoption of large-scale geophysical investigative techniques by government agencies, already referred to, is a testimony to the growing appreciation of the practical importance of this type of study.

In the commercial field of seismic prospecting, the most impressive change has been the practical eclipse of the refraction method, and the substitution for it of the reflection technique, a typically American development. Although the reflection method had been in use for some time prior to 1934, yet it was in that year that dip-shooting scored its first success as a purely reconnaissance procedure. Reflection shooting increased spectacularly, and by 1936 refraction shooting had all but disappeared. By 1938 the peak in seismic prospecting in North America seems to have been passed, and the current use of something over one hundred crews is apparently an optimum. The decrease in the United States was largely made up, before the outbreak of war, by increasing activity abroad, in which mainly American crews were employed.

Technical improvements effected in recent years include a compounding of geophone energy, better automatic volume control, improvements in the arrangement of receptors, and the use of motion-picture films to record oscillograph signals, together with an electromechanical analyzer for the interpretation of the record thus obtained. Also, the procedure for determining the thickness of the weathered layer has been simplified, and less time is now spent in this task, greatly speeding up field operations.

Electrical Methods

Electrical methods receive some attention in the oil fields, and the Eltran technique is credited with mapping prospects in West Texas where usable reflections were well-nigh unobtainable. It is achieving great depths, and may come in for increasing emphasis. Continuous electrical profiling is also a new development, brought about by the devising of a "mobile electrode," consisting of a tractor with one wheel heavily spiked, to maintain continuous contact with the ground. A great depth range is also claimed for this technique.

A new electrical technique has recently been developed, which may

prove important in studying the disposition and configuration of basement igneous complexes. It has been found that variations in time of the earth's natural electrical currents bear a relationship to the electrical resistivities of underlying structures. By suitable observations of these telluric current variations, it becomes possible to locate buried resistant formations, such as salt domes, and to delineate roughly the igneous basement surface.

Soil-gas Analysis

One authority has stated that, if they want to continue finding petroleum, the oil companies may soon have to turn to electrical methods and soil-gas analysis. The latter is one of the newer, spectacular developments, and the first geophysical technique giving promise of locating oil deposits directly, by surface observations on the hydrocarbon contents of the soil. Progress has been made in analyzing for solid hydrocarbons, in the correlation of anomalies thus observed with anomalous distributions of gaseous hydrocarbons, and in relating these to oil fields. Inorganic constituents of surface soils can also be used as petroleum indicators.

Geothermal Measurements

Geothermal measurements are receiving more attention, and the possibilities are being recognized of a wider field of applicability. They may throw light on the nature and age of geological structures, on the flow of fluids through rocks, and on the influence of oil and gas seepages. True geothermal gradients in wells are important in studies of reservoir content and in correlation, while transient temperatures influence drilling procedures. By combining temperature and gravity studies, the possibility is foreseen of widening the attack on problems of deep structure.

Magnetic Methods

Magnetic techniques remain a minor factor in petroleum exploration, but seem to be slowly growing in importance. Their speed and cheapness recommend them for reconnaissance work, for which they are increasingly employed. One of their particular fields of application is the study of the configuration of basement igneous rocks. They are used for this not only by commercial companies but by some state geological surveys, as already mentioned. In mining work magnetic methods are extremely important, particularly for the delineation of concealed structures. No particularly important improvements have been effected, except possibly the production of a photoelectric recorder for magnetic balances.

Geophysics Education

The accelerating progress and expanding recognition of geophysics have brought with them a keener desire to understand the relationship of this science to the field of geology, and a realization that educational institutions must recognize the need of training men to practice in this new field. In 1938, the Mineral Industry Education Division of the A.I.M.E. established a committee on geophysics education, which marks the first concerted and sustained effort to study the problem of the educational preparation needed for a career in geophysics. The Committee's findings to date are summarized elsewhere in this volume. The most impressive is the practical unanimity among employers of geophysicists and the high-ranking practitioners of the art that the educational preparation should *not* concentrate on instruction in the use of specific instruments and techniques. They insist rather on thorough training in geology, physics, mathematics, chemistry, and allied subjects; in other words, the emphasis should be on the fundamental sciences, not on the minutiae of application—the latter will come with field practice. This reflects a growing trend in education circles, away from "shop courses" and back to a sound training in the fundamentals.

With these requirements in mind, it seems evident that most universities are equipped to give their students adequate preparation for becoming geophysicists. They need only to permit the necessary grouping of the required courses, and grant credits toward a degree to the students that follow what, in most schools, would now be an unorthodox assemblage of lectures.

Geology and Physics of the Earth

It should be kept firmly in mind that not only is geophysical exploration included in the work of a geophysicist, but also that far wider field to which it is closely tied, geophysics considered as the physics of the earth. All too often is the practitioner in one field ignorant of the progress in the other, yet their instruments, techniques, and direct objectives are closely related. In an effort to overstep this false barrier, the American Geophysical Union invited the Geophysical Exploration Committee of the A.I.M.E. to prepare a symposium for presentation at the Annual Meeting of the Union, held at Washington in April 1939. This symposium summarized the techniques and progress in the geophysical search for ores and oil, and described the United States Government's activities in the field of geophysics. This was followed by an invitation extended by the A.I.M.E. Geophysical Exploration Committee to officers and members of the Sections of the American Geophysical Union, not as representatives of the Union but as leaders in their respective lines of geophysical research, to participate in a symposium

on the "Everyday Importance of Geophysics." This was held at the Annual Meeting of the Institute in 1940. The symposium emphasized the little-realized role played by geophysics in our everyday life, particularly in agriculture, commerce, navigation and communication, and brought out the close relationships of these other lines of geophysical research to geophysical exploration and to geology.

In the autumn of 1939, American geophysicists were given an unusual opportunity to meet with their fellow workers from all parts of the globe. For the first time in its history, the International Union of Geodesy and Geophysics held one of its Triennial Meetings on this side of the Atlantic. The American Geophysical Union, a member of the International Union, acted as host for the guests from abroad, at the gathering in Washington, D. C. Many papers were presented and printed, on all phases of earth physics, although the brilliance of the meeting was marred by the onslaught of war in Europe, declared almost at the moment when many of the geophysicists from across the sea were landing on these shores.

The profound importance to geology of this growing science of the physics of the earth is obtaining but slow recognition in geological circles—yet the realization is increasing that we may soon find it necessary to revise completely the traditional approach to geological problems, and attack them from the viewpoint and with the weapons of physics, chemistry, and physical chemistry. It is safe to predict that the time is not far distant when to be a competent geologist a man will need to be also a geochemist and a geophysicist. In only a few academic halls is this already appreciated.

Progress and changes such as these induce speculation as to how our sciences, especially geology and geophysics, will appear to our distant descendants. The Time Capsule, buried by the Westinghouse Electric and Manufacturing Co. at the New York World's Fair, contains many records of this day and age, including an article by the writer on present-day geophysics, to be preserved deep in the ground for 5000 years. The book of directions for finding the Capsule, and for reading what it contains, also includes instructions for discovering its location by geophysics. When the time comes to apply these methods, they will doubtless appear childishly rudimentary—or will geophysics be a lost art by then?

Acknowledgments

In the preparation of this summary, I have drawn heavily on the material provided in past years by many co-workers, and especially by members of the Committee on Geophysical Exploration, for use in the annual reviews of progress that have appeared each January for several

years past in MINING AND METALLURGY. These collaborators are too numerous to name, and to them I express my deep appreciation for their indispensable help. If any errors of fact or interpretation are found in these pages, I apologize to them and to our readers, for the fault will be mine.

SHERWIN F. KELLY, *Chairman*,
Committee on Geophysical Exploration, 1937–1939,
and Geophysics Courses Committee, 1938–1940.

September, 1940

A Perspective of Geophysics

By Sherwin F. Kelly,* Member A.I.M.E.

(New York Meeting, February, 1938)

In presenting this brief historical perspective, it is not my purpose to address myself to the geophysicists, to most of whom the story is already well known. My objective is to draw the attention of those who hold the belief that geophysics is something new, typically twentieth century, to the fact that fundamentally, whether it was known by the name of geophysics or not, the science is an ancient and an honorable one. It began many centuries ago, and our modern civilization and its commerce owe a great debt to its founders, who today are likely to be overlooked and their names forgotten.

The word "geophysics" means the physics of the earth, and is descriptive of a science that in many ways bears the same relationship to geology that astrophysics does to astronomy. When dealing with distant stars or deeply buried rocks, the experimenter cannot reach them to obtain samples, which can be taken to his laboratory for study. He therefore exhibits the resourcefulness of Mahomet in the mountain episode, and makes the terrestrial globe or the cosmic universe his laboratory, studying the effects of distant bodies on certain "fields of force" which come to him across interstellar space, or through the rocks that make our earth. The application of these techniques of the physics laboratory to the study of the stars is the science of astrophysics, and to the investigation of the earth is geophysics.

Yet the origin of the most valuable contribution to the world's trade and commerce ever made by geophysics remains lost in deep obscurity. Unknown even is the country wherein lived the unsung genius who invented the first geophysical instrument, the mariner's compass! Worse yet, it is not certain whether this invaluable guide of man had its beginnings in Europe or in Asia. A Chinese legend tells of the Emperor Hwang-ti, who attacked his enemy Tchi-yeou on the plains of Tchou-lou in the year 2634 B.C., whereupon the latter magically raised such a fog that the Emperor could not pursue the retreating troops. Not to be outdone, the mighty ruler constructed a chariot that indicated the cardi-

Manuscript received at the office of the Institute May 17, 1938. Issued as part of T.P. 950, August, 1938.

* Chairman of A.I.M.E. Committee on Geophysical Methods of Exploration and Chairman of Geophysics Curricula Committee of the Mineral Industry Education Division.

nal points, and thus guided through the fog, captured his discomfited enemy. Because it is known that later the Chinese mounted compasses on their carts, frequently in a pivoted doll-like figure with an arm pointing to the south, this has been assumed by some to indicate that the Chinese knew the principles of the compass at that earlier date. Others ascribe the first recorded "south-pointing chariot" (Fig. 1) to the emperor Hian-Tsoung (806 to 820 A.D.), although a dictionary of 121 A.D. refers to the fact that south-pointing polarity may be conferred on a piece of iron by a lodestone.

Fig. 1.—Japanese south-pointing cart. Of the same general type as the Chinese carts. From The Earth's Magnetism, by D. L. Hazard. U. S. Geol. Survey *Spec. Pub.* 117.

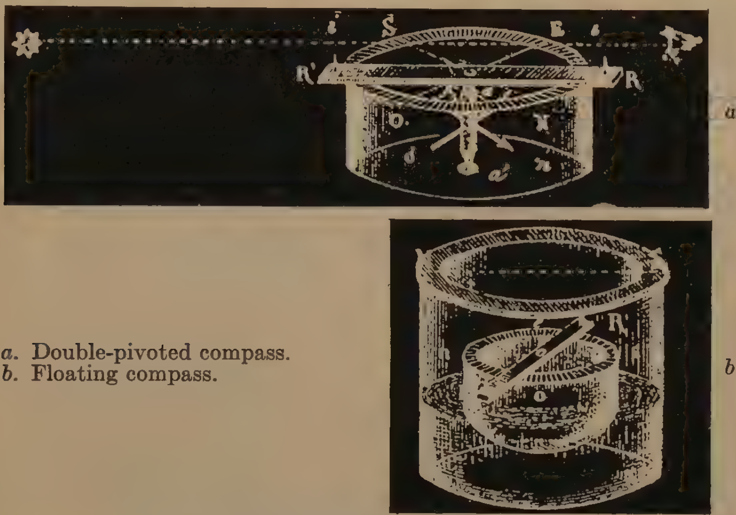
Magnetic

The lodestone's magnetic power of attracting iron was known to the ancient Mediterranean civilizations; it is mentioned by Thales, Plato, Aristotle, Theophrastus, and others. There is no definite reference to the use of a magnetized bar of iron as a compass, however, until the twelfth century, when Alexander Neckham described a pivoted needle, used in ships, to guide mariners when the pole star was hidden.

Nevertheless, nobody seems to have left any records of experiments with magnets for more than 100 more years, until, in 1269, Peter Peregrinus, a student of Roger Bacon, recorded his discovery of some of the laws of attraction and repulsion of magnets. He employed a sphere of lodestone in some of his experiments, and coined the use of the word "poles" with reference to magnets. He thought, however, that the mysterious force acting on the magnetic needle turned it toward the pole of the sky, and was a property of the heavens! To Christopher Columbus belongs the credit, some 200 years later, not only of discovering a new continent, but of making an outstanding scientific discovery, the declination of the compass needle or its deviation from true north. After the lapse of about a century a London instrument maker, Robert Norman, in 1581 studied and described the dip of the needle.

Little other progress was made in the science of magnetics, however, in the 300 and more years since the days of Peter Peregrinus until William Gilbert, court physician to Queen Elizabeth, carried on experiments that have led to his being dubbed the "father of modern electricity" and the "Galileo of magnetism." To these we may add "the first geophysicist."

Gilbert also used a lodestone globe, which he called a "terrella" (Fig. 3), but from his experiments with it he deduced the magnificent generalization that the earth itself is the magnet that turns the compass needle! He described his studies of the dip or inclination of the magnetic needle, of its declination or divergence from true north, and of the irregularities in geographical distribution of this declination. He published his experiments and theories in 1600 in "De Magnete," and for 150 years the science of magnetics could not pass beyond the milestones he had set. In



a. Double-pivoted compass.
b. Floating compass.

Fig. 2.—Compasses used by Peregrinus.
From The Earth's Magnetism, by D. L. Hazard. U. S. Geol. Survey *Spec. Pub.* 117.

fact, his conception of the earth as a spherical magnet was the starting point in the development of the science of terrestrial magnetism. What a debt navigation, exploration, trade and travel owe to this man, the first experimental geophysicist!

William Gilbert's work had stood for nearly 200 years, an unheeded and lonely signpost pointing the way to experimental investigation of this earth, before man began to ask effectively how much our globe weighs, what it is made of, and how its constituent substances affect such forces as magnetism, gravity, earthquake vibrations and electricity. The pull of our earth, which we call gravity, is bound up with its weight and its density, and to these properties Henry Cavendish turned his studies about 1798.

Gravitation

Cavendish designed a special kind of balance, called a torsion balance, to measure quantitatively the force of attraction between two masses. His apparatus was adapted from that of the Rev. John Michell, who

previously had modified a similar instrument invented by Coulomb to study gravitational and magnetic attraction. The Cavendish balance consisted of a horizontal rod suspended by a fine fiber, and carried a 2-in. sphere of lead on each end. When he placed two 12-in. lead balls near these small ones, one at each end and on opposite sides of the rod, the small lead masses were attracted towards the larger ones. This caused the bar to swing a little and twist the suspending wire. With this balance Cavendish obtained a value for the mean density of the earth of 5.45, which is very close to the figure later adopted as a result of more refined experiments. The principle of his balance is now so fundamental in much of the prospecting for oil-bearing structures by gravitational methods that it should be kept clearly in mind. But it was not until about 100 years had elapsed that the Cavendish principle was applied to the investigation of local variations in gravity caused by varying densities of near-surface rock masses.

FIG. 3.—GILBERT'S TERRELLA. From *Applied Geophysics*, Science Museum, London.

The machine capable of accomplishing this is a modification of the Cavendish balance, designed in the 1880's by Baron Roland von Eötvös (1848–1919). He was Professor of Mathematical Physics at the University of Budapest and President of the Hungarian Academy of Sciences. The study of physics and geodesy occupied much of his attention and he designed his balance primarily for the accurate and more extensive study of gravity. Instead of having both weights attached to the bar, von Eötvös suspended one of them from the end of it by a fine wire about 30 cm. long. Thus, to the difference in gravitational pull at the two ends of the bar, he added that at the different levels of the two weights. It may seem incredible that the earth's attraction could vary a measurable amount in such a short space, but it should be remembered that since the gravitational force of a body is proportional to its mass, a body of dense rock has a stronger pull than a light one. In other words, when standing in a region underlain by light rocks, such as limestone, you actually weigh less than if the bedrock is a heavy basalt! Infinitesimal though this difference may seem, the Eötvös torsion balance can never-

theless detect and measure the stronger pull exerted on the weight nearer the denser rock.

Although Professor von Eötvös conceived his instrument as one principally adapted for geodetic research, nevertheless he demonstrated in the early years of the present century that it could be used for interpreting buried regional geological structure through a study of gravitational anomalies. This idea was adopted and elaborated upon by Hugo V. Boeckh, a geologist, in 1917 and by Eugene W. Shaw, an American geologist. In the following year Schweydar and Boeckh delineated a German salt deposit by the use of the Eötvös instrument and technique.

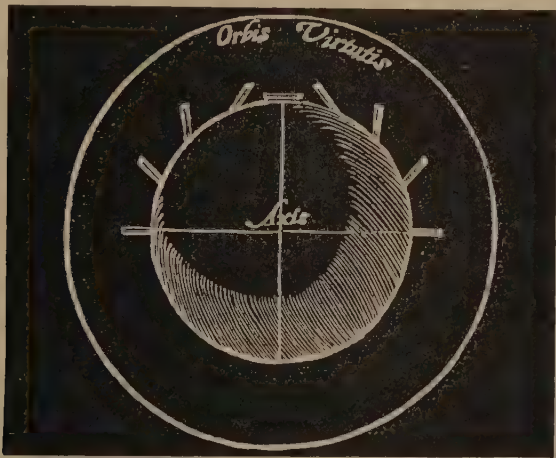


Fig. 4.—Diagram of deductions by Gilbert from his experiments with the Terrella, dip of magnetic needle varying with latitude and magnetic field of earth extending into space.
From The Earth's Magnetism, by D. L. Hazard. U. S. Geol. Survey *Spec. Pub.* 117.

This was the first step in the application of von Eötvös' design, or modifications thereof, in the campaign that has been so successful in discovering oil-bearing formations yielding millions of dollars' worth of petroleum.

The weight of the earth was again subjected to study by von Eötvös and others, giving a value of 5.527 for its mean density. The surface rocks are known to be much lighter than this, having an average density of only about 3, so it follows that the deep, inaccessible interior of our planet must consist of a different material, far more compact than the usual surface formations. Speculating on this phenomenon, geophysicists have come to the conclusion that the outer portion of the globe is composed of rocks varying from dominantly granite near the surface, through increasingly heavy and more "basic" rocks (i.e., containing more iron and magnesium minerals, and less silica), to an interior core of probably metallic iron and nickel with a density of about 12.

The outer crust, too, has been made to yield some of its secrets to the geophysicists studying the vagaries of gravity. Measurements in moun-

tain chains and over ocean deeps, by Bouguer in South America, Maskelyne and Airy in Great Britain, Everest and Pratt in India, Dutton, Hayford and Bowie in the United States, and Meinesz in the East and West Indies, revealed the astounding fact that the mountains apparently consist of lighter rocks, and the ocean deeps lie over heavier ones than normal for the crust! So, like empty barrels, the mountains float higher on some denser substratum, which, solid though it be, can yet yield like inconceivably thick molasses, under the high temperatures and great pressures prevailing miles below the surface. On this substratum our continents are pictured as floating like rafts.

Seismic

Still more knowledge of the earth beneath our feet has been wrested from its rocks by the effects they exert on passing earthquake waves. That earthquake shocks travel by a wave motion in the rocks was recognized as long ago as 1760 by the Rev. John Michell (the inventor of the magnetic torsion balance), but really scientific understanding of earthquakes was first promulgated by Robert Mallet (1810–1881), some 85 years later. He was an Irish engineer, physicist and geologist, whose fame rests as much on the structures he built, such as the Fastnet Rock Light House, as upon his distinguished research in earthquake phenomena. He studied the great Neapolitan earthquake of 1857, including in his publications thereon his conception of the principles of seismology, and established that the focus of that earthquake lay some 8 to 9 miles beneath the surface. Collaborating with his son, he published the Earthquake Catalogue of the British Association in 1858. He was also a great student of volcanic energy. He was a Fellow of the Royal Society, and in 1877 received the Wollaston medal of the Geological Society of London.

It was Robert Mallet who first made the suggestion that artificial quakes be generated by charges of gunpowder, so as to make possible the study of the speeds of the resultant vibrations in various known geological formations. He also conceived the idea that information concerning the geology of the inaccessible ocean bottoms could be obtained from the study of the time of transit of earthquake shocks across the abyssal ocean floor. His vision of the possibilities of investigating earthquake shocks, together with his own researches and experiments in that field, probably entitle him to be considered the "father of modern seismology."

The causes and effects of great earth-shaking tremors became the subject of intensive investigation about this time, especially in Japan, by Milne, Ewing and Omori. In that country seismometry, or measurement of earthquake disturbances, may be said to have been first extensively developed. There, too, the study of earthquake damage to buildings resulted in the formulation of construction rules and precautions

for the guidance of architects and engineers, a realm in which geophysics touches civil engineering.

Such work in Japan, and by Oldham in Europe, revealed the complicated character of earth waves in the rocks. The paths of these waves through the earth can be deduced from the seismic records of a given quake as received at a number of widely separated stations. But since

FIG. 5.—EARLIEST RECORDED SEISMOSCOPE, DEVISED BY CHOKO, A CHINESE, IN 136 A.D. (From *Applied Geophysics*, Science Museum, London.)

When the seismoscope was jarred by an earthquake, one or more of the balls balanced in the dragons' mouths would fall into the mouths of the frogs beneath, depending on the direction and severity of the quake.

such waves are reflected and refracted at rock boundaries, their crooked paths and the speeds of the waves along them tell the geophysicist much about the deep-lying formations that have interfered with the orderly progress of the vibrations. This was worked out by Wiechert in 1907, and his studies, with those of other investigators, have shown that our earth has several layers, or "skins"; the crust itself, some 40 to 60 km. thick, is divided into three layers. The first, about 10 to 13 km. thick, is

mainly of the composition of granite; then follows a layer about twice as thick, more "basic" in character, like basalt or "traprock." The bottom layer is yet more basic, and from it we pass to a subcrustal zone extending down to a depth of about 2900 km., of increasing content of iron compounds (i.e., increasingly basic), until we come to the core—inelastic, highly heated, probably of metallic nickel and iron, but whether solid or not is still to be discovered.

The data thus obligingly furnished by earth tremors about the rocks beyond the reach of man's machines are paralleled by the invaluable information given by artificial earthquakes concerning strata nearer the surface, within reach of drilling tools, from which man draws his supply of that indispensable "black gold"—petroleum. This idea of investigating the nature of buried formations by studying the transmission times of shocks generated by explosions was first put forward about 1900, by Belar, in Germany. Thirteen years later Fessenden, in the United States, patented a scheme of detecting orebodies by sound waves propagated through the rocks; but it was not until Mintrop patented his seismic method in Germany in 1919 that a practicable method was evolved for determining the nature and position of buried rock masses by their effects on artificially created earth waves. Now millions of dollars are spent yearly in this manner in the search for oil-bearing strata.

Electric

It is in this search for oil, worldwide and of such vital importance, that the seismic and gravitational methods have had their widest application and phenomenal success. The magnetic technique has played a lesser role in this field but is of more importance in the search for the equally valuable metallic resources. All three of these methods had their inception in the study of the major structure of our globe, but the fourth geophysical technique, the electrical one, had its beginning as well as its major subsequent development in the mining industry. Its history, too, dates further back than is generally realized, as the electrical phenomena natural to sulphide deposits were first noted in the copper mines of Cornwall by Robert Fox about 1830. He lived most of his life (1789–1877) near Falmouth, England, and was noted in his time for his scientific researches on such matters as high-pressure steam, geothermics, electricity and terrestrial magnetism. His magnetic studies led him to design the first dip circle for the determination of magnetic dip and intensity on board ship. It is interesting to recall that his father, although not a citizen of the United States, was appointed to a consulship by President Washington, and this consulship remained in the Fox family until the post was abandoned some years ago.

Robert Fox and some of his contemporaries made extensive observations of the natural electrical currents flowing in certain types of sulphide

mineral deposits and speculated as to the cause thereof. The idea that the currents could be generated by chemical decomposition going on ceaselessly within the veins was first put forward by A. Von Strombeck. In spite of the interest aroused, no practical application was made of this phenomenon, and the matter was neglected for nearly 50 years. The subject was revived in 1880 by the late Dr. Carl Barus, who, at his death a few years ago, was Professor Emeritus of Physics at Brown University. His work was published in Becker's "Monograph on the Comstock Lode," although Barus had been unable to detect any natural currents in that mine. However, when he removed to Eureka, near by, he was able to verify the presence of the spontaneous currents that had been noted by Fox in Cornwall. Barus was, I believe, the first to suggest that this phenomenon could be used in the prospecting for concealed ore bodies. R. C. Wells, of the U. S. Geological Survey, made an exhaustive study of the chemical reactions involved in this phenomenon, and published his results in 1914 (U. S. G. S. *Bull.* 548: Electric Activity in Ore Deposits). The time was not yet ripe, however, and the idea remained dormant until the late Conrad Schlumberger (1878–1936), Professor of Physics at the School of Mines in Paris, perfected the necessary instruments and technique for practical application of this phenomenon in commercial prospecting. It was the introduction of this method by the author into the United States and Canada in 1921 that marked the first practical geophysical work in North America.

This natural battery action, however useful it might be in prospecting for sulphide deposits, is limited to that field and cannot be applied in wider studies of geological structure. Several experimenters had conceived the idea that the electrical resistances of the rocks could be utilized for this purpose by passing a current through them and then measuring earth resistances at the surface. Like so many new inventions, this plan seems to have occurred to numerous investigators at about the same time. In the United States as early as 1915, F. Wenner, of the United States Bureau of Standards, had published the fundamental principles upon which such investigations must rest. O. H. Gish and W. J. Rooney, of the Carnegie Institution in Washington, developed a practical method for applying these ideas to geological investigations at shallow depth. In France, Conrad Schlumberger was working along the same line and actually carrying out geological studies by means of his resistivity methods. Hans Lundberg was doing the same thing in Sweden, using methods of his own devising, with the encouragement and cooperation of the Swedish Geological Survey. All these methods depend on actually passing an electrical current through the ground, Schlumberger using direct current and Lundberg using alternating current. The idea of inducing subsurface currents by electromagnetic induction occurred to Sundberg in Sweden and Harry Conklin in Missouri. About 1917 Conk-

lin was granted a basic patent on this idea, which has a very definite field of application, although the resistivity methods employing a current passed through the ground are much more widely used.

Practical Application

The magnetic, gravitational and seismic methods of prospecting were outgrowths from earlier studies directed toward elucidating questions of shape and constitution of our earth. Practical application was quick to follow, however, because even the earliest developed geophysical science, that of magnetics, was turned to commercial account very shortly after Gilbert's work, since we learn that in the seventeenth century Swedish miners employed a special form of compass to locate magnetic iron ore. In the electrical method, however, the process was reversed, as it was utilized first in the study of small details, specifically directed toward the search for commercial deposits. Only recently have efforts been made to apply electrical phenomena on a larger scale, with the thought that they may be used to cast light on the deeper structure of the earth's crust. The late Professor Schlumberger probably went farthest in this direction, having extended his studies to a theoretical depth of some 30 miles.

Evidently this application of physics to the study of our planet is no twentieth century development, but extends back to the very beginning of experimental science. Yet, old as is this marriage of geology and physics, it was not recognized as a department of knowledge worthy of distinctive appellation until nearly 1853. In that year the word appears for the first time in a lexicon published in Germany. Thirty-five years later it is found in English, in a review of a geology textbook. The reviewer applies it to problems concerned with the earth's crust and the primitive state of the earth; wherein, of course, geophysics borders on cosmic physics, and the origin of the solar system and of the planet whereon we dwell. The first publication definitely devoted to the physics of the earth was Gerland's "Beitrage zur Geophysik," which appeared at the University of Strasbourg in 1887, and has been followed since by a regular series of volumes. In his preface to the first volume, Dr. Gerland showed the relation of geophysics to other fields of knowledge and emphasized the fact that there is no department of earth lore that does not have its origin in that science.

For example, to start at the beginning of earthly time, the origin and history of the earth, the determination of its age and the changes it has undergone, are geophysical problems. So are the matters of its present movements, its mass, shape, composition, temperature and internal pressure. Here geophysics begins to be allied more closely with geology as we usually picture the latter, especially when the geophysicist investigates questions of crustal deformation and mountain building, drift of the continents, volcanism, and tides on land and sea. When the study

turns to the distribution of mass in the earth, the seismic behavior of core and crust, and the gravitational, magnetic and electrical fields of the earth, we come to the phases of geophysical investigation that have laid the foundations for applied geophysics—physical methods of determining the disposition of geological formations in the crust for the purpose of turning them to economic use.

As a matter of fact, geophysics skirts or overlaps so many earthly sciences that it resembles the ads of the brand of paint for which the proud claim is made that "It covers the earth." It even spreads into interplanetary space at times.

By showing the origins of this science of geophysics and how it is related to the study of our earth, I have attempted to lay a foundation upon which to base an evaluation of the discussion to take place here. This symposium has been organized to give us a preliminary view of how the teaching of this fundamental science is handled at various institutions, and whether it is considered an integral and basic part of geology or treated as a sort of poor relation of that science. Possibly with the discussion to follow, outlined against this historical background, it will be easier to envisage a program for the future, to encourage the teaching of geophysics with a standing befitting its importance.

Appendix

Since this paper was written, my attention has been drawn to one or two points which deserve inclusion in the discussion of gravitational methods. In 1917 and prior thereto, Dr. David White, then Chief Geologist of the United States Geological Survey, and Major William Bowie, now retired as Chief of the Division of Geodesy of the U.S. Coast and Geodetic Survey, had been working on the problem of correlating gravitational anomalies with geological formations and structures. Dr. Bowie had recognized that differences in density of strata in the outer part of the earth's crust would affect the gravity anomalies, and Dr. White had expressed the opinion that such gravity observations could be of great aid in solving some of the problems of areal, historical and economic geology.

For the solution of some of these problems of economic geology, the torsion balance seems to have had its heyday, and in the intensive gravitational search for oil deposits is gradually being replaced by gravimeters of astounding sensitivity. Actually, the use of a gravimeter for measuring minute variations in gravity preceded the development of the Eötvös torsion balance, for M. Perrot was using a gravimeter for this purpose as early as 1862. For other names connected with the development of this important instrument, see the paper by C. A. Heiland beginning on page 196 of this volume.

[For discussion, see page 74.]

The Place of Geophysics in a Department of Geology

By M. King Hubbert*

(New York Meeting, February, 1938)

The growth of human knowledge is an evolutionary process. Historically our separate sciences came into existence as people became interested in various apparently unrelated domains of phenomena, and in this manner we acquired the sciences of physics, chemistry, astronomy, biology and geology. As various of these sciences became established, cognizance of them came to be taken by the universities and in this manner there arose the traditional departments of physics, astronomy, chemistry, biology and geology of our American universities. As there was no particular rationality in the boundaries separating the various sciences, in the university departments the jurisdiction of each has assumed quite arbitrary limits.

In the domain of earth phenomena the first science to grow up was that of geology, which arose originally from the activities of the collectors of rocks, minerals and fossils. Thus geology evolved as a descriptive science dealing with directly observable terrestrial phenomena for which elementary rational explanations were offered. The traditional treatment of phenomena by the science of geology has rarely been with more than an elementary regard for the fact that these phenomena were also phenomena of physics and chemistry.

Since the latter part of the nineteenth century, a new approach to terrestrial phenomena has been emerging and becoming of so great a significance that it can no longer be ignored. This approach has been made by people trained primarily in the sciences of physics and chemistry, who have investigated terrestrial phenomena as constituting phenomena of their respective sciences. In this manner we have acquired the so-called "borderland sciences" of geophysics, the physics of the earth, and of geochemistry, the chemistry of the earth. Neither of these fields is new, having arisen chiefly at the hands of European investigators during the latter part of the nineteenth century. The European universities have long since given official recognition at least to geophysics, by the establishment of departments or institutes of geophysics, which are on a par with the university departments of the other sciences.

Manuscript received at the office of the Institute Dec. 8, 1937; revised April 25, 1938. Issued as T.P. 945, in Mining Technology, May, 1938.

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Logically the problems of geochemistry should be discussed with those of geophysics, and it is only because our interest here attaches primarily to geophysics that geochemistry has apparently been slighted. Let it be emphasized, however, that the omission of geochemistry is in no sense to be interpreted as indicative that it is less important than geophysics. On the contrary, the remarks that are made regarding geophysics are equally valid with respect to geochemistry.

In our American universities, with their arbitrary departmentalization of the whole domain of science into the fields of physics, chemistry, astronomy, etc., each occupying a certain conventional domain, no allowance has been made for the inclusion or recognition of such domains as geophysics or geochemistry. Consequently the rather spectacular development in these two fields has occurred almost entirely outside the American academic scientific departments. Up until the last two decades geophysics was an almost unknown term in American universities, though the leading journal in the field, *Gerland's Beiträge zur Geophysik*, was founded in the year 1887 and is now in its fifty-first volume. In America the development of both geophysics and geochemistry has been almost entirely carried out by such nonacademic institutions as the Carnegie Institution of Washington, with its Geophysical Laboratory, its Department of Terrestrial Electricity and Magnetism, and its Seismological Laboratories; the United States Government bureaus such as the Geological Survey, the Coast and Geodetic Survey, the Bureau of Standards, and the Weather Bureau; the state geological surveys; the several oceanographic institutes; and the commercial companies.

It has been an increasingly anomalous fact that such subjects as seismology, meteorology, geodesy and hydrology are either totally absent from the curricula of most of our best universities or else receive perfunctory attention on the most elementary level. Within the last few years, in recognition of this state of affairs, a few of our universities have made the first timid moves at establishing instruction in the domain of geophysics. In some instances geophysics has been included in the department of physics. In other instances attempts have been made to give the instruction in departments of geology. In still others, instruction in geophysics has been established in very nearly independent departments with close liaison with both the departments of geology and of physics.

These attempts have not been very satisfactory, in general, owing principally to the failure on the part of the university authorities to appreciate the interrelations among the several sciences and the true significance of the so-called "borderland fields." If a department of geology is considering the initiation of a division of geophysics, confusion may be avoided if sufficiently careful consideration be given to the fundamental question of what is geophysics, and what is the relation of geophysics to the remainder of geology. The policy to be pursued with

respect to geophysics will depend almost entirely upon which of two conflicting points of view is adopted.

Relations among the Sciences

Sciences as Separate, Independent Domains.—One of these points of view is that the various sciences constitute separate, equal, sovereign domains of phenomena and of scientific investigation, and that the evolution of each science is occurring jointly by an increase in the intensity of investigation of its conventional subject matter and by outward expansion into new and hitherto uncultivated domains.

According to this conception, the increase of knowledge within each science leads to segregation and specialization, merely because one man can no longer master all that is known of the whole science. The outward expansion of neighboring sciences narrows the undeveloped territory between pairs until there arises a series of "borderland fields" between the various pairs of the sciences, which may be cultivated by workers from either of the neighboring sciences or cooperatively by both. A borderland field constitutes, therefore, according to this conception, merely another subdomain of about the same magnitude and importance as one of the specialized subdivisions within either of the parent sciences.

We may show this diagrammatically (Fig. 1) if we designate one of the parent sciences by the symbol A and the other by B. At time t_0 the two sciences are at an embryonic stage of development. At a later time, t_1, they have expanded and subdivided into the subdomains A_1, A_2, A_3 . . . and B_1, B_2, B_3. . . . At a still later time, t_3, the expansion has continued until the two sciences have begun to overlap, giving rise to the borderland field A-B.

Sciences as Dependent and of Unequal Rank.—On the other hand, the point of view might be adopted whereby it is maintained that the separate sciences are not equal and sovereign domains dealing with different kinds of subject matter, but that all sciences deal with the same subject matter, and differ from one another only in the permutations and combinations of its arrangement.

According to this conception, the common subjects with which all sciences necessarily deal are the movements and changing configurations of matter and the accompanying transformations of energy. But some sciences deal with this in a less restricted and more fundamental manner than others. The science that treats of this common subject matter in the most general, or least restricted manner may be defined as the most fundamental. Of two sciences, that one is the less fundamental whose phenomena constitute special cases of the material dealt with by the other, or which requires material gained by the other science to elucidate its own problems. Sciences are of equal orders of dependence if their mutual dependencies are reciprocal or nonexistent.

Orders of Dependence among Sciences.—We may now regard the several sciences as belonging to various ranks or orders in a scale of dependencies. The science that is so fundamental as to constitute a special case of no other science we may assign to the order 0. Sciences that have a unilateral dependence only upon that of order 0, let us assign to order 1. Sciences belonging to order 2 are dependent with respect to both those of orders 0 and 1, and so on to higher orders.

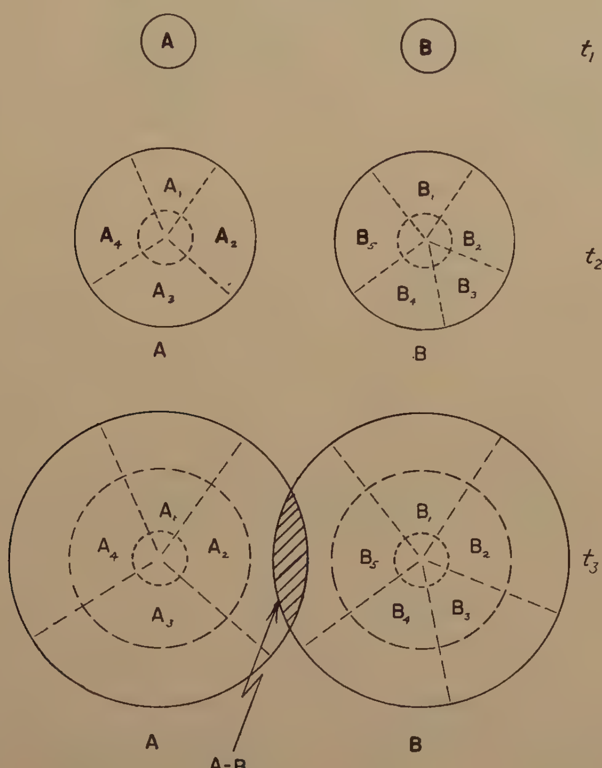

FIG. 1.—THE EVOLUTION OF TWO NEIGHBORING SCIENCES ACCORDING TO THE FIRST CONCEPTION.

Orders of Dependence of the Major Sciences.—With these definitions in mind, it is instructive to examine the several major existing sciences with regard to their accepted scope and subject matter and to arrange them into a diagram according to their orders of dependence, where each successive row from top to bottom represents a higher order than the one next above. This diagram becomes even more significant if between pairs of sciences we draw an arrow from the less to the more dependent of the two.

To do this, let us define the scope of each of the several major sciences in such a manner that our definitions agree approximately with the

limits as currently conceived. Let us define the domain of physics to be the whole unrestricted field of matter and energy; that of astronomy to be the movements and configuration of matter and the transformations of energy on a celestial scale; that of chemistry to be the changes in the combinations and configurations of matter, and the accompanying energy transformations on an atomic and molecular scale; that of biology, the transformations of matter and energy involved in or affecting organisms; that of geology, the configuration and movements of matter and the accompanying transformations of energy of the earth; and finally, the domain of the so-called social sciences we may define as the matter and energy transformations directly involved in or affecting the activities of the human species.

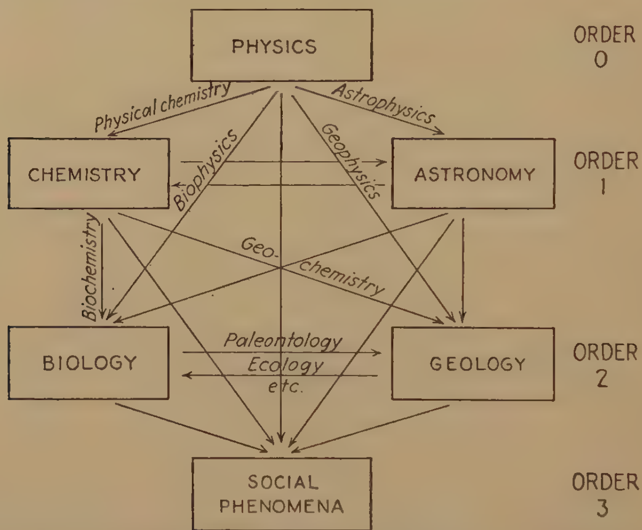

FIG. 2.—RANKING OF THE SEVERAL SCIENCES ACCORDING TO ORDERS OF DEPENDENCE AND THEIR MUTUAL DEPENDENCES.

To be sure, the above are not the conventional definitions that are ordinarily given of the scopes of the several sciences, yet there are no major discrepancies between the scopes as defined above and those given in the more conventional manner. The erosion of a mountain and the feeding and clothing of a human population are phenomena of matter and energy, although they may not customarily be so regarded by the students of these respective subjects.

We are now able to place these several sciences in the diagram (Fig. 2) according to their orders of dependence. In the highest row, of order 0, comes physics, the most general and least restricted of all the sciences, its problems constituting special cases of no other science. To order 1, in the next row down, belong chemistry and astronomy, both being dependent upon physics and having a slight mutual dependence upon each other.

That is to say that the chemical composition of the celestial bodies is arrived at by a knowledge of the data of chemistry, while the stars may be regarded as chemical laboratories wherein experiments are performed that cannot be duplicated in a terrestrial laboratory.

To order 2, in the next lower row, belong biology and geology, both of which, besides having a mutual dependence upon each other, constitute special cases of, or are dependent upon, the phenomena of physics, chemistry and astronomy. Both involve chemical reactions and physical transformations, and depend for their principal energy supply upon solar radiation. Besides, the earth is an astronomical body.

Finally, as the most dependent of all, we may assign to order 3 at the bottom of the list the study of the human species. This is only a particular organic species depending for its food supply upon a complex of other species. It employs materials derived from the earth, obtains its energy from sunshine, past and present, utilizes chemical reactions both physiologically and industrially, and employs every known type of physical transformation that can be effected on the earth. Thus a science of the human species, when one arises, must unavoidably employ materials borrowed from physics, from chemistry, from astronomy, from biology and from geology.

In the diagram of the sciences as shown in Fig. 2 no explicit recognition has been given to mathematics on the ground that it represents only the formalized rules of symbolic logic, depending for their validity upon the realities of experimental science, and that when mathematics goes beyond the realities of experimental verification it becomes merely an amusing form of solitaire.

Relations among Sciences of Different Orders.—If we now study the interrelations among the several sciences as depicted in Fig. 2 a number of things of great significance become apparent:

1. Of two sciences of different orders in the scale of dependencies, a worker in the science having the lower order (the less dependent science) is in need of no special knowledge of the science having the higher order of dependence. On the contrary, a worker in the more dependent science may require a profound knowledge of the less dependent science, of which the phenomena of the more dependent science are but special cases. Thus a chemist may do very well without any knowledge of biology, yet many of the phenomena of biology require for their solution an elaborate knowledge of chemistry.

2. The first stages in the history of any dependent science must be devoted principally to direct observation, recording, describing, mapping, naming, and the offering of elementary rational explanations for the phenomena of that science. If, however, any dependent science is to progress beyond the elementary level of this initial stage, it can do so only through the efforts of workers who, in addition to the essential phenomena

of the dependent science, must also be familiar with the sciences of which its phenomena constitute special cases; or else its workers must independently discover the fundamental relations involved—the latter being both highly improbable and wasteful.

3. The higher the order of dependence of any given science, the larger is the number of other more fundamental sciences whose materials must be comprehended and utilized if the investigations within the dependent science are to progress beyond the elementary stage.

4. What have been mistakenly called the "borderland sciences" such as *A-B* are actually not separate sciences or subdivisions of sciences at all, but are instead the processes whereby a more dependent science such as *B* incorporates the data and techniques of a less dependent science such as *A* for use in solving the problems of *B*.

In the light of this, the directed arrows in Fig. 2 acquire an additional significance in that they represent actual or potential flow of data and techniques from the more to the less fundamental sciences, which flow constitutes the so-called borderland sciences. Of these potential transfers of data and techniques only those of *physical chemistry*, *astrophysics*, *biophysics*, *geophysics*, *biochemistry*, *geochemistry*, *paleontology* and *ecology* have as yet advanced far enough to be dignified by names.

In passing it is interesting to note the position of extreme dependence of the domain of the social sciences with respect to all the other sciences. Yet, with the exception of a slight amount of biology and geology, almost no active recognition has so far been taken of this fact.

Evaluation of the Two Points of View.—It is difficult to overestimate the importance of ascertaining which of the two opposed points of view outlined above is the more nearly correct, because the conclusions drawn and the policies adopted with respect to matters of the greatest scientific importance on the basis of the one are almost diametrically opposed to the corresponding conclusions and policies on the basis of the other.

According to the first point of view—which appears to be much the more widely accepted—it may be deduced that since the capacity of an individual to acquire and retain scientific information is finite and limited, and since the sum total of scientific knowledge already far exceeds that limited capacity and will doubtless increase with time, the ratio of what one man can possibly know to the whole of science must already be small, and can only decrease with time. From this a further deduction leads to the conclusion that it is impossible for one man to become authoritatively informed in any but a small domain of scientific phenomena, and that he must necessarily forego all other domains because of lack of time.

It is thus argued that the specialists in any given field must necessarily know more about the problems in that field than it is possible for an outsider to know. Consequently, on this basis, if an outsider who makes no pretense of being a specialist in the field under consideration offers an

opinion that is contrary to that held by the proper authorities, he can be—and commonly is—damned for his trouble, no matter how accurate or well founded his assertion may have been. This forms the basis for the rigid proprietary right of learning which so largely pervades our universities and professions at the present time. It may also be remarked in passing that it has been upon precisely this basis that almost every major innovation in scientific thought from Galileo onward has been condemned.

If one accepts this point of view he finds himself at a loss to explain how it is possible for a man in one science to solve problems that have baffled the experts in another, yet science is replete with just such occurrences—witness the problems of chemistry, and incidentally of petrology, solved by the mathematical physicist Josiah Willard Gibbs, equipped with nothing more than pen and paper and a profound knowledge of the relations of matter and energy as expressed by the laws of thermodynamics.

If on the other hand one accepts the second point of view, no paradox is involved, for if a man be informed, as Gibbs was, in the relations of thermodynamics, he unavoidably knows the restrictions under which matter and energy must behave, regardless of whether that matter occurs in a test tube, a blast furnace, an organism, or the earth. In general it is impossible for a scientist to be informed in one of the more fundamental sciences without at the same time having knowledge of the sciences whose phenomena constitute special cases of his own science. What may be lacking is a knowledge of the particular configurations of the matter and the related energy in the dependent science, but once this knowledge is supplied, the more fundamentally trained scientist is in a better position to solve the problems in a dependent science than its own experts not so trained.

Apparent exceptions are to be found, where workers trained in the more fundamental sciences such as physics or chemistry make sweeping generalizations contradicting direct observations of phenomena in a less fundamental science such as biology or geology. In all such cases, however, it can be shown that an erroneous statement violates the tenets of the science from which it is supposed to have been derived, or else that either those tenets or the deductions employed are erroneous. Thus if a geological phenomenon can ever be found that violates the laws of thermodynamics, the laws of thermodynamics themselves will have to be revised. Yet there is no sure way of preventing physicists from making statements that, if true, would constitute such violations. Distinction must therefore be made between correct and erroneous deductions from a more to a less fundamental science.

While the more advanced analysis in a dependent science must necessarily come from the hands of workers familiar with the more fundamental science being employed and with the particular configuration of matter

involved in the problems of the dependent science, it may result from close liaison (as with Chamberlin and Moulton) between the workers in each field. This latter type of procedure has the serious disadvantage that the results obtained, in order to be useful to the workers in the dependent science, must be written in the elementary language of that science, which means essentially that rigorous proofs cannot be given, leaving the workers most concerned in the unsatisfactory position of having to accept without adequate proofs the word of an authority who might be mistaken.

Fallacious Attempts at Being Scientific.—Before leaving the general subject of the relations among all of the sciences for the specific one of the problems of geology, a word of caution should be uttered against a prevalent type of error—mistaking the trappings of science for science itself. There was a time when so great an emphasis was placed upon the necessity for accurate measurement that it came to be rather widely supposed that the difference between science and nonscience was principally one of the use or nonuse of mathematics. An investigation that did not employ mathematics was said to be *qualitative* and relegated to a position of inferiority with respect to a *quantitative* one that did.

While this distinction is to a certain extent well founded, it does not follow, as seems to be somewhat commonly supposed, that if mathematics is liberally applied to fallacious premises of prescientific origin a science is thereby created. For were this true we should experience no difficulty in making respectable sciences of alchemy and astrology.

Another common type of error arises when the workers in one field, impressed by the successes of the workers in another, have set about to find an *analogy* between the phenomena of the two fields, so that analogous relations could be established, or, as sometimes happens, when the successful worker in a certain field attempts to solve the problems in another by means of an incorrect analogy. While there do exist among physical phenomena many perfect analogies, the attempt by a famous historian* to establish an analogy between the "degradation of energy" in physics and the "degradation of the democratic dogma" in human history, or the attempt by a distinguished physicist† to establish upon the preconceptions of economics a "mechanics of prosperity" are alike abortive and foredoomed to failure.

The error in all such attempts is the failure to see that the phenomena dealt with, instead of being *analogous* to the phenomena of physics, *are* the phenomena of physics. The degradation of energy occurs in human history in quite as exact an accordance with the laws of thermodynamics as it does in the laboratory of the physicist, and no analogy is required.

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* H. Adams: The Degradation of the Democratic Dogma. New York, 1919. Macmillan.

† H. C. Dickinson: The Mechanics of Prosperity. 1937. Williams & Wilkins.

Recent Geological Trends.—Returning now to the specific problem of the science of geology, it is instructive to consider the advances that have been made since the later editions of Lyell's "Principles."* Lyell had an excellent grasp of the origins of rocks, the evolution of organisms, the principles of stratigraphy, stream erosion, earth history, diastrophism, vulcanism, and glaciation, in so far as direct observation and elementary explanations were concerned. If we contrast this with our contemporary textbooks of general geology there is astonishingly little to be found that Lyell would consider either new or strange. Aside from merely more data on familiar subjects, probably the most important innovations would be the modern theories of the origin of the earth and the elementary chapters on seismology.

If, on the contrary, we look outside the textbooks of geology, and outside the curricula of our geology departments, for treatises of subjects that once were regarded as parts of geology, a strikingly different situation is encountered. Where Lyell was accustomed to descriptive accounts of earthquakes we now have treatises of *seismology* written principally in the language of mathematical physics; where Lyell was accustomed to elementary consideration of the atmosphere and of surface and underground water, we now have highly technical treatises of *meteorology* and of *hydrology*. The same may be said of *oceanography* and of a number of other subjects, nearly all of them characterized by the fact that they have arisen in response to the application of physics and are written in the language of the physicist.

Stated in another way, most of the advances in the theory of geological subjects in the last 50 years have been made by the direct application of physics, chemistry and astronomy to geological problems. In many instances the geologists themselves have played distinguished roles in these advances and have incorporated them into geological thought. Many of these advances, however, have been made principally by people outside the geological profession, and in such a manner as to constitute a virtual "capture" in that the present technical state of those subjects has advanced beyond the present comprehension of the geological profession. Furthermore, there is every indication that this advance will continue, whether departments of geology take part in it or not.

Geophysics in a Department of Geology

Now, returning to our original question of what is geophysics and what should be the status of a division of geophysics in a department of geology, it should be clear that the conception of geophysics, if one accepts the second of the two points of view outlined, is vastly different from that based upon the acceptance of the first point of view.

* Sir C. Lyell: Principles of Geology, Ed. 12. 1875.

According to the first point of view, geophysics is a minor and relatively unimportant subject dealing principally with certain methods of prospecting. Its scope is distinctly limited, and it is commonly assumed that one or two university courses in a department of geology or of physics would adequately take care of the subject.

According to the second point of view, geophysics, as its name implies, is the study of all the physical processes of the earth with the explicit recognition that they are physical processes. The physics of the atmosphere, of the ocean, of running water, of glaciers, of mountain making, of earthquakes, as well as of the earth's gravitational field, thermal field, electrical field, and magnetic field are all geophysics. Hence, far from being a narrow specialty, as the first point of view implies, geophysics is as broad as the problems of geology and as broad as the methods of physics; encompassing, in fact, almost the total breadth of both subjects taken together.

As to the place of geophysics in a department of geology, we have but to recall that geological phenomena are also physical phenomena but that as yet little official recognition has been taken of that fact in the curricula of the departments of geology. The result is that the conventional type of professional training for a geologist continues to be descriptive and elementary, the student rarely acquiring a respectable knowledge even of mechanics.

The question of greatest importance to the geological profession, and more particularly to a department of geology, when confronted by a situation such as this is the very practical one of what is to be done about it.

If a department of geology in a university prefers to retain its present *status quo*, it may of course do so; but if it does, the discrepancy between the physical knowledge of earth phenomena and that of the geology department in question can only increase with time.

If it be objected that the physics of the earth is outside the domain of geology, and that geology itself encompasses only those subjects currently dealt with in academic departments of geology, no logical objection can be made, since this constitutes a definition of "geology" and all definitions are arbitrary. Accepting this definition of the scope of geology, a student of earth phenomena would do well not to devote too much time to a subject that deals so exclusively with descriptive details, if he is to have any time left for the mastery of fundamentals.

If, finally, the department mistakes the name "geophysics" for its actuality and attempts to meet the situation by introducing into its curriculum a trivial course or two, designed for students who can confidently be expected to possess hardly a working knowledge either of elementary mechanics or of trigonometry, the solution is no nearer than if nothing had been done at all.

If the situation is actually to be met, the department is left then only with the third possibility of recognizing geophysics for what it really is—the application of physics to all earth phenomena. In this event the division of geophysics in a department of geology would be staffed and equipped for the joint purposes of carrying out physical investigations of earth problems and of instructing geological students in earth physics.

Qualifications of a Geophysicist.—From what has been said it follows that a geophysicist must be both a geologist and a physicist of the broadest possible training. As regards physics, his concern is primarily with classical as opposed to modern subatomic physics. He must be both a theoretical physicist and an experimentalist, being able to deal analytically with problems of gravitational, electromagnetic, thermal, hydrodynamic, elastic, and strain fields. He should have a thorough grounding in analytical mechanics. He should be able to design, construct and operate mechanical, electrical, and optical equipment for use in experimentation.

On the geological side, he should be grounded in the fundamental facts of rocks and minerals and of geologic processes such as erosion, diastrophism, and vulcanism. He should also be familiar with regional geology, historical geology, stratigraphy, structural geology, and economic geology, and have besides some experience and competence as a field geologist.

Courses of Instruction in Geophysics.—It is suggested that courses in geophysics be given at two separate levels of technicality, an elementary level and an advanced level. The elementary level would be for geology students, both graduate and undergraduate, who have had elementary courses in general geology, a full year of college physics, and mathematics through integral calculus. Furthermore, this should be required of all geological students as a prerequisite for a higher degree in geology, with a possible exception of students specializing in paleontology and stratigraphy.

The advanced courses in geophysics would cover the same ground as that covered in the elementary courses except that the students would have had more advanced work in physics and mathematics as a prerequisite. This course would be primarily for those students interested in specializing in the physical aspects of earth phenomena.

With regard to the scope of the subject matter actually to be covered by courses in geophysics, a certain amount of arbitrariness must be exercised, because to cover the field completely a staff of several full-time men would be required. For geological purposes the field can be narrowed somewhat without too great a loss if meteorology and oceanography be omitted, and the work be confined to the solid earth and to the liquid and gaseous parts thereof only in so far as the latter are more directly involved in processes affecting the solid earth. Thus hydrology would need to be included.

Assuming all the instruction to be done by one man, the subject matter to be covered in both the elementary and the advanced courses might be somewhat as follows:

I. Geodesy
- The astronomical setting of the earth
- Shape and size of the earth
- Motions of the earth
- Moments of inertia of the earth

II. The Earth's Gravitational Field
- Theory of Newtonian potential
- Measurement of gravity field
  - *a.* Intensity
  - *b.* Gradient
- Mass of the earth
- Distortion of gravity field as revealed by
  - *a.* Gravity measurements
  - *b.* Geodetic measurements
- Mass distribution of earth as revealed by gravity
- Prospecting by gravity

III. Seismology
- Earthquakes
- Seismographs
- Seismograms
- Elastic wave theory
- Physical properties of earth's interior
- Seismological method of prospecting

IV. Earth Thermal Field
- Thermal measurements
- Theory of heat conduction
- Sources of heat
  - *a.* Initial heat
  - *b.* Contraction
  - *c.* Radioactivity

V. Radioactivity and Geologic Time
- Theory of radioactivity
- Methods of determining geologic time

VI. Terrestrial Magnetism and Electricity
- Methods of measurement
- Space and time variations of magnetic and electrical fields
- Magnetic methods of prospecting
- Electrical methods of prospecting
  - *a.* Resistivity methods
  - *b.* Electromagnetic methods

VII. Physics of Diastrophism
- Facts of diastrophism
- Mechanics of deformable bodies
- Possible causes of diastrophism

VIII. Tides
- Ocean tides
- Earth tides
- Rigidity of the earth

IX. Hydrology
- Mechanics of running water
- Mechanics of underground water
- Mechanics of erosion
- Mechanics and thermodynamics of glaciers

X. Thermodynamics of Earth Processes
- Flow of matter
- Transformations of energy
- Unidirectional and irreversible nature of earth processes

This is only a rough sketch, and it is quite probable that important subjects have been omitted, but it gives an idea of the scope of the field to be covered.

It is practically impossible to cover such a schedule as this on the elementary level in less than 100 hours of lectures, and it is desirable that ample laboratory work be given in addition to the lectures.

On the advanced level there is no upper limit to the time that could be spent. Single subjects such as seismology, gravity, geodesy, or hydrology alone could easily occupy all of an advanced college course and still not be exhausted. Again, the very minimum of lecture time should be not less than 100 hours, and more should be utilized if available.

Equipment and Facilities for a Division of Geophysics.—The division of geophysics will require lecture space with an ample and convenient blackboard. It will require a laboratory space for student experiments, and suitable equipment both for the laboratory and the field. A part of this equipment can be bought. Most of it, however, will have to be designed and built specially for the purpose.

The most important equipment of the division, aside from the lecture room, will be a machine shop and research laboratory. Both of these are absolutely essential if any original investigations other than theoretical work and routine field work with already existing equipment are to be conducted. The shop will be a light machine shop for both metal work and a limited amount of wood work. The tools required will be one or two lathes, a milling machine, grinding wheel, drill press, hand saw, circular saw, and a miscellaneous array of hand tools and attachments.

If very active research is to be conducted a full-time mechanician will be required, and should be provided for just as though the work were being conducted in a department of physics.

Relation of the Geophysics Courses to the Other Courses in Geology.—It was stated that the elementary level of geophysics courses should be required of all candidates for higher degrees in geology, with the possible exception of the students specializing in paleontology and stratigraphy, and that the prerequisites for this level should be certain introductory courses in general geology, with a full year of college physics and mathematics through integral calculus.

This is nothing more than saying that all geologists ought to be trained in fundamental mathematics and physics in order to understand problems in geology, and that they should then be shown how to attack geological problems by these methods. The elementary courses in geophysics would serve just this function, giving the student a glimpse of the vast amount already known of earth physics.

The advanced courses in geophysics, while covering the same ground as the elementary courses, would do so in a much more advanced manner. For these courses the student should already have had courses in physiography, historical geology, structural geology, and regional geology as prerequisites, besides having acquired a fairly advanced knowledge of physics and thermodynamics. Then the advanced geophysics would become in a sense an integration of both the geological and the physical data bearing upon earth problems. This would place the student in command of what is known already and reveal to him those problems remaining to be solved, while providing him with the means for undertaking such a solution.

It should be emphasized that, while no physical earth problem is outside the domain of geophysics, and the phenomena dealt with are the ordinary phenomena of geology, there is no duplication between the geophysical approach to these problems and the geological approach. Take ground water, for example: The geology courses describe in elementary terms the modes of occurrence of ground water and something of its known flow and geological action, with especial emphasis on nomenclature. The geophysics course would take the geological knowledge of ground water as a starting point and then inquire into the physics of ground-water motion. This would involve such fundamental relations as D'Arcy's law, and a general derivation of the field theory of ground-water motion, including sources and sinks of ground water, the porosity and permeability of the media, and the derivation of a potential function of ground-water flow. After this has been done the flow of ground water into wells, into canals, the boundary between fresh and salt water, and numerous other problems of great practical importance can be investigated.

The same kind of difference between the geological and the geophysical discussions would characterize all other subjects dealt with by the two in common.

Effect of Program upon Professional Standards of Geology.—It will doubtless be wondered what the effect of the adoption of a program such as has been outlined here would be upon a geology department, and probably it will be objected that such a program will serve to frighten away prospective students because of its requirement that nearly all of the students of geology should have a knowledge of elementary mathematics and physics. This raises the question as to whether geology

students are able to comprehend mathematics and physics. If they can comprehend mathematics and physics, there is no reason why they should not be required to do so. If they are unable to comprehend mathematics and physics, evidently there exists at present a process of natural selection whereby out of all of the students that come to a university the superior ones are attracted to fields of science involving mathematics and physics, while the inferior ones are left to take geology and other subjects less exacting still. For there is ample evidence that students who are attracted to fields involving mathematics and physics are not repelled by the problems of geology, and in fact are being attracted to them in increasing numbers. If it be maintained that the students of geology are not inferior to the students of physics, there is no reason why a student of geology should not be able to comprehend at least elementary physics, and the introduction of such a requirement into the geology curriculum should not diminish the number of students taking geology. If the inferiority of students of geology be admitted, the introduction of such a requirement would serve both to attract the superior and repel the inferior student. This might temporarily diminish the departmental enrollment but would greatly improve the quality of the finished product.

Summary and Conclusion

It has been indicated that the phenomena of the earth with which geology deals are also phenomena of physics, of chemistry, and of astronomy. These phenomena can be adequately dealt with only by workers who, in addition to the geological knowledge involved, are also equipped with an appropriate knowledge of the physics, the chemistry, or the astronomy as the case may be, that is required. It has been shown that already there has occurred a great advance in the knowledge of the physics of earth phenomena, and that not only has this advance been made principally by workers outside academic departments of geology but that as yet very little recognition of it has been taken by such academic departments. The result is that students now studying according to the curricula of the departments of geology emerge as professional geologists with only the vaguest comprehension of the enormous body of knowledge now in existence dealing with the physics of earth phenomena.

It has been shown that geophysics, far from being the trivial subject it is commonly erroneously assumed to be, is the study of all earth phenomena with the explicit recognition that they are also physical phenomena. Moreover, the physical study of a number of major subjects that once were regarded as being parts of geology has now progressed so far as to constitute virtual capture, in that these have come to be discussed in such physical language as to be no longer comprehensible to the majority of the members of the geological profession.

This is an evolutionary progression that can neither be stopped nor reversed and probably not even retarded. It places the departments of geology as now constituted in the position of being left farther and farther behind as the scientific procession moves onward.

To deal with this anomalous situation, it has been herein suggested that departments of geology install divisions of geophysics in the true sense of the word, allowing the geophysicist the scope and freedom that his subject requires and such physical equipment and research funds as are necessary for his purposes. An outline has also been given of the type of subject matter to be dealt with in courses of geophysics, with suggestions as to the time required and the types of physical equipment needed.

It has been shown that the introduction of such a program would require some major adjustments of the geology curriculum, the most important of which would be the requirement of elementary courses of geophysics, preceded by an elementary knowledge of college physics and mathematics, as a prerequisite to higher degrees. This adjustment of the curriculum would serve both to repel the inferior type of student who takes geology when he would not dare attempt to meet the requirements of a more rigorously exacting science and to attract the superior type of student who prefers a more rigorous science.

There seems little reason to doubt that ultimately all students of earth phenomena will come to speak a common basic language, which will be a composite of the present languages of the physicist, the chemist, the astronomer, the biologist and the geologist. It is probably of slight scientific importance which of several possible evolutionary paths be taken to this goal. One possibility is that the geological profession may come to a recognition of the problems that have been herein discussed, and so adjust its educational procedure as to give the younger members of the profession the requisite knowledge of the more fundamental sciences. Or it is possible that students of physics and chemistry may proceed, as they are doing already, to investigate the remaining terrestrial phenomena that have never satisfactorily yielded to the conventional geological approach.

While it may not be of great ultimate concern in the evolution of science which of these two possible procedures is to be the dominant one, it is of vital immediate concern to the geological profession. If the geological profession is to regain the position of dominance and vitality that it held during the eighteenth and nineteenth centuries with respect to the study of terrestrial phenomena, it is imperative that the members of that profession become conversant, to a far greater extent than is at present customary, with the application of the methods of physics, of chemistry, and of astronomy to the solution of terrestrial problems. This, however, is possible only if the academic instruction of students of

geology be drastically overhauled so that they may become conversant with the sciences of physics, chemistry, and astronomy.

If, on the other hand, the members of the geological profession do not see fit to make these necessary adjustments, there can be no doubt that the invasions into the territories that have been traditionally regarded as belonging to the geologist will continue to be made by workers who have been trained in the fundamental sciences until little of what once was geology will remain.*

Bibliography

For the benefit of those who may not be familiar with the immensity of the present technical literature and knowledge in the field of geophysics a brief and somewhat random list of representative journals and technical treatises on the subject is herewith appended.

Journals

Gerland's Beiträge zur Geophysik. Founded in 1887 and now in its 51st volume. Deals with all phases of geophysics.

Zeitschrift für Geophysik. Founded in 1924 and now in its 13th volume.

Bulletin of the Seismological Society of America. Now in its 27th annual volume.

Transactions of the American Geophysical Union. Issued annually since 1919.

Geophysics, a Journal of General and Applied Geophysics. Issued by the Society of Exploration Geophysicists. Now in its second year.

Treatises

Geodesy, by A. R. Clarke. Clarendon Press, 1880.

Lehrbuch der Landkarten Projecktionen, by Norbert Herz. 312 pages. Leipzig, 1885. Teubner.

Die Mathematischen und Physikalischen Theorieen der Höheren Geodäsie, by F. R. Helmert. 2 vols. Leipzig, 1884.

Handbuch der Messungskunde, by Wilhelm Jordan. 3 vols. Stuttgart, 1888–1890. Metzler.

Handbuch der Geophysik, by Siegmund Günther, Ed. 2. 2 vols. Stuttgart, 1899.

Physik der Erde, by M. P. Rudzki. 584 pages. Leipzig, 1911. Tauchnitz.

Handbuch der Geophysik, edited by Beno Gutenberg. This work is now about half completed and off the press. It is planned to comprise 10 large volumes of about 1000 pages each, covering approximately the following subjects:

Volume I. The earth as a planet. Origin of the solar system. Shape and density of the earth. Pressure inside the earth. Earth tides. Precession of the earth. Theory of gravity and isostasy.

Volume II. Structure of the earth. Deals with the structure, history, and physical composition of the earth. Also earth temperature.

Volume III. Alteration of the earth crust. Diastrophism, vulcanism, and erosion.

Volume IV. Earthquakes and seismology.

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* Recently there appeared a short article by Beno Gutenberg [Geophysics as a Science. *Geophysics* (1937) **2**, 185–188], in which the definition of the scope of geophysics is practically the same as that given here.

Volume V. Magnetic and electrical phenomena.
Volume VI. Geophysical prospecting methods.
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[For discussion, see page 74.]

Organization of a Department of Geophysics

By C. A. Heiland,* Member A.I.M.E., and Dart Wantland†

(New York Meeting, February, 1938)

There once was a little kid, whose lot was a very tough one until he grew up. His parents did not have much in common; from all indications, it is probable that the child was not wanted. His father Geology and his mother Physics separated shortly after his birth, which occurred during the World War amid a smoke screen of mathematics. He was kicked around a great deal. His parents could not agree on a name, and they finally christened him "Geophysics," for short. His relatives insisted that the name should be "Geophysical Prospecting," "Geophysical Exploration" or "Exploratory Geophysics"; though the youngster has grown up by now, this battle is still going on. His father did not think the child would amount to much until he began to show some unexpected talents, which were acquired, no doubt, from his mother Physics. His progress caused his father no little worry, and he began to wonder whether his offspring would help him in his business when he grew up or would set up shop in competition with the old man. His aunt Petroleum watched his growth with benevolent interest; his uncle Mining remained rather skeptical and cousin Civil Engineering not only assumed the attitude of the man from Missouri but was not much convinced even after he had been shown.

Making a long story short, the kid decided that he had to stand on his own feet and get along as best he could. Having come from a good family he possessed some outstanding characteristics such as resourcefulness and a reasonable amount of business ability. He therefore decided that it would be to his advantage to follow in his father's and mother's footsteps as much as possible, and develop the good qualities of both. As a result, the youngster did fairly well for himself, and his relatives decided that it might be to their advantage to be on better terms with him. Father Geology had come to the conclusion that the boy could be very helpful to him and now wanted him to live in his house. His mother, who had brought him through the teething and measles stages, felt quite capable of bringing up the boy without the help of his father; and, as the fairer sex will do, would confide to a neighbor every now and then that his

Manuscript received at the office of the Institute Feb. 11, 1938. Issued as part of T.P. 950, August, 1938.

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father was not of much account. In his dilemma, and having acquired some experience in life, the boy decided it would be best for him not to take sides, and to retain his respectful attitude toward both parents. He had learned that both of them had their good points, although they could not get along together.

Differences in Interpretation

It is highly probable that a study of the recent geophysical literature would have prompted our old friend Aesop to tell us this fable. It seems that the geologists and physicists have decided to move into opposite camps and that neither is willing to give much ground. There have been strong expressions by the physicist that he is quite capable of interpreting his findings without the help of the geologist. The geologist has countered with the statement that Geophysics is nothing but a "tool" of Geology.

No doubt, the spectacular development of the reflection method has been largely responsible for the expression of such extreme thoughts. It is true that the physicist can sit in relative comfort in a seismic recording truck, push a few buttons and turn a few switches, yell "Fire!" at the fellow on the other end and pull out a record, count a few time lines and calculate the depth to a reflecting bed with little more than a slide rule. Thus the temptation is great to say that the geologist is a thing of the past, and that physics, and nothing but physics, has done it again. Yet, it was not so long ago that the confidence of our reflection seismologists was rudely shaken when they found out that depth-mapping of inconsistent or lensing beds could be wholly in error, and that it was necessary to adapt their procedure to the requirements of geologic conditions in the area.

An equally extreme view is held in the geological camp, that geophysics should be and is nothing more than another "tool" for the geologist, like a plane table. It is quite true that the interpretation of geophysical data is largely a matter for the geologist, but that does not mean that a geologist without any training in physics and mathematics would be in a position to perform or direct geophysical operations to the best advantage.

Geophysics has often been likened to the X-ray, with the geologist as the physician and diagnostician and the physicist the X-ray operator. Some doctors operate their own X-ray machines, but the majority prefer to send their patients to an X-ray specialist. It would appear, therefore, that the X-ray technique is not of such nature as to enable doctors to take routine pictures and to obtain what they want. It is well known that the X-ray technician has to modify his technique to suit conditions. He has to place his apparatus in position to obtain the clearest pictures

of the organs desired, and has to feed the patient certain special chemicals to bring out certain organs at all.

However, we question whether the comparison of Geophysics with X-rays is at all a good one. Geophysical results do not appear in such simple manner as the picture on a fluorescent screen, on which a layman can recognize the position of his organs (when they are pointed out to him). The technique of adapting geophysical procedure to varying geologic situations is vastly more complicated, and the interpretation of our findings requires a much more profound knowledge of physics than the mere inspection of an X-ray picture.

Cooperation Required

The intricate nature of geophysics requires the closest cooperation between physicist and geophysicist. The field is large enough to give both of them ample diversion (and occasional headaches), and there is every opportunity for both geologist and physicist to specialize in the phases of the work for which he has been trained. The design and operation of instruments can be handled largely by physicists; the adjustment of technique to meet geologic conditions is common working ground for both physicist and geologist, and in the interpretation of the results they should again get their heads together and the physicist should pay careful attention to what the geologist has to say, and vice versa.

With so much common ground for cooperation there is no need for either specialist to tread on the other's toes. As a matter of fact, there is definite need for men who combine the trainings of the physicist and the geologist. We are dealing here with a highly specialized technique, applied for a very definite purpose and one purpose only, that of locating oil and ore. This may well be called "Engineering" and there is no reason why the application of physical methods to the location of oil or ore should not be called "Geophysical Engineering." The term "Geophysicist" or "Geophysical Engineer" would apply then to men with fundamental training in both physics and geology. A geologist or a physicist working in special phases of geophysics would thus not necessarily be a geophysicist. It is really regrettable that the word is not long enough to get a little more geology into it.

Department at Colorado School of Mines

It was with this idea of a substantial engineering background and a proper balance of the prerequisite subjects of mathematics, physics and geology that the Department of Geophysics at the Colorado School of Mines and its courses were organized. To be frank, we have felt at times that we might have bitten off a bit more than we could chew. However, we have had every encouragement and cooperation from the School administration and the other departments concerned, so that after eleven

years of work in this direction it is believed that a well balanced curriculum for geophysical engineers is possible of attainment.

It is probably true that an engineering institution is somewhat better suited for an undertaking of this sort than a university, because of the possibility of more rigid requirements and correlation of courses from the point of view of fundamental engineering education. One question that has confronted us from the beginning has been whether advanced geophysics courses should be offered to graduate students only. Without going into all aspects of this question, it may be stated that the best solution has been found in a compromise; that is, fourth-year students are permitted to take, in one year, but *two* of the four major courses offered. They have to return for another year if they wish to take all of the graduate courses. In addition, two courses of a general nature are open to third-year students and are required of all students of geophysics, which prevents overspecialization in only one branch.

During the first years, we were also faced with the following reaction from the outside: Why offer any special or advanced geophysics courses at all, in view of the greatly changing character of methods, instruments and technique? Why not leave the training of geophysicists to operating companies and confine instruction to the basic subjects of geology, physics and mathematics? The answer to this question is found when considered in a perspective of the geophysical picture 10 or 15 years from now. Despite earlier rapid changes, certain methods and their technique have become more and more crystallized. Their fundamentals will not change and are involved regardless of what changes in details of technique may be made. "Geophysical Engineering," to use a term we prefer, shows a definite upward trend and is developing into a definite field of its own. The industry employing geophysical engineers has a right to expect them to be trained in *geophysics*, not merely in geology, physics and mathematics. An oil company hiring a petroleum engineer today expects him to be familiar with methods of petroleum production and refining and does not hire a man for such a job if his education has been confined to chemistry or basic engineering only. Training in details of methods applied by the company will always be necessary, especially for men just out of school, and is common practice in other fields of engineering.

In planning the courses of instruction in Geophysics, two pitfalls had to be avoided by preserving a reasonable balance between basic theory on the one hand and the details of instrument operation and field technique on the other. Overemphasis on the first would tend to turn out theorists with their feet off the ground as to the objectives of a particular method; stress on instrument operation would lead to a form of tradesmanship instruction not requiring the facilities of an institution of higher learning.

The training in geophysical engineering offered at the Colorado School of Mines is built around the fundamental engineering training of the

TABLE 1.—*Important Courses Required in Study of Geophysics at Colorado School of Mines*[a]

Year	Civil Engineering	Mechanical Engineering	Mathematics	Physics	Geophysics	Geology	Chemistry	Languages	Economics
Senior 2nd Semester	Contracts Specifications 4	Design 8	Potential Functions 9	Radio 9	Torsion Balance or Seismic Prospecting 9 Seminar 3	Maps 3 Stratigraphy 4		German 6	Principles 4
Senior 1st Semester			Vector Analysis 9	Thermo-dynamics 6	Magnetic or Electrical Prospecting 9	Ore Dep. 7 Opt. Min. 7 Pet. Geol. 5		English 3 German 6	Principles 6
Junior			Partial Differential Equations 6	Electrical Measurements 10	Applications 4	Structural 6 Historical 4 Paleontology 3 Field 20	Physical Chemistry 9		Elements 6
Junior	Strength of Materials 9	Machinery 7	Differential Equations 8	General 7	General 4	Petrology	Physical Chemistry 9		
Sophomore	9		24	General 23 Mechanical 10		12	12	8	
Freshman	Surveying 20		30			11	26	15	

[a] Numbers indicate credit hours inclusive of preparation

geological engineer, with supplementary courses in mathematics and physics. Table 1 gives a summary of the more important courses required of geophysical engineers, together with the number of credit hours. Details on courses are omitted for the freshman and sophomore years. The tabulation shows courses in Geophysics, Geology, Physics, Mathematics, Civil and Mechanical Engineering, Chemistry, Languages and Economics. In addition to the regular courses in geology as given in the tabulation, geophysicists take Ordinary and Partial Differential Equations in their junior year (as required subjects), and Potential Functions and Vector Analysis in their senior year (as electives). In Physics, they take General Physics and Electrical Measurements in their junior year, and Thermodynamics and Radio in their senior year. Additional courses (Electronics and Electrical Engineering for Geophysicists) are contemplated. In languages, German has been required of geophysicists during the past years.

Two general courses in Geophysics are given in the junior year, one dealing with principles and methods and the second with applications to type geologic problems.

As previously noted, two of the four graduate geophysics courses are open to seniors. In the first semester, Magnetic and Electrical Prospecting are offered; in the second, Gravity Methods and Seismic Prospecting. Each course consists of three weekly lecture periods, three laboratory periods and field work either at week ends or at the end of the semester. The lectures are generally arranged as follows: (1) fundamental principles of method, (2) physical properties of rocks and formations and methods for their determination, (3) instruments, theory and operation, (4) theory of subsurface effects, interpretation, (5) discussion of results obtained with the method under discussion in the field of oil exploration, mining and civil engineering. Laboratory work consists of measurements on rocks, instrument calibrations and small-scale model experiments. Field work is done on oil structures in eastern Colorado (two weeks reflection and torsion balance work), on placer and mine prospects, on ground-water location in the vicinity of Golden.

For each major course, mimeographed notes are available to students. A set of such notes generally comprises about 400 pages.

In addition to the four major courses, two courses in Geothermal and Radioactive Investigations are offered when there is sufficient demand. One seminar and two research courses, and one general course for mining engineers complete the scope of the work.

The department of Geophysics at the Colorado School of Mines was established in 1926 and the first course was given in January 1927. Looking back, it was nothing to be particularly proud of, but somehow we managed to keep always one lecture ahead of the class. In the period from 1927 to the end of 1937, altogether 339 students have taken the general geophysics courses. Of these, 142 have taken, in addition to the

general courses, at least one of the graduate courses offered by the department. Of these 142 men, 47 (33 per cent) had degrees from other institutions before they entered the School of Mines and 15 of these came from foreign countries: four from Canada, two from Australia, one each from Czechoslovakia, Belgium, Poland, Norway, Colombia, Venezuela and China and two from Japan. Some of them were sent by their respective governments or foreign operating companies; two were Commonwealth Fellowship men.

The distribution of the men taking graduate courses may be of interest (Fig. 1). In considering this diagram it should be recalled that one man may have taken, and usually did take, more than one graduate course. The magnetic course seems to have been in greatest favor; then follow, with about equal shares, gravitational, electric and seismic methods; radioactive and geothermal courses attracted only a small number of students.

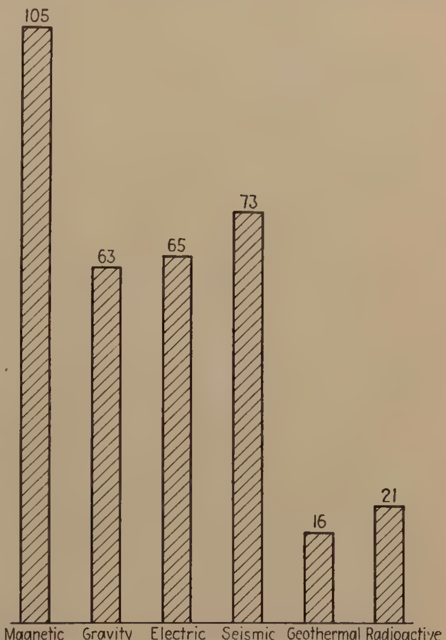

FIG. 1.—DISTRIBUTION OF STUDENTS AMONG GRADUATE GEOPHYSICS COURSES AT COLORADO SCHOOL OF MINES IN ELEVEN-YEAR PERIOD, 1927–1937.

Fig. 2 shows the trend in student attendance through the 11-year period from 1927 to 1937, as well as the number of advanced degrees (M. Sc. and D. Sc.) granted on the basis of research work to advanced students during the same period.

Because or perhaps in spite of our efforts, these men, who have been out of school on an average of 5½ years, now occupy or have occupied the positions of which a percentage distribution is shown in Fig. 3, according to information available. The approximately equal percentages of party chiefs and supervisors, which together equal 21.8 per cent, and of observers and computers at 23.9 per cent, would tend to show that the training received fits men for supervisory technical positions as well as for the somewhat less highly paid posts. While there are always economic factors involved when a man becomes a consulting geophysicist, it is notable that the training received is apparently sufficient to permit men to occupy such positions.

A final question pertaining to the problem of suitable training for geophysicists is that of the practical experience of instructors in geophysics. While the field of geophysics as such is not new, and while

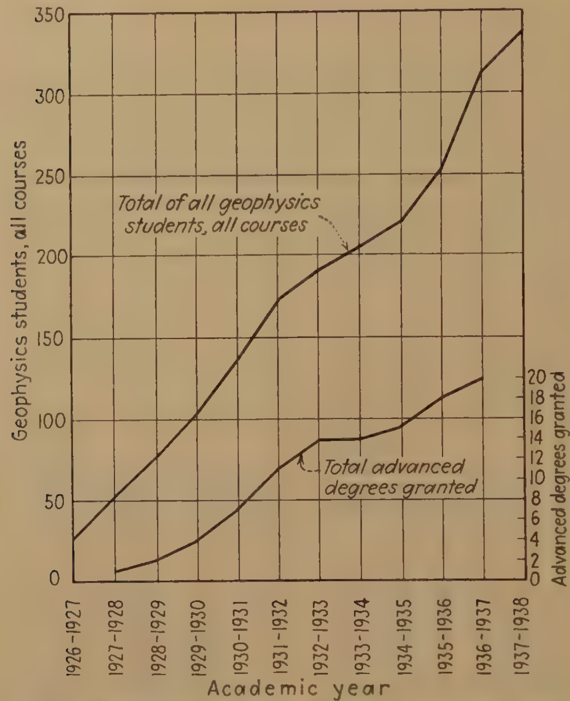

FIG. 2.—TREND IN STUDENT ATTENDANCE AND ADVANCED DEGREES GRANTED, COLORADO SCHOOL OF MINES.

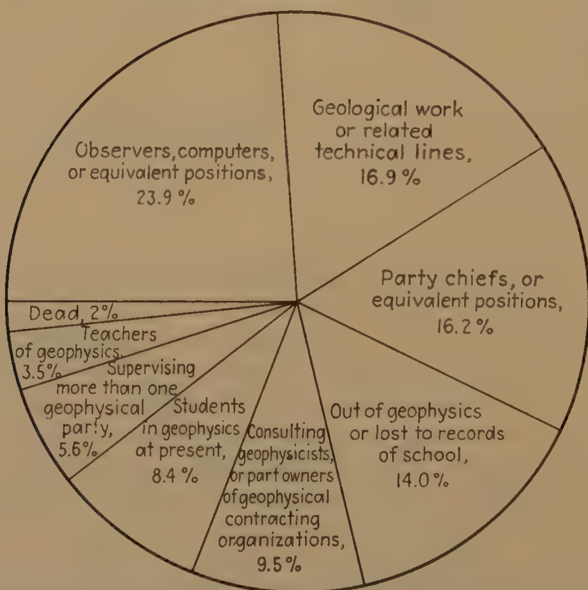

FIG. 3.—PERCENTAGE DISTRIBUTION OF 142 STUDENTS THAT TOOK AT LEAST ONE GRADUATE COURSE IN GEOPHYSICS AT COLORADO SCHOOL OF MINES.

technical information on various methods is being published at an increasing rate (more so than many geophysicists realize) in this country and abroad, the fact remains that a great deal of information on instrument construction, interpretation methods and geologic applications can be acquired only by practical experience. The authors of this paper attained commercial experience before going into teaching; most of the teaching assistants employed in the past 11 years came from the commercial field. Furthermore, we have made it a definite point to carry on, with the consent of the administration, a definite amount of consulting work. Although conflicts naturally will come up on occasions, they have not been allowed to assume serious proportions; we believe that the effect of commercial activity on our teaching has, on the whole, been beneficial.

In presenting this material on Geophysics Education we have perhaps, though not intentionally, laid ourselves open to the charge of being didactic, not an uncommon failing of professors. We do not consider that ours is *the* best way or the *only* way. We welcome suggestions for improvement and change. In fact, we have in the past quite materially changed the order and nature of our courses to better fit the needs of our students to enter industry. At the same time we have not departed from the fundamentals of a broad engineering training with a groundwork in geology, supplemented by instruction in mathematics and physics as correlary to courses in the various geophysical methods.

[For discussion, see page 74.]

Teaching Geophysics in a Department of Physics

By David A. Keys*

(New York Meeting, February, 1938)

Applied geophysics is the youngest child of that old branch of learning that has been known from Aristotle's time as physics—the constitution and laws of nature. The mother science, with the help of mathematics, has given birth to many children, who, on becoming of age in the scientific sense, wedded more practical applications which later became known as electrical engineering, physical chemistry and acoustical engineering. Some vigorous sons still retain the family name, among which the most recent to take its place as a division of science is geophysics, which deals with the physics of the earth. Geophysics in its broad sense has many departments—meteorology, oceanography and seismology, to mention only three. Within the past 20 years, however, a special interest has arisen in the application of physical principles to problems in geology, mining and civil engineering, with the consequent rapid growth of what has now been called "applied geophysics."

Applied geophysics in this narrower sense is the application of experimental and theoretical physics to determine variations and discontinuities in underground structures that cannot be ascertained from surface inspection. It seems evident, then, that the physics department is the natural place in a university where the principles and methods involved should be taught and developed, for the physicist will also have had the requisite training in mathematics that the proper understanding of many methods now used will require. A knowledge of the procedure and principles, however, is not sufficient for the successful application of geophysical methods. One may know the dynamics of flight of golf balls and of impact and yet not play a par game of golf! The proper selection and use of the right club to suit the circumstances of the lay of the ball and an acquaintance with the possibilities and limitations of the strokes as well as the practical execution of the play is essential to a successful game. So, also, in the application of geophysical methods, it is important for the geophysicist to use the best device under the conditions presented, to know the limitations of the conclusions deduced from his physical measurements and to make the proper interpretation of the results. A knowledge of geology will be of great aid in coming to the correct con-

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Manuscript received at the office of the Institute Feb. 15, 1938. Issued as part of T.P. 950, August, 1938.

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clusions but ignorance of the physical limitations has in the past led even a good geologist to wrong conclusions from a geophysical survey.

Even under the best cooperation between the physicist and geologist there will often be considerable uncertainty in interpretation, whether the conclusion be made by one well acquainted with physics and geology or by the consultative accord of the geologist and physicist. Usually the geologist has had too meager a training in physics and mathematics to appreciate properly the fundamentals and limitations of the physical methods employed and the physicist is deficient in his knowledge and field experience of geology. In order to rectify this situation it is important that every mining engineer and geologist should be given instruction in the theory, interpretation and limitations of various geophysical methods, in order that full advantage may be taken of the service offered by this new division of physics. The experienced geologist today understands very well the use of the diamond drill, the interpretation to be placed on the results of the core analysis and the limitations of their tool in developing a prospect. The same cannot be said of the geologist in regard to a similar knowledge of the methods of applied geophysics.

It has already been stated that a physics department is a proper place in which to give instruction in applied geophysics. The physicist should have field experience, however, as the more practical work he does the better will he be able to present the limitations of the methods. It is not a science that can be properly learned in an armchair; practical work is essential. The students should also have some practical experience, how much will depend upon the time allotted to the subject and the location of the university in which the subject is taught. Apart from the few institutions in North America that have separate departments of geophysics, the instruction in this subject is limited to a single course, and students in such a course should have had adequate instruction in the preliminary fields of physics—that is, besides a general course in college physics, a more advanced course in electricity and magnetism; electrical measurements, particularly a knowledge of modern alternating-current bridge methods, some advanced hydrodynamics, sound, theory of potential, and an acquaintance with electromagnetic theory. This necessarily implies a knowledge of the elements of differential and integral calculus, differential equations and solid analytical geometry. One could easily add to this other topics such as the theory of functions and the higher branches of applied mathematics and physics, which have already been used by many investigators[1] to interpret geophysical measurements, but such fields of learning are not usually studied by geologists and mining engineers. This only confirms the opinion already expressed by others[2]

[1] See the investigations by King, Hummel, Königsberger, and others.

[2] See, for example: Physics in Industry, published by the American Institute of Physics, 1937.

that our engineering students require more physics and mathematics than they did formerly in their undergraduate training.

Instruction at McGill

At McGill University, applied geophysics is given to all mining engineers and geologists in the fourth (senior) year. The preliminary training of these students in physics has been one college course in general physics in their pre-engineering year, followed by two full courses in their first and second years. In their third year they have a course in electrical engineering. These courses are all accompanied by laboratory work and the students will have acquired some familiarity with magnetic measurements, bridge methods using direct and alternating current and the effects of inductance and capacity in circuits. They are familiar with elementary calculus, plane analytical geometry and linear differential equations. They know little of potential theory, advanced hydrodynamics and electromagnetic theory. With this preliminary training in physics they come to their course in applied geophysics.

The course in applied geophysics covers the usual fields of magnetic, electrical, seismic and gravitational methods.[3] By means of lecture-room demonstrations and by the use of models and laboratory experiments, many of the principles may be adequately illustrated. Since the electrical and magnetic methods have been employed more frequently in Canada than either seismic or gravitational, more time is devoted to these devices. Some field apparatus is also demonstrated on the campus, so that the student is given opportunity to see the actual equipment under working conditions. The theory can be fairly well covered in this way but the interpretation of the results of readings taken under different conditions is still desirable. To meet this the students are given as problems sets of readings actually made in the field by various methods[4] and are required to make the calculations and to draw their conclusions. The limitations can then be pointed out, as the actual conditions either were known at the time of the survey or were determined by subsequent mining or drilling. One practical example of each method is not sufficient; several obtained from different localities bring out the fundamentals and also exemplify the possibilities. Even sets of reading taken without due care are useful, especially when a repetition is made with greater refinement, which indicates a difference in result. Such problem work is not equivalent to field experience but is a fair substitute when time and expense do not permit the latter.

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[3] Subject matter as covered in Applied Geophysics, by Eve and Keys.

[4] The author is indebted to several geophysicists (some connected with operating companies), for examples of their field results, which help very much to broaden the extent of the student's knowledge.

Widening Understanding and Application

Since geophysical methods have proved of value as an aid to geologists and mining engineers, it is essential that all such students should be acquainted with at least the elements of the theory of methods available and the practical results that may be obtained. Such knowledge will have a twofold influence. First, it will eradicate the ignorance still exhibited by some of our geologists as to the purpose and value of geophysical investigations. Not long ago, an outstanding geologist questioned entirely the value of any geophysical surveys and added that in his opinion they were carried out mainly to entice an ignorant public to invest its money in worthless prospects. Such a statement showed a complete lack of knowledge of geophysical principles. Overenthusiastic reports have been made, but when those to whom the report is submitted know more about the subject they may demand more details and make a more accurate appraisal of the results, and thus prevent any such accusation. Secondly, as the geologist or mining man becomes more familiar with the subject, either he will apply the methods more intelligently himself or he will call in the professional geophysicist for consultation.

Special emphasis has been given to the teaching of geophysics to geological and mining students, but it must not be overlooked that this new branch of physics has its important applications in civil engineering also. Much progress has been made in the use of geophysical methods in foundation work for road building, dam-site surveying and railway construction. There is also wide scope for research by physicists and mathematicians in developing new methods, improving existing ones and applying other physical principles to new problems, both in the applied and in the cognate fields. Such investigations as that of the propagation of radio waves, terrestrial magnetism, isostasy and oceanography have long been the subject of research by physicists.

The instruction in geology is usually better given by those more competent than the physicist to do so. The more geology a geophysicist knows, the better, but for developing, applying and properly interpreting results of geophysical surveys, a knowledge of physics and mathematics is paramount. Most students who are taught geophysics have already had some instruction in geology and many are geologists by profession. Often it is the inadequate preliminary training in the advanced branches of mathematics and physics that causes a misconception of the possibilities and limitations of geophysical methods, and not the lack of a knowledge of geology. Many of the successful methods of geophysical surveying in use today have been devised and operated by physicists who have had no geological training and whose knowledge of geology is very limited indeed.

It is obvious that a geophysicist may be trained in different ways. He may be a geologist who learns something of physics; he may be a

physicist (or engineer who has had a thorough training in physics) who acquires some knowledge of geology, or he may be that rare person who combines in one head expert knowledge of geology, physics and mathematics. Many of the latter can hardly be hoped for and so it remains for geophysicists to be trained from one of the remaining groups. Opinions will differ as to how this should be accomplished. The physicist can certainly help in this training; indeed, his part is perhaps the most important one.

[For discussion, see page 74.]

Geophysics Education and Exploratory Geophysics as a Career

By Donald C. Barton,* Member A.I.M.E.

(New York Meeting, February, 1938)

Exploratory Geophysics as a Career

Geophysical methods of prospecting taken as a whole do not seem to offer much promise to a young man planning to enter them in the future. They have come to stay, to be sure, and they will continue in at least moderate use for a long time. Domestic use of those methods presumably is at a maximum at the present time. Geophysical prospecting is expensive; the effects of the law of diminishing returns in efficiency in finding prospects can already be recognized. Small oil companies, independent oil operators, and royalty buyers have been doing much geophysical work. But in face of the increasing costs of discovery of prospects and in face of the probably decreasing favorability of the prospects discovered, I expect to see a sharp drop in their use of geophysics within the next 10 years. The major oil companies and the larger small oil companies will continue to use geophysics indefinitely, but it seems doubtful whether the very extensive domestic campaigns of a few companies will be maintained for many years, certainly not for 10 years. The use of the geophysical methods in foreign countries has not yet reached its peak, and probaby will continue to increase for several years. That peak level of activity will probably be held for 10 years, although the site of maximum activity will shift from country to country; by the end of 25 years from now, I should expect the foreign use of the methods to have leveled off to one-fourth to one-half the peak level.

Foreign service in geophysics does not seem to offer an attractive career, except for men of exceptional disposition. Young men just out of college, who are willing and who can afford to spend three or four years having a fling at adventure before settling down, will find it attractive, but when their tour of foreign service is over, they probably will have harder work finding a job than when they were seniors, and their foreign experience will count for little, except for further foreign service. A few men, mainly the higher men, will have headquarters at civilized or semi-civilized cities and towns or at permanent oil-company towns, and can

Manuscript received at the office of the Institute Feb. 15, 1938. Issued as part of T.P. 950, August, 1938.

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have wives and families with them. Most of the men in foreign service in geophysics will be out of civilization for 11 months out of the year, and if married, even though they may be permitted by the company rules to have their wives "with them," they may not be able actually to be with their wives more than twice a year for a total of perhaps five weeks. Most of the personnel in foreign service is hired on contracts running for a specific number of years and is not regarded as being on the (so-called) permanent payroll. There is a tendency on the part of the companies to regard themselves as not having any responsibility for the men after the termination of their contracts. Competent party chiefs—men who are technically competent and who also are good linguists, who can handle men under psychologically trying conditions, and who can organize and handle a party working away from civilization—will probably have jobs as long as they wish them. A few interesting careers are offered in the higher positions, but most of the men in them will have to stay permanently in foreign service; and only a few of them can expect promotion or transfer to positions in the United States.

Advancement for a man entering the profession in the future will tend to be extremely slow because of the youth of the personnel and the inclusion in it of many men of exceptionally good training and ability who already will have had years of experience. The present personnel is predominantly young; even the men in the higher positions are almost all in their thirties or early forties. To obtain advancement, a young man of training and ability above the average entering the profession now or in the future will have to compete with many men who have equally as good or better ability and training, and who have the advantage of many years experience and of established position in the science; and furthermore, as deflation of the domestic use of geophysics takes place, able and experienced men will have to accept demotion to lower ranking positions, if they wish to stay in domestic work. The newcomer will have to compete with them. Of the men entering geophysical prospecting in the future, only a very few will be able to win advancement to the higher positions, although a man having a good training and exceptional ability, combined with pleasing personality, common sense and a disposition to work hard, will always have a reasonable chance of advancement.

Effect of Training

The future for a career in geophysical prospecting differs according to the type of training of the geophysicist. The types of exploratory geophysicists may be loosely classified according to training as follows: (1) men having a degree of Ph.D. in physical geophysics, (2) those having Ph.D. in geological geophysics, (3) men holding only a bachelor's degree in physics, engineering, geology or geophysics.

The future for a man having a Ph.D. in geological geophysics will be as good as for a Ph.D. in geology. Geophysical prospecting is weak in men who have combined good training in geology, in physics and in mathematics. As such a man is both geologist and geophysicist, he is eligible for promotion to geological as well as geophysical positions. As the reduction in domestic exploratory geophysics takes place, he will be able to transfer to geology without loss of rank. He will have a reasonably good chance for promotion to the highest executive positions. Men whose duties bring them frequently into conferences in regard to important company problems and policies have the best chance of promotion to those positions. The geologist-geophysicist's duties bring him into frequent conferences in planning geophysical programs and in regard to activities resulting from the discovery of geophysical prospects. His chance of promotion, therefore, is much better than that of the physicist-geophysicist who rarely gets into a conference except with his immediate chief or the chief geophysicist in regard to his own narrow problem.

For the Ph.D. in physics-geophysics, the future is fairly good, but is more restricted than for the Ph.D. in geological geophysics. Petroleum engineering and probably refinery engineering also offer considerable field of activity for him.

The reduction in research geophysical activity probably will be very much slower than the reduction in domestic exploratory geophysical activity. And if and when the physicist geophysicist becomes involved in the reduction of exploratory geophysics, he stands a chance of being able to transfer over into research for the production engineering department or for the refinery department. Unless he becomes a petroleum engineer, he stands little chance of promotion except within the geophysical service.

The future for men holding only a bachelor's degree seems distinctly poor, except for a few men who take exceptionally well to foreign work. They may be able to find lower ranking positions, but their chance of promotion to higher positions will be extremely small; and the chance is good that in 10 years they will not be able to get work in geophysics and will not be able to transfer elsewhere within the oil-company organization without complete loss of rank, even if they succeed in transferring. Geophysical prospecting, in my opinion, is a dead-end line of activity for these men, with the end not too far in the future.

Geophysical Education

Sharp distinction should be made between geophysical education for men who intend to become professional geophysicists and for those who have only a collateral interest in geophysics.

Geophysical Education for the Nongeophysicist

All geologists and mining engineers should have an elementary knowledge of the geophysical methods of mapping geologic structure.

Mining engineers, petroleum geologists, and geologists specializing in geology in connection with civil engineering should have a sufficient training in geophysical prospecting to understand the professional geophysicists with whom they may have to consult, and to make intelligent use of geophysical work. Those men do not need to know how to handle geophysical instruments, or how actually to take or interpret geophysical observations; rather, they have greater need to know other things. Geophysical methods are only one set out of the many sets of tools that those men must learn to use; in general they will not be quite as important to them as the standard methods of field and subsurface geology and will be far inferior in importance to a sound knowledge of other phases of geology, such as structural geology. The time given to instruction in these methods should be apportioned as part of the total time given to field and office methods in geology. Each one of the four methods of exploratory geophysics is essentially as complicated a subject as petrography; and they all appeal to me as considerably more complicated subjects to teach than the standard field methods. Slightly more time, therefore, may have to be allocated to them than their relative importance in reference to the other geologic methods would justify. For programs leading to the bachelor's degree, a two months course of two hours lecture and three hours laboratory work per week would seem to be sufficient. For five-year engineering programs, and for geological programs leading to the master's degree, the course should be longer but should not exceed four months. The allotment of time between the several methods should vary between the courses that are not specialized, those designed primarily for mining engineers, and those designed primarily for petroleum engineers, approximately according to the following scheme:

Course	Percentage of Time for Each Method			
	Electric	Gravitational	Magnetic	Seismic
Course not specialized	33	22	10	33
For mining engineers	55	23	15	7
For petroleum geologists	20*	30	10	40

* Two-thirds of that time should be devoted to electrical logging of wells.

Calculus should not be required as a prerequisite for these courses; and, except for mining engineers, the course should be given in terms of elementary mathematics.

Men planning to become professors of general geology or of dynamical and structural geology or to take graduate degrees in field geology or in dynamic and structural geology should have a fair elementary theoretical knowledge of the geophysical methods of mapping geologic structure. The professors of general geology will need an elementary knowledge in their teaching and in their understanding of general geology. The geophysical methods of mapping geologic structure are an important type of method of mapping structure, therefore should be known to any one specializing in field mapping or in structural geology. An understanding of geologic structure and of dynamical geology involves the mechanics of the dynamical processes producing geologic structure; mechanics is a phase of physics and mechanics as applied to geology comes under the head of geophysics (although not under the head of geophysical prospecting). Most of the subject matter of dynamical geology should be overhauled by geologists well trained in geophysics, for much of our geologic theory in regard to dynamical and structural geology has been formulated by men whose knowledge of physics has not been adequate. The course should be either one-half year or a full year. Ideally, this course should be divided into two half courses. The time of the first half course should be devoted approximately two-thirds to geophysical methods of mapping geologic structure and one-third to pure geophysics. The second course should be devoted wholly to advanced pure geophysics, which, in this case, would be essentially advanced dynamical geology. (This however, is almost a virgin field.) A good course in physics and at least an elementary course in differential and integral calculus should be prerequisites for this course.

All students majoring in geology should have geophysics and the geophysical methods of mapping geologic structure brought to their attention. A series of two to four lectures in a half-year course and three to six lectures in a full-year course covering general geology would seem to be sufficient. Students interested in a more detailed introduction to geophysics can take one of the preceding types of geophysical courses.

Education for Professional Exploratory Geophysicists
More Highly Trained Men

For *physicist-geophysicists*—the men who should do the research on the improvement of instruments and instrumental technique and on the invention and perfection of new methods and instruments—any well rounded, sound, thorough training in physics and physical mathematics will be sufficient. Men concentrating in various of the special fields of physics can find a place in geophysical research and it is not important that they should have heard of geophysical prospecting before entering it. But the following direction of his course will be advantageous to a man if he wishes especially to train for geophysics:

1. His physics should be strong in electricity and electromagnetism, light, and wave motion.

2. His mathematics should include good familiarity with the potential function and the theory of statistical probability.

3. He should have the beginning whole-year course in geology and should follow it with a summer field course in geology, preferably in the Rocky Mountains.

4. He should have two summers as a helper on geophysical field crews, preferably one summer with a seismic crew, and one summer with a gravity or electric crew.

Basic courses in physics, mathematics and field geology (in areas of good, deep, continuous exposures of sedimentary rocks) will be of far more importance to him than courses on geophysics. He will have plenty of time to study up on geophysics after he gets into it. But if he has the opportunity, he might attend as a listener the course on geophysical prospecting recommended for mining engineers, petroleum geologists and geologists.

The field courses in geology will be most important to the physicist-geophysicist in moderate degree if he deals with the experimentation with the development of methods in the field; in high degree if he wishes to work into interpretation. The most important thing for the physicist-geophysicist to learn is how rapidly sedimentary beds vary, how complex most fault systems are, how complex most "structures" are, how complex most mineralized terranes are, and how complex most mineral deposits are. Two elementary field courses in successive years in different areas under different instructors will be as good as, or better than, an elementary course one year followed by an advanced course the second year. The position of companion assistant to a geologist or a graduate student in geology working out of Professor Thom's Red Lodge (Montana) Camp on a predominantly sedimentary terrane would be most excellent for one of the summers. Those are standard geological courses. A valuable laboratory course could be arranged for the physicist-geophysicist; it would consist of a series of studies of maps of all the "structures" that are well known geologically; and of maps of mineral deposits and terranes in which mineral deposits occur. All these courses point to the complexity of actual geology. A physicist can fairly easily acquire a reasonably good idea of idealized, diagrammatic structures by taking an elementary course on geology, or by reading elementary textbooks. But it is only by a field course or by a laboratory course such as I have suggested that a physicist can begin to realize the complexity of geological features and the crudeness of the assumed simplifications which the geophysicist must make to handle the mathematics.

For *geologist-geophysicists*—the men who should handle the geological interpretation of the routine field work of mapping structure—the need

is underestimated by the men who have come into geophysics from physics. The latter believe that the physicist-geophysicist should do all the interpretation and if admitting an excuse for existence to the geologist-geophysicist they limit his field of activity to the passive one of contact man who interprets the language of the geophysicist to the supervising geologist. This view is a mistaken one and seems to be based on: (1) a natural prejudice in favor of one's own science, (2) a dogmatic oversimplification in the concept of geology; and (3) a reaction from experience with geologists who had little sound insight into geophysics and who have tended to underrate geophysical indications not in accordance with the geologists' concept of what the geological situation ought to be. Until geophysics can directly and positively identify oil, gas and sought-for minerals, the geophysical results will have to be turned into geological concepts before the results can be used, because our knowledge of the laws of occurrence of oil and gas is in terms of geologic concepts. The physicist must become enough of a geologist, or the geologist must become enough of a geophysicist, to take the physical concepts resulting from a geophysical survey and turn them into geological concepts. But better yet, it seems to me, is to have a man who is both geophysicist and geologist.

They should major in general or structural geology and should be able to qualify for the U. S. Geological Survey. They should not be required to take the long, detailed basic courses in paleontology, mineralogy and petrography. They should have sound field training in areas having good, continuous, deep exposures of sedimentary beds.

They should have strong minors in physics and physical mathematics. Their mathematics should extend through differential equations and should include the Newtonian potential function, and the theory of probability as applied to statistics.

Their training in geophysics need not go beyond the type of course recommended a few paragraphs earlier for men planning to become professors of general geology or of dynamic and structural geology, etc. But if that course is not available, a man competent to take two years of graduate work should be able to read up on the subject under the direction of one of his professors. If he can get a summer's work with a field geophysical crew before his junior year, well and good; later summers would better be spent in field work in areas in which he can get a firm idea of the habit of sedimentary rocks and of the structures superimposed on them.

But within the triangle of physics, physical mathematics, and geology plus a minor in mathematics and physics, any able, soundly trained man is well trained to enter geophysical prospecting. For the men who are to do the actual interpretation of the geophysical results into geologic language, men who cannot qualify as geologists are severely handicapped

by their overdogmatic picture of geology and their resulting drastic oversimplification and idealization of geologic features.

I am in hearty disagreement with the recommendation that geophysics should be done by cooperative work between a geologist and a geophysicist. That arrangement would be comparable to doing petrography by having a physicist at the microscope and a geologist sitting by his side. The interpretation of the geological significance of geophysical data can be done efficiently only by a physicist who has become also a good geologist or by a geologist who is also a good geophysicist.

Less Well-trained Men

The rank and file, composing perhaps two-thirds of the personnel of exploratory geophysics, is not, and should not be, composed of highly trained men. The routine, stereotyped character of the work and the impossibility of promotion of more than a few from these lower ranks even under the best circumstances justify companies in filling the bulk of these rank and file positions with men having only a bachelor's degree at the most.

There can be a wide range of training for such positions, although certain lines of study are preferred for certain kinds of work, and different employers have different preferences. For work in gravitational and magnetic prospecting, first preference will be for men having geological training and second for men with civil engineering training. In seismic work, men with geological training are preferred by some employers for computers; physicists, electrical engineers and all whose technical education has covered radio circuits are preferred for the other technical positions.

Collegiate training in geophysical prospecting brings no preference and by many employers is given a slightly inferior rating. One large company seldom hires men who have just graduated with a major in geophysical prospecting. This prejudice is based on the reasoning that much of what is learned in college is obsolete or obsolescent, that most of the techniques learned in college will have to be unlearned, because different from those used by the employing company, and that in the long run additional courses in basic physics, mathematics and geology are of more value to a man than the courses on geophysical prospecting.

DISCUSSION

[This discussion refers to the five papers beginning on page 23 and ending on page 74.]

L. W. Blau,* Houston, Texas.—I have had some rather sad experiences with forecasting. For example, I could not foresee 10 years ago my present occupation, and I do not profess to be able to look ahead 10 years and tell anyone what he will be doing then. Referring to the point raised in the discussion, I do not know, for example, what the price of oil will be 10 years from now. If an appreciable number of

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* Geophysics and Production Research, Humble Oil and Refining Co.

prospects should go sour, resulting in a failure to maintain reserves, the price of oil might go up and consequently prospecting might increase and more work might be done 10 years hence than is being done now.

It has been said that if no more oil were found, there would be a shortage of oil within five years, because we have only about 13 years supply in reserve, and this reserve could not be produced in 13 years. I gather from conversation with others and from observation that at present prospects do not look very good on the Gulf Coast. It has been stated that the prospects now being found look less attractive on the map than those found four or five years ago. However, one major oil company made the following experiment: it took some 200 prospects and rated them *A*, *B*, *C* and *D; A* being the most attractive and *D* the least attractive on the map. The maps were made by geologists from geophysicists' and other information. The company then compared the production records of the fields, which had by that time been drilled in. It found that there were as many good oil fields on the *D* prospects as there were on the *A* prospects. So, again, we cannot say that these sour-looking things being discovered now will be sour when they are drilled in a few years from now. On the other hand, there are many areas where no prospecting has been done. But there are relatively few red spots (meaning outcrops of igneous rocks) on the map of the United States, and I suppose that if oil is not found fast enough to maintain reserves, if a shortage of oil should confront us, we shall probably look at all the places that are not marked in red now. An increase in prospecting activity would be the result.

Of course, this means that we have to look into places that do not look very attractive now and places where we now have an agreement with the geological authorities that there cannot be oil, because there are no source beds or some other things are wrong. But in the past, we have drilled and found oil and, when we found it, the geology was all right and the source beds were found. Maybe that will happen again.

I do not know how to look into the future because of the fact, as I said, that I do not know how much oil we shall find. If we do not find much, we will have to look for it more enthusiastically and we will have to hire more men. If we find more oil, it will not pay to hire more men and spend more money, but now we are finding barely as much as we are using.

I might say one more word about field work for students in colleges. I am not a great admirer of engineering schools of the old kind. It is more important, in my opinion, to learn what makes the motor go than it is to find out where the switch is. We can tell a man where the switch is if he has already learned what makes the thing turn around.

I do not care much for students that have been trained by somebody else in the use of a geophysical method as that person or that instructor thought the thing was used five years ago. He does not know how we are using it now, and we are not telling him just now, either. But I would like the instructor to give the student a good training in the fundamentals, and if the latter knows potential functions, if he knows something about gravitation, if he knows something about the earth's magnetic field, then, in one or two months he can be trained to use a torsion balance or a magnetometer. As a matter of fact, when we get into a pinch, we can train a man who has had only a high school education to use a magnetometer or a gravity meter. I think, therefore, that a student is wasting his time when learning to operate a magnetometer according to the formula of some person that cannot possibly know how we are using it or how he is going to have to use it after he gets out of college. In fact, the instructor does not even know how it is being used now.

I think there should be more work in the fundamental subjects and less work in the applications. We have electrical engineers that learn very carefully how light

plants were operated five years ago or ten years ago, at the time the book was written or when the instructor had a summer job with General Electric, but not how they are going to be operated three years from now when they get jobs—and they do not need to know either, provided they know their physics and chemistry, their mathematics, their fundamental subjects. The applications come easily to those who know the fundamentals.

It is much easier to train a man to look up information in a manual than it is to train him in the fundamentals. He can be taught how to look up constants in a very short time, but it takes too long to give him a fundamental course in physics. We have to assume that he knows it.

A. F. Buddington,* Princeton, N. J. (written discussion).—Dr. Hubbert has presented in a most stimulating and fascinating way the relationship of geophysics to geology and the necessity for most geologists of a thorough grounding in physics, chemistry and mathematics. As the author has indicated, it is imperative that adequate courses in geophysics and geochemistry (and shall we include geostatistics?) shall be added to the curriculum available for geologists. But he has not indicated whether it is his idea that this is to be at the expense of some other material now given, or whether the period of training is to be lengthened. A prospective geologist who seeks to acquire in his undergraduate career all the science, mathematics and engineering which we may think it desirable for him to have, will find little time for work in the social sciences and humanities, and will be at a definite disadvantage in this matter with respect to men in most other scientific fields. If we advise the potential geologist to take mostly the sciences other than geology as an undergraduate, the chances are very strong that he will end up in one of those fields rather than in geology. The problem is a complicated one.

Geochemistry and geophysics have made great contributions to geology and undoubtedly will make more at an accelerated pace. But they have so very far yet to go that the writer predicts that the geologist who has had a training to understandingly watch their progress, absorb what seems their valid conclusions, and continue the old-line geologic research, still has a long future of great usefulness ahead of him.

Dr. Hubbert no doubt recognizes that there are more aspects to the problem than he has discussed, and he has done a real service in presenting so clear and outspoken a paper on behalf of revision of the curricula for geologists.

L. B. Slichter,† Cambridge, Mass.—I wish to present a simple picture of what seems feasible in the four years of college work in giving the student a good fundamental start in geophysics. As an example, let us consider the course that has been given for some years at M. I. T. I want to analyze this very briefly, and to point out the possibilities of fitting an adequately broad foundation into this limited amount of time. The course offers an example of some of the things that Dr. Hubbert has so ably emphasized.

For the purpose of analyzing any type of engineering training, it is useful to group the subjects studied into four major classifications: (1) mathematical and analytical subjects; (2) experimental or laboratory techniques; (3) directly professional subjects, which give the student knowledge of the actual material with which he works and the problems to which he must apply his basic tools; and finally, (4) the humanities and nontechnical subjects.

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The accompanying table shows a complete schedule of the undergraduate course in geophysics in the Department of Geology at M. I. T. In the first and second

Course of Study at Massachusetts Institute of Technology for Students Majoring in Geophysics

NUMBERS INDICATE TOTAL HOURS PER WEEK IN CLASS AND PREPARATION

1	2	3	4	5	6	7	8
Year	Term	Mathematical Subjects	Physics, Chemistry, Electrical Engineering, etc.	Geology	Required Humanities and Non-technical	Elec-tives	Total
1	1	Calculus (M11) 9	Physics (8.01) 11 Chemistry (5.01) 11 Engineering Drawing (D11) 6		English 8 Military Science 3 Physical Training 1		49
	2	Calculus (M12) 9	Physics (8.02) 11 Chemistry (5.02) 11 Descriptive Geometry (D12) 6		English 8 Military Science 3 Physical Training 1		49
2	1	Calculus (M21) 9	Physics (8.03) 10	Engineering Geology 7 (12.321)	Literature and History 8 (E21) Military Science 3 (MS21)	10	47
	2	Differential Equations 9 (M22)	Physics (8.04) 10 Machine Tool Laboratory (2.854) 3	Engineering Geology 6 (12.322)	Literature and History 8 (E22) Military Science 3 (MS22)	10	49
3	1	Advanced Calculus 9 (M36)	Electrical Engineering Principles 10 (6.00T)	Structural Geology (12.70) 9 Mineralogy (12.01) 10	Foreign Language 6	6	50
	2	Advanced Calculus 9 (M37)	Electrical Engineering Laboratory 7 (6.75T)	Metamorphic Geology 9 (12.75)	Foreign Language 6	16	47
4	1	Introduction to Theoretical Physics 12 (8.461)		Sedimentation 7 (12.851)	Economic Principles (Ec11) 8	22	49
	2	Introduction to Theoretical Physics 12 (8.462)	Introduction to Geophysical Prospecting 4 (12.87)	Economic Geology 9 (12.40)	Economic Principles (Ec12) 8	15	48
Totals.......		78	100	57	74	79	388
Per cent.....		20.1	25.8	14.7	19.1	20.3	100

columns are indicated the year and term; in the third are listed mathematical subjects; in the fourth, subjects involving the basic laboratory techniques; in the fifth, basic

geological subjects; in the sixth, the humanities and nontechnical subjects; in the seventh, the time free for electives; in the last, the totals. The associated numbers mean the total hours per week devoted to the respective subjects, in classroom and laboratory and in preparation. In the bottom two rows are the totals and percentages for the several categories.

Several features of the schedule are evident. There is an approximately equal division of effort among the four major subjects mentioned; each accounts for about one-fifth of the time, with the remaining fifth supplied by the electives. Within each basic category, continuity of study is provided throughout the four years; thus the student obtains four full years of training in mathematics or mathematical physics, combined (except in the first year) with an equal emphasis upon the fundamentals of geology. Nominally, only one subject from each of the four categories is specified at one time, but flexibility is imparted by the elective subjects in each year. Thus the special needs, talents and interests of the individual are readily taken into account.

Field work in practical geophysics has been held down almost to the vanishing point. It is believed that the practical operation of instruments is best learned "on the job," and that the large amount of time involved in such instruction can, with more profit, be devoted to other matter. It is common experience that bright young high school graduates can quickly be trained to make excellent field operators. Valuable time in college should not be wasted upon instruction of a detailed or routine type.

A natural impression is generally prevalent to the effect that the study of a borderline field, such as geophysics, entails an unusually heavy load upon the studentt This opinion is not well supported by experience, nor, as I believe, does the presens schedule indicate any undue or severe burdens. The schedule, of course, represent. merely a *preparation.* For an adequate training in geophysics (as in geology and physics) three years of graduate study are highly desirable, if not essential.

It will be noted that the course is essentially a fundamental training in the basic tools of science. Indeed, it is sufficiently fundamental to provide an excellent foundation for graduate specialization in either geology or physics alone. The student is protected against danger of overspecialization, and receives a professional training of sufficient scope to give him adaptability and potential usefulness in a wide field of applied science.

Geophysics itself is represented in the schedule by only one course, an introductory and orientation course in the last term of the fourth year. This is needed, if only to justify the name "geophysics" for the course, and to provide a proper label.

I should like to add a brief remark about the need for more well trained young geophysicists. It has been our experience that not enough young men are preparing themselves for responsible jobs in this field. Certainly we have always had difficulty in finding enough good men to fill good positions available; this shortage of suitable candidates persisted even during the 1932 depression, and seems to be still with us. I am speaking, of course, only about well trained men, with several years of graduate work.

C. A. HEILAND,* Golden, Colo.—The foregoing papers and discussions on geophysical education will undoubtedly prove to be a very valuable guide not only to those engaged in the teaching of geophysical exploration and geophysics, but also to prospective students of the subject.

I fully realize the importance of geophysics in oil exploration but do not believe that those with any experience in the subject would wish to consider it from that viewpoint alone or to predict its future on such narrow basis. As in other lines of

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science and technology, forecasts appear more logical when based on an analysis of the possibilities in fields that are, at the time of the forecast, at the beginning and not at the height of their development.

In some respects, opinions expressed in the foregoing discussions have been harmonious. One point on which most writers have agreed is that *both* physics and geology are important and that therefore a balanced training in both gives a man the greatest flexibility and adaptability in the geophysical profession. Another point, clearly emphasized and agreed on by most, is that a sound training in fundamental subjects is a prime prerequisite for geophysical courses.

A number of other issues have remained controversial and it is with these that this discussion is chiefly concerned. To some extent some of the divergent opinions appear to have resulted from a confusion of geophysical science with geophysical exploration. Secondly, there has been some misapprehension about geophysical education resulting from not differentiating between *principle* and *detail*.

In regard to the distinction between geophysical science and geophysical exploration, probably I have been guilty myself of not pointing out their relation sufficiently in our paper. The paper deals only with geophysics in the sense of geophysical exploration and makes no mention of geophysical science. By way of addition, I may mention that from 1928 to 1933, we offered a course "Physics of the Earth" in the second semester of the senior year, totaling five credit hours. This course consisted of four parts. In the first, we covered the physical aspects of the various cosmogonical hypotheses, of paleogeography and paleoclimatology, with special reference to the Theory of Continental Drift. The second part consisted of a discussion of constitution and dynamics of the atmosphere and dealt primarily with meteorology, atmospheric electricity and other physical atmospheric phenomena, such as the propagation of sound, etc. In the third part we covered constitution and dynamics of the hydrosphere; i.e., oceanography and hydrology. The fourth and major part of the course covered the lithosphere; in particular, such topics as shape and mass of the earth, gravity, isostasy, terrestrial magnetism and electricity, radioactivity, movement of the earth in space (variations of latitude, precession, nutation, pole migrations, etc.), tides produced by sun and moon, volcanism, seismology and epeirogenic movements.

As the material to be covered in the courses devoted to geophysical exploration increased and room had to be made for more prerequisite courses in mathematics and geophysics, it was found increasingly difficult to allot time to the course in earth physics and the course was dropped. A contributing factor was the impossibility of keeping up with advancements in earth physics and giving up-to-date instruction in geophysical exploration at the same time; the choice had to be made in favor of the latter.

I believe that in a curriculum containing geophysical courses, a distinction between earth physics and geophysical exploration should be made. Notwithstanding the fact that most of our present techniques in gravimetric, magnetic and seismic prospecting have been derived from observational methods in earth physics, development in geophysical exploration has reached a point where it is justly classed as an engineering subject as contrasted to the science of earth physics. Hence, the two subjects should be offered as separate courses; the tremendous amount of material alone makes this necessary.

I fully agree with Dr. Hubbert that at least a course in earth science should be a part of a geologist's training. However, I would require, in addition, a course in the elements of geophysical exploration, with the emphasis on applications and interpretation of results in oil, mining and engineering. A two-semester course is now required at our institution of all juniors in geology, irrespective of whether they enroll in the geophysical option. Moreover, a three-credit hour course in geophysical exploration

is offered as an elective to mining seniors, with emphasis on electrical and magnetic methods.

A second point is the misunderstanding of the significance of engineering and engineering principles in geophysical education, evident in some of the preceding papers and discussion. One author has confused engineering with "techniques" and another goes so far as to say that "electrical engineers need not know how light plants are operated provided they know their physics and chemistry, their mathematics, their fundamental subjects." From other remarks made in the same discussion, it appears that Dr. Blau must have confused an "engineer" with an "operator." Granted that in the belief of many an engineer is simply one that runs a locomotive, and granted further that there may have been engineering schools of the "old kind" that placed too much stress on "operations" and "techniques, nevertheless, an engineer is *definitely not* an operator; rather, he is the intermediary between the scientist and the craftsman. I agree thoroughly with Dr. Blau in stressing fundamental mathematics and physics for the engineer, but I do not agree with the statement that a man trained in the fundamentals only can master their applications to engineering problems without additional training, in competition with men so trained. If this were true, we would hire a mathematician or a physicist instead of a civil engineer to design a bridge; we would hire them instead of mechanical engineers to construct a steam engine, instead of mining engineers to direct underground operations, instead of electrical engineers to design a power plant, and instead of an automotive engineer to design an automobile—all of which is not done for very good reasons.

We have emphasized in the available literature and have repeated in our paper (p. 53, this volume) that we aim at a balance of theory and application and do not train our men to be "operators" of geophysical instruments. Being a part of an engineering school, we deal with the subject from an engineering point of view. For each geophysical method, we cover the theory of the method, theory of instruments, discuss properties of rocks and formations, derive the theory of the effects of subsurface bodies on the method at hand, discuss numerous examples of results obtained in oil, mining and civil engineering under all types of geologic conditions, and allow the students to take their instruments out on known geologic structures and ore bodies and to do their own interpretation. In this connection, we look at field work from a point of view different from that of Dr. Slichter, who contends that "practical operation of instruments can best be learned on the job and the large amount of time involved in such instruction can with more profit be devoted to other matters." We consider field work primarily as a means to demonstrate to the students the relationship between geology and geophysics under actual working conditions. For this purpose we select structures with known stratigraphic and structural conditions and ore bodies that are known from mining operations or geologic mapping. In this work, operation of instruments is secondary; if we were to instruct students in the mere handling of instruments, we could set them up somewhere on the campus. Dr. Slichter will admit that the same reasoning applies to the civil engineering curriculum at the Massachusetts Institute of Technology, which includes a course in plane surveying—not for the purpose of making a surveyor out of the student, but to teach him the applications of surveying to problems of highway, railway, and building construction. Operation of special types of instruments is secondary; chances are the student will use a Gurley transit in school and will have to use a K. & E. instrument when he gets out on the job. Similarly, it would be unthinkable to teach aeronautical engineering without demonstrating to the student the operation of a wind tunnel and showing him its application to the design of aircraft, both student and professor knowing full well that in the airplane manufacturer's plants a different design of wind tunnel may be in use.

It is no new experience to teachers in engineering schools to have former students tell them about a certain method or formula which they have used with a company and to have them perhaps imply to others that the professor is five years behind the time because he did not tell them about it. Very likely, the professor knew about the method but purposely refrained from telling the student in order not to crowd the course with details of only temporary value. On the other hand, if a fundamental principle is involved, I do not share Dr. Blau's objections to including discussions of methods used five years ago. For instance, the theory and formulas for use of the torsion balance as developed by Eötvös 50 yr. ago are still being applied almost unchanged by all users of the torsion balance. In regard to principle, it is wholly immaterial whether the calculation is made on form A or form B, or whether nomographs are used. We see no objection whatever to teaching a student the derivation of these formulas as Eötvös did 50 yr. ago, and see no objection, in demonstrating these principles, to using an instrument purchased 10 yr. ago. We know from experience that students so prepared have no trouble in applying the torsion balance to all kinds of geologic problems as independent operators or in following any oil company's special form of calculation.

I do not believe that a geologist or geophysicist with actual magnetometer experience will agree with Dr. Blau's statement that its application is now totally different from what he has been told by his instructor when he was in college. I am frank to say that the cause of geophysics and geophysical education is not helped if simple issues like this are surrounded in this manner with unnecessary mystery. After being stripped of the trimmings, any of the "superspecial" magnetometer formulas come down in the final analysis to the simple relations derived by Ad. Schmidt 25 yr. ago. After all, the reading of a magnetometer simply expresses a condition of equilibrium between the moment of gravity and the torque produced by the magnetic vertical intensity on a magnetic system in the magnetic prime vertical. At our institution, the student learns to derive the mathematical expression for this condition of equilibrium, which leads immediately to the fundamental relation that the anomaly in vertical intensity, with respect to an arbitrarily chosen base value, is equal to scale value times deflection. In teaching the derivation of this relation, we do not hesitate to go back to fundamentals developed 25 yr. ago. From the viewpoint of teaching principles, it is immaterial how the calculation of the results proceeds in detail, whether on white or on pink forms; with curves taking into account variations in the shape of the knife-edge; by multiplication with a constant scale value; by nomographs, or charts involving an automatic determination of averages in accordance with the probability function; whether the corrections are applied in scale divisions and then converted into gammas, or the anomalies are calculated in gammas and corrections applied in gammas, and so on ad libitum. Indeed, we could add to this list numerous varieties of formulas and procedures from our collection of forms and graphs used by various companies in this country since 1925. I have worked with the first magnetometer constructed for Ad. Schmidt by Toepfer in 1915, and have had an opportunity to work with most models that have come out since, and can assure Dr. Blau that the fundamental method of calculating the results has not changed. Hence, we see no objection to using magnetometers purchased 5 and 10 yr. ago in demonstrating applications of geophysics to geologic problems. Owing to the extreme variability of the magnetic properties of formations with their geologic history, it is far more important that the operator learn to *adapt his technique*, the sensitivity of his instrument and the accuracy of his calculation *to the geologic problem at hand* than to seek a cure-all in some highly specialized formula which has been developed to give the maximum possible accuracy.

I believe that this very example demonstrates sufficiently the need for *engineering in geophysics*, and the need for men trained in an appreciation of what is fundamental and significant and what is not.

Summary of Reports by Committee on Geophysics Education, Mineral Industry Education Division

By Sherwin F. Kelly,* Chairman of Committee

The Geophysics Education Committee was organized in 1938 and presented its first report at the A.I.M.E. annual meeting in February, 1939, at a session held jointly with the Committee on Geophysical Methods of Exploration.

REPORT OF 1939

The first task the Committee set itself was to find out in which universities the subject of geophysics was being taught, and the manner in which such instruction was given. A questionnaire was drawn up and sent out to 147 universities in the United States, Canada, and abroad. Of these 100 replied. Their answers were then tabulated and analyzed, yielding the information summarized in the following pages.

Two universities, Colorado School of Mines and St. Louis University, actually maintain departments of geophysics, while 3 Canadian and 31 American universities give courses on the subject. In other institutions, 45 in the United States and 5 in Canada, more or less extended reference is made to geophysics in lectures on other subjects. Altogether, there are at least 86 universities in the United States and Canada at which students may obtain instruction of some kind in geophysics.

To enter the courses in geophysics given by 33 American universities, certain prerequisites are demanded, varying with the level at which the subject matter is treated. In analyzing the prerequisites, some assumptions had to be made: one year of college mathematics was taken to mean up to, but not including, calculus, while three years was presumed to embrace a course in differential equations. On this basis, 16 per cent require only elementary college mathematics, 40 per cent require differential and integral calculus, and 31 per cent require differential equations. A few demand even more advanced mathematics, while some list no prerequisite mathematics beyond high school. Evidently a student contemplating the study of geophysics should ground himself in calculus and differential equations.

In physics, similar assumptions have been made as to time required in the subject. All of the universities giving geophysics require their students to take physics first, and 34 per cent require a standing in

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advanced physics. This is a fairly small percentage, so most of the instruction in geophysics must be on a fairly elementary level.

Advanced chemistry is required by 28 per cent (12 per cent demand that physical chemistry be included), and 34 per cent require only elementary chemistry. The remainder do not mention it.

Geology is another 100 per cent prerequisite, but a majority of the institutions (60 per cent) demand only elementary courses for the geophysics students. Of advanced courses, historical geology, structural geology, and economic geology were most frequently mentioned as required.

Evidently, physics and geology are universally demanded as prerequisites for studying geophysics, followed closely by mathematics and then by chemistry. The statement by over 80 per cent of the universities that they insist on a *thorough* grounding in geology or physics is open to question, since only 37 per cent require advanced geology and only 34 per cent require advanced physics.

Most of the geophysics courses (94 per cent) deal with the subject as a prospecting tool. Just half of these institutions also lay claim to the teaching of geophysics as the physics of earth phenomena as well, while only 6 per cent teach it the latter way and ignore the prospecting aspects. These figures are doubtful however, since some confusion apparently existed in the minds of a few of the respondents as to the distinctions the Committee desired to make.

Geophysics instruction dates back to 1885, in Stockholm, where a course in magnetic prospecting was instituted at that city's Technical University. In the United States the way was led in 1910 by St. Louis University and the University of California, where courses in geophysics, but not as a prospecting tool, were established. In 1926 the Michigan College of Mining and Technology, the University of Colorado, and the University of North Carolina, set up geophysics courses. The following year the Colorado School of Mines founded its Department of Geophysics. Other schools followed, the peak years being 1930, 1931 and 1938, during each of which four new universities instituted instruction in the subject.

Aside from the 2 institutions with Geophysics Departments, the geophysics courses are given by 12 schools in the geology department, by 12 in the physics department, by 1 in both departments, by 6 in the engineering department and by 1 in both geology and engineering departments. Most universities have but one instructor devoting time to the subject, a few have 2 or 3, and one has 4. Use is frequently made of visiting lecturers, usually commercial geophysicists. Altogether, these institutions seemed to have a total of about 560 students taking some instruction in geophysics during the academic year 1938–1939. An average of 2 undergraduate courses with 6 credits is given, and 2 graduate ones with 7 credits, but institutions that emphasize the subject naturally give more courses.

Viewpoints on Geophysics

When asked to define their viewpoints on geophysics, 10 universities in the United States, or 30 per cent of the 33 with courses or departments, consider it to be a prospecting technique only. It is regarded purely as a fundamental science by 21 per cent, while 12 per cent consider it inclusive of both these phases. One university looks upon geophysics as dealing only with the major physical characteristics of the globe, but 24 per cent consider that it properly embraces all three of these aspects. Three universities expressed no opinions. These figures probably do not accurately reflect the situation, however, since the question seems not to have been clearly understood in many cases.

In reply to the question of how geophysics is brought into harmony with other sciences, particularly geology, some helpful comments were received. Prof. F. G. Tickell, of Stanford University, pointed out that geophysics commonly means two different things, hard to combine in a single course; one of them is more particularly related to seismology, structural geology, vulcanism, etc., and should be dealt with in physical geology, while the other is a prospecting technique properly taken up in technology courses. Prof. Perry Byerly believes that integrating courses should be given, showing students how their knowledge of physics and mathematics can be used to obtain geological information from seismic, gravitational, magnetic and electrical data.

The fundamental importance to geology of geophysical data on gravity, seismology, radioactivity, etc., was pointed out by M. King Hubbert and Prof. G. L. Shue. The distinction between the use of geophysics in a general study of geological phenomena, and its use in working out structural details in an exploration program, was emphasized by Clyde H. Wilson and by the late Dean F. H. Probert. The first use might be considered the science of geophysics, and the second the art of applied geophysics, although these terms are by no means accurate definitions of the subject matters involved.

In the teaching of applied geophysics, 8 universities, or 24 per cent, confine their instruction to the general principles of prospecting techniques, while in 20 universities, or 61 per cent, this is accompanied by practical instruction in the use of field instruments. This trend toward instruction in specific techniques is contrary to the expressed desires of those employing geophysicists, who state specifically that they do *not* want men trained in particular methods.

Scope of Courses

The replies to the question as to what was taught in the geophysics courses were difficult to tabulate. In general, however, it seems that seismology is the most common subject, being given in 21 universities. Electrical and gravitational subjects are found in 20 schools, and magnetics in 18. Courses in radioactivity and geothermics are less popular,

but still apparently rank ahead of the subjects proper to the physics of the earth, such as geodesy, meteorology, hydrodynamics, or allied subjects.

Of these schools, 21 have equipment for field and laboratory work. Magnetic apparatus for field experiments is found at 17 universities, and for laboratory use at 13. Electrical field instruments are owned by 16 institutions and laboratory instruments by 9. Gravitational instruments are used by 11 schools for the field, and by 8 in the laboratory. Seismic outfits are found in 10 universities. Only one is equipped for geothermic measurements. Radioactive apparatus is missing from the field lists, but 3 laboratories have equipment. Four universities have ordinary physics laboratory instruments, and 4 have other, or unspecified apparatus. Only 7 institutions in the country seem to have adequate equipment for teaching geophysics. Of the remainder, 18 want more field and 11 more laboratory supplies, 8 of these universities being found in both groups.

Of the few graduates regarding whom we could get information, 17 have gone into commercial geophysics, 5 into universities, and 2 into government work.

What prospective employers expect from the universities in the way of training for geophysical work was covered in one of the questions. It brought forth remarkably unanimous replies—that the employers insisted on the fundamental training in mathematics, physics and geology. This is one of the most important features of the replies to our questionnaire, that the universities should stress the fundamental sciences, not the mechanical applications.

Future of Geophysics

As to the future of geophysics, there is some difference of opinion, although the majority of those replying see a promising and important future for it. A few believe it is near the saturation point, but others that it is bound to be an expanding field. However, to take advantage of it, a man must be well trained, and should have obtained a Ph. D. degree.

Possibly the best considered comment is that of M. K. Hubbert, who says: "As a profession it is one of the broadest and most important branches of science, and has the advantage that most of the big problems are still awaiting solution. As a means to a livelihood, it is about in employment equilibrium with other branches of science and technology—more people want jobs than find them. This condition will continue and probably get worse." From this it might be concluded that care should be taken to turn out annually only a few men, but very well trained in geology, physics and mathematics.

Impressions from Experience

In addition to answering the questionnaire, a few foreign correspondents were kind enough to write their impressions, gained from experience in the field or in teaching.

From Sweden, Helmer Hedstrom writes that meteorology is taught at the University of Uppsala, and oceanography at Gothenburg. Plans have been made for an institute in Stockholm to give complete courses in "pure" geophysics, but nothing has yet materialized.

Prof. John G. Koenigsberger sends word from Germany that, in his view, applied geophysics, if experimental, belongs to applied physics; but if it is theoretical it belongs in applied mathematics. A man gifted for mathematics will be the one to formulate the problems, whereas the one liking to work with apparatus and to construct new instruments becomes the applied physicist, or the "physicist-geophysicist." On the other hand, the field man, the man that plans the geophysical survey, controls it, and handles the geological interpretation, is the "geologist-geophysicist"—but if he turns to helping to make geology into an exact science, he becomes a "geophysicist-geologist." Professor Koenigsberger believes the time has come for geology no longer to lag behind the movement toward experimental and mathematical verification of hypotheses. For the geophysicist that knows geology well, the future is good.

Also from Germany, Dr. Mintrop sends a description of the difficulties he encountered early in the development of geophysics, in getting geologists and geophysicists to work together harmoniously. He believes that the types of scientists that developed instruments and methods in the past—mathematicians, physicists, engineers, etc.—should continue to do so, for they cannot be replaced by geologists—nor can the geologist be replaced by them. Today there are many different kinds of men working in geophysics, and he does not believe it possible to educate one man to combine all the qualities and knowledge necessary for the efficient application of all the various procedures. Geologists should, however, take some courses in geophysics, and geophysicists should study some geology, in Dr. Mintrop's opinion.

Prof. Mario Bossolasco is planning to establish courses in geophysics at the Geophysical and Geodetic Institute of the University of Messina. Another such Institute exists at Naples. The applied geophysics he plans to include will be the first offered in Italy, as heretofore instruction in that country has been principally along the lines of general geophysics.

Probably the most pertinent remarks of all, applicable to any educational endeavor, came from the Director of the Ecole des Mines, in Paris. He says: "We seek rather to develop the qualities of reflection and observation than to multiply the instruction in technological details."

American Comments

F. S. Hudson, of the Shell Oil Co., specifies two types of men considered desirable for the work of his company. One is the theoretical geophysicist, needed for developing geophysical methods, who should be trained in physics and mathematics. The other is the practical geo-

physicist, who should have studied geology, physics, and mathematics through differential equations. The physics should include the potential theory of gravitational fields, and wave propagation in elastic media.

G. C. Gester, chief geologist of the Standard Oil Company of California, comments on the importance of geophysics to the petroleum geologists, and notices that geophysics, aside from being a useful exploration tool, has also forced them to change many of their ideas regarding the subsurface configuration of some of the relatively large sedimentary basins. He observes that his company usually has one or more geologists attached to each geophysical party, whose business it is to interpret, in terms of geology, the records obtained by the geophysical instruments.

The comments of Clyde H. Wilson, himself a geophysicist who is concerned with the employment by his firm of other geophysicists, are particularly interesting because they combine the two viewpoints. He has found that, without exception, the men that get ahead in geophysics are the ones with a thorough grounding in fundamentals. Instruction in operating details, instruments, and other specialties, he believes should be employed only as a means to illustrate principles. Only a few men will acquire the well-rounded training needed to equip them to handle the complete details, from research and development to the final interpretation and expression of results in terms of geology. Consequently, it will probably remain the physicist's job to handle research and development of methods and equipment, while the geologist with good fundamental training (in physics, electricity and magnetism, and some engineering) will handle the interpretation of the field data. The two working in close cooperation should give the best results.

Dean F. H. Probert, giving the fruit of his own reflections on this question, said:

> The answers to our questionnaire are all in agreement as to the necessity of sound, fundamental training in mathematics, physics and geology. There can be no argument as to the validity of this conclusion. I, however, sense a difference of opinion as to whether our educational institutions should stress basic subjects or espouse the cause of geophysics as such in curricula offerings. It has long been my contention that technical schools can never turn out a finished product, no matter how long the training. We may give our students much of knowledge and perhaps a little of wisdom, but judgment comes alone from experience. I feel that we fully serve our purpose as educators if we give a clean-cut exposition of principles from and on which practices have been developed to meet particular problems.
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> In our profession, as in nearly all others, the trend is toward specialization. Geophysics certainly falls in this category. It is not a subject that all can or should master. With appreciation of its usefulness we should seek the services of those who have devoted themselves to the subject. I cannot bring myself to feel that instruction in geophysics should be offered by all technical schools. In my opinion it would best serve the ideals of higher education if one or two of the ranking universities sufficiently endowed, well equipped with personnel and laboratories, would exclusively sponsor the highly specialized instruction in geophysics. In State-sup-

ported institutions it would be, in my way of thinking, a gross waste of public money for each and every one of them to attempt the work.

He furthermore suggests: " . . . that Federal or State bureaus, or maybe institutional or industrial research laboratories, should devote themselves to writing the alphabet of the new and promising science of geophysics."

Discussion of Report

Following the presentation of the above report, some valuable comments were made in the ensuing discussion. (W. R. Chedsey and Sherwin F. Kelly, Associate Chairmen, presiding.)

H. W. Straley, III, brought up the point that some institutions failed to list in geophysics such subjects as hydrology, given in the Department of Civil Engineering, or meteorology, given in the Geography Department. He suggests that the curricula in geological engineering are about the only ones where the mathematical and physical prerequisites, deemed essential by geophysicists, can be found strictly adhered to.

The fact that most institutions are divided by impenetrable walls was stressed by M. King Hubbert, who pointed out that in many cases a student that tried to get an adequate preparation in geophysics by taking physics, mathematics, geology, engineering and chemistry, would sacrifice his chances for a degree. He also mentioned the influence that is being exerted in academic geological circles by the discussions of the geophysicists. Some universities are already instituting courses for geologists in which mathematics, physics and chemistry are required.

H. T. F. Lundberg expressed the opinion that we are leaning too far towards the theoretical, and have forgotten that what the geophysicist does in a day's work is more to be compared with the work of the geologist. For this reason the actual application of geophysical methods tends to pass from the hands of the physicist to those of the geologist. The physicist will nevertheless always occupy an important role, chiefly in the designing of new apparatus and new methods.

The fact that some of the questions were misinterpreted was also mentioned by Prof. H. E. Landsberg. He also brought up the point that some instruction in field work is advisable from the standpoint of educational technique, and gives the student more satisfaction.

Sherwin F. Kelly replied that there was no intention of deprecating the use of instruments, if they were employed purely to illustrate principles. What the report intends to condemn is the concentrating on instrument manipulation. He then mentioned the debt of gratitude the committee owed to M. King Hubbert for having provided, in his paper the previous year, the original argument on which the committee's work was based.

Prof. W. T. Thom, Jr., pointed out that geophysical research has been mainly concerned with exact quantitative studies of the form, composition, and make-up of the earth. Structural geology, on the other hand, has been mainly qualitative in its study of forms, space patterns, and time patterns of geological formations. Now, in order to become really scientific, structural geology is having to become quantitative, and lean heavily upon geophysical procedures and findings. He believes that blends of sound physics, sound geology, and good engineering training are best calculated to produce effective new workers in geophysical-geological research. The engineering schools are, in his opinion, the logical places in which to give the foundation work needed alike by men who will specialize in geophysical exploration, or in structural geology, or both.

Readers interested in obtaining the complete report, with tabulation of the results of the questionnaire, should write to the A.I.M.E. headquarters. A few copies are still available.

REPORT OF 1940

At a joint meeting on Geophysics Education, held by the Geophysics Education Committee of the Mineral Industry Education Division, and the Committee on Geophysical Methods of Exploration, William R. Chedsey, *Chairman*, and Sherwin F. Kelly, *Associate Chairman*, copies of the "Appendix to the Report of 1940" were first passed to the audience. This appendix contained tabulations of the figures derived from the questionnaires circulated by the Committee in 1939. These questionnaires were designed to reveal how graduate, practicing geophysicists and their employers evaluated the available academic preparation for a career in geophysics. In sending out these questionnaires, the Committee circularized the fields of both geophysical exploration, and what, for lack of a better term was called "pure" geophysics, or the physics of earth phenomena. The latter includes such subjects as meteorology, hydrology, oceanography, geodesy, seismology, volcanology and terrestrial magnetism, as exemplified in the Sections of the American Geophysical Union.

One of the outstanding results of this past year's investigation is the demonstration that practically none of the geophysicists now in professional practice has been adequately prepared for the career he has adopted. This inadequacy of the educational institutions involved is due apparently in part to the newness of the demand and in part to a fossilization of the theory of instruction. More than 50 universities, scattered all over the country, have sent graduates into geophysical careers, and in most instances without specialized courses preparing them for such a profession. There is no single institution in this country offering a completely rounded introduction into a career in either exploration geophysics

or in the broader field of the physics of the earth. Some respondents, particularly in the latter field, apparently do not realize what aspects of earth science *are* embraced in geophysics. If this Committee is to fulfill its purpose it would seem to be necessary not only to serve as an advisor to aid in setting up geophysics courses but also to educate the technical public as to just what the science really includes.

The notable deficiencies in the academic preparation for their careers have been summed up by the geophysicists themselves. These deficiencies resolve into the conclusion that those who specialized at college in physics, mathematics, and electrical engineering wish they had had more geology; the geologists wish they knew more physics and mathematics; and the civil and mining engineers wish they had studied more of all three. On the whole, however, the outstanding lack is geology.

The figures on which the above conclusions, and also those to follow, were based were obtained from answers to the previously mentioned questionnaires. These questionnaires were distributed by all the members of the Committee from personal knowledge of consultants, graduates, employers, etc., from professional cards and advertising found in technical and trade journals, and from data contained in replies to the previous year's questionnaires. (See the 1939 report of this Committee, commencing on page 82.) In the 1939–1940 period, 290 of our questionnaires were sent to employers of geophysicists, of whom 40, or 14 per cent, answered. Of these 40, replies relating to geophysical exploration numbered 31, while 11 related to other fields of geophysics (preoccupation with both phases of geophysics on the part of two employers explains the apparent sum of 42 instead of 40). Circulars were sent to 1100 graduate professional geophysicists and 145 answered (13 per cent of the total). Of these replies, 122 related to geophysical prospecting and 23 to earth physics. While the method of circularization is admittedly not highly scientific, certain trends are emphasized with such persistence as to arrest attention, and warrant the consideration of our results as corresponding to a representative, bulk sample.

From the list of universities granting bachelor degrees to our various respondents in geophysical exploration, a chart was drawn up to show in what years most of them were graduated, and the cumulative totals each year. The highest number graduating in a given year was 8, a figure reached in 1927 and 1928, since when the average has been about 5 per year. The cumulative totals give a curve rising at the rate of about 5 men per year; or, on the basis of roughly 100 replies, a present yearly increment of about 5 per cent of the total involved in this study.

In the other fields of geophysics, the physics of earth phenomena, the graduates now employed therein date back to 1894, but not more than 2 have entered the field in any graduation year. The cumulative totals indicate a yearly increment of about 2.5 per cent of those now employed.

Thus 5 per cent can be considered the absorption capacity of the geophysical exploration profession, and 2.5 per cent that of the earth physics phase of the science. These figures reduce to an average of nearly 4 per cent.

If a yearly increment of about 4 per cent can be absorbed by the geophysics profession, what does this represent in actual numbers of new graduates? From a consideration of the membership of the American Geophysical Union, of the Society of Exploration Geophysicists, and of the distribution of geophysical TECHNICAL PUBLICATIONS to interested A.I.M.E. members, a rough estimate was made of 1000 practicing exploration geophysicists and 600 in research and the professions concerned with earth physics. (Subsequent information indicates that these figures may be too low.) The yearly increments mentioned would then mean that about 65 new men per year could expect to find geophysical jobs.

Opinions of Employers

What about the future of this profession? Over 80 per cent of the exploration geophysicists think that both geophysical exploration and earth physics offer good to fair futures, an opinion in which 100 per cent of the earth physicists agree.

Admitting that the future is good, and that men need to be trained for this comprehensive earth science, it becomes necessary to study the faults of present academic preparation and devise improvements therein. Attention will be given first to geophysical exploration. The principal fault is inadequate geological preparation, according to 78 per cent of the answers from employers or consulting geophysicists. Next in order comes the lack in training for interpretation. Only 28 per cent of the employers found consulting geophysicists inadequately instructed in physics. Lack of training in theoretical bases of geophysics came next (22 per cent), followed by inadequate knowledge of instrumental techniques (17 per cent). Then came poor mathematical preparation (11 per cent). Other lacks mentioned were engineering, chemistry, and economics.

Among employers in oil exploration that have their own geophysical staffs, lack of geological training on the part of their employees ranked second, with 51 per cent, being exceeded (58 per cent) by deficiency in geophysical theory. Lack of field practice came third (37 per cent), followed closely by the need for better instruction about geophysical instruments. The need of improved preparation in physics, mathematics, and electrical engineering was also mentioned several times.

In view of the opinions expressed by most employers that they want men well grounded in the fundamentals of geology, physics, and mathematics, but *not* in specific geophysical techniques and methods, it is interesting to see how large a number complain that their employees are deficient in training with geophysical instruments and in field practice!

As might be anticipated, those who specialized in geology were found lacking in physics and mathematics and geophysical theory. None of those specializing in electrical engineering had enough geology, and the majority were deficient in geophysical theory. The physicists and mathematicians also were short on geology.

In view of these inadequacies in academic preparation, as brought out in opinions expressed by employers, it is well to see how the geophysicists concerned oriented their college courses. The principal subjects studied as majors for a bachelor degree were, in order of decreasing number: physics, geology and mineralogy, mathematics, electrical engineering, and chemistry, with various other engineering courses and some geophysics and seismology following in lesser amounts. (Seismology is listed separately as distinct from exploration and unspecified geophysics.) The most numerous minors were mathematics, chemistry, physics, geology and mineralogy. For graduate degrees, the principal majors were: physics, geology and mineralogy, geophysics, and electrical engineering, with chemistry, various engineering courses, and seismology following in smaller numbers. The most numerous minors were mathematics, physics, geology and mineralogy.

It is dangerous to draw too sweeping a conclusion from these figures without analyzing carefully and individually all the returned questionnaires. It would seem, however, that all the subjects specified as desirable by the employers of these geophysicists are available (though not always at one university), and that the deficiencies in academic preparation result in no small part from an unbalance in the courses followed by the men in their university years. This is an important conclusion, commended to the attention of instructors charged with advising students contemplating careers in geophysics.

Men that followed specific geophysics courses in school classified them as follows: Applied, 16; General, 14; "Pure" (physics of the earth), 3; Theoretical, 2; and Research, 1. There are overlaps in these replies, as several men took two or more courses thus characterized.

Opinions of Practicing Geophysicists

What do the practicing geophysicists themselves think of their university careers? Subjects voted essential, with the percentage of respondents voting for them, were as follows: physics, 100 per cent; mathematics, 96; geology, 89; electrical engineering, 43; chemistry, 30; civil engineering, 12; mining engineering, 6; and petroleum engineering, 6. The subjects actually found most useful in practice, in the main follow the same order: physics, 79 per cent; mathematics, 72; geology, 49; electrical engineering, 26; with 7 per cent each for chemistry and civil engineering, and also for seismology. One man found camp cooking his most useful subject! When we come to the additional sub-

jects, which our respondents *wish* they had taken, we find a distinctly different order: Geology ranks first with 43 per cent; it is followed by geophysics and advanced physics, tied for second place with 34 per cent each; advanced mathematics was named by 20 per cent; electrical engineering by 11 per cent, and radio engineering by 7 per cent. Seismology also received a 7 per cent mention. Chemistry was named by 5 per cent, civil engineering by 3 per cent; interpretation was mentioned by 2 per cent.

These figures were further broken down and individualized, and grouped according to the major subjects studied. Space prevents detailing of these results, for which reference should be made to the mimeographed "Appendix to Report of Mineral Industry Education Division Committee on Geophysics Education," which may be obtained for a limited time from the office of the A.I.M.E. in New York.

One point brought out therein is worth mentioning, however. Among the men that studied electrical engineering, a few expressed no desire for geology. They were specializing in research on instrument construction for seismic work.

The fields of geophysics in which the men that answered our questions were practising may be tabulated as follows, keeping in mind that there are many overlaps, as one man may have practiced or be practicing in several branches.

Seismic prospecting	102	Terrestrial electricity	5
Gravitational prospecting	59	Meteorology	4
Electrical prospecting	56	Physics of rock formation	3
Magnetic prospecting	41	Geodesy	3
Seismology	15	Hydrology	2
Gravimetry	11	Volcanology	2
Radioactivity	10	Oceanography	1
Terrestrial magnetism	9	Electrical well logging	2
Geochemistry	9	Teaching	2
Geothermics	5	Miscellaneous	2
Physics of rock deformation	5		

The types of work in which these men engage are: with field crews, 102; in interpretation, 99; in instrument research and construction, 49; in theoretical studies, 45; in the laboratory, 43; supervising, 11; executive, teaching, and miscellaneous, 4 each. Here again there are numerous overlaps, because of one person's preoccupation with several types of activity, but the figures give a rough idea of how the geophysicists are distributed.

The universities from which these men came are too numerous to list. Those that granted bachelor degrees to four or more graduates answering these questionnaires include: Texas University, California University, California Institute of Technology, Rice Institute, and De Pauw University. Those granting four or more doctorates were: California Institute

of Technology, Texas University, Harvard University, Rice Institute, Chicago University, and California University. Of all the graduates replying, 37 received only a bachelor's degree, 34 no higher than a master's, and 40 a doctorate.

Pure Geophysics

The field of what was called, probably inadvisedly, "pure" geophysics, produced answers somewhat more difficult to tabulate and to summarize, and again reference should be made to the mimeographed Appendix to Report mentioned above. The branches of earth physics covered in this part of the questionnaire were: seismology, volcanology, gravimetry, terrestrial electricity, terrestrial magnetism, geothermics, radioactivity, geochemistry, physics of rock formation, physics of rock deformation, geodesy, hydrology, meteorology, and oceanography. Eleven institutions, specializing in one or more of these fields, replied. Several suggested that academic training could be improved through more work in advanced physics, mathematics, geology, thermodynamics, and geophysics theory. Some suggested biology and oceanography, and one or two mentioned seismology, terrestrial magnetism, gravity, meteorology. The necessity for more laboratory work was also spoken of, as well as training in drafting, and construction of equipment.

The principal subjects taken for a bachelor's degree by the men answering these queries were: physics, mathematics, civil engineering, and chemistry as majors. Aeronautical, electrical, and mining engineering, and biology, geology, and metallurgy were each mentioned once. The minors were: physics, mathematics, chemistry, and languages. Geology, biology, and hydraulics were each named once. For graduate degrees, the most numerous majors were: physics, mathematics and oceanography. Mentioned once each were electrical engineering, geophysics (unspecified), geology, chemistry, and soil technology. The minors were: mathematics, physics, and geology, with one mention each for chemistry, biology, and economics. There is a striking unanimity here for placing physics and mathematics at the top of the list in each case, and geology well down on it. Among the exploration geophysicists, however, it will be recalled that physics and geology headed the list of majors, although in the minors it was mathematics and physics or chemistry that led.

In addition to the majors and minors listed, some men had taken specific geophysics courses, listed as: geodesy, meteorology, oceanography, hydrology, and general geophysics.

The tabulation of the subjects believed essential, most useful and which they wish they had taken, provides some interesting parallels and comparisons with the same kind of analysis of the replies from the exploration geophysicists.

Of the workers in earth physics, 100 per cent believed physics and mathematics essential; 83 per cent named geology, and 52 per cent mentioned chemistry. Civil engineering followed with 26 per cent, and electrical, mining, and petroleum engineering each received a 13 per cent vote. In this list chemistry ranks well above electrical engineering, the reverse of the situation in the previous tabulation. Three new subjects appear here: modern languages, 9 per cent; biology, 9 per cent; and geodesy, 4 per cent. The ranking of subjects found most useful is in much the same order: physics, 78 per cent; mathematics, 74; geology, 26; chemistry and civil engineering, each 17; electrical engineering, 4; (in the exploration list electrical engineering stood above both chemistry and civil engineering). Modern languages received a 9 per cent vote, outranking electrical engineering, and also biology (4 per cent), meteorology (4 per cent), and hydrology (4 per cent). The latter subjects did not appear at all in the listing of the exploration geophysicists' choices.

When it comes to the subjects they wish they had taken, interesting comparisons appear with the prospecting geophysicists' choices. Geology still heads the list, with 35 per cent. It is followed by advanced mathematics (30 per cent), and advanced physics (17 per cent). But then come hydrology, hydrodynamics, and meteorology with 13 per cent each; seismology and geophysics (unspecified), 9 per cent each, the same vote as for chemistry and radio engineering. Geodesy, soil physics, interpretation, and mechanical engineering, each received 4 per cent mention.

The types of work in which these men engage are: theoretical studies, 15, interpretation, 13; field crews, 12; laboratory, 11; instrument research and construction, 8; teaching, 2; meteorology, 1; administration, 1. As for the exploration geophysicists, multiple interests produce duplicate listings and overlaps. It is interesting to note that field crews took first place in the exploration geophysicists' statements, but are in third place here.

The fields of activity of the respondents in this group are:

Seismology	7	Terrestrial magnetism	2
Oceanography	5	Terrestrial electricity	1
Hydrology	5	Gravimetry	1
Meteorology	5	Radioactivity	1
Geodesy	3	Volcanology	1
Electrical prospecting	3	Petrology	1
Magnetic prospecting	2	Geomorphology	1
Geochemistry	2	Astronomy	1

The universities granting bachelor's degrees to two or more of these geophysicists are Stanford University, Cornell University, and Johns Hopkins University. Two or more with graduate degrees came from California University, Cornell University, Pennsylvania University,

and Leland Stanford University. Five received only bachelor's degrees, 5 no higher than a master's, and 13 a doctorate.

Word from the Two Groups

In both this group and the exploration geophysicists we find outstanding the desire to be better instructed in the fundamentals—physics, mathematics, and geology. Then come different subjects in the two groups, but all pointing to the same conclusion—a wish for courses dealing specifically with the particular field of geophysics in which the man is practicing, whether it be geophysical prospecting, geodesy, hydrology or some other branch. This wish so definitely expressed should not be overlooked when giving weight to the statements of employers, emphasizing *only* the fundamentals. The fundamentals would seem to need some rounding out, at least with orientation courses covering various fields of geophysical activity.

Some of the opinions written by the geophysicists that returned these questionnaires were read, as they provided interesting commentaries on the subjects under discussion. Several men emphasized the necessity for better instruction in English, spelling, and geography, indicating that our universities are lax in their standards of what should be fundamental requirements for any well-educated person. Others stressed the need for closer cooperation between physics and geology departments, in the interest of better coordination of the instruction for prospective geophysicists. This is such an obvious necessity that it is surprising that mention of it should be necessary. Some comments indicated a belief that a sound, basic engineering education would give adequate preparation for geophysical careers. Others mentioned the special need for courses in electrical and radio engineering. A desire to have instructors in geophysics keep up with progress by spending their summers in field work was voiced by some. A good many correspondents expressed the belief that students should be taught the use of instruments in the field. This attitude is in contradiction to the opinions of others, well summarized by one man, who said: "A thorough training in physics, geology, and mathematics is essential, and in no case should the students' time and energy be wasted on descriptive courses that may appear more closely related to geophysics than those fundamental subjects." On the other hand, one geophysicist demands integrated courses for professional geophysicists, survey courses for geologists, and some field practice.

An oceanographer suggests thorough grounding in physics and mathematics, with supplementary training in biology, geology, and meteorology. A meteorologist believes that courses in his subject should be brought into consonance with actual practice. A hydrologist calls attention to the rapidly expanding field in this branch, as yet largely unrecognized by universities.

An interesting suggestion is made by another oceanographer, that more emphasis be placed on the study of the earth as a whole.

Severe condemnation was expressed in one reply for "coffee grinder mass production," and a plea was made for a system that would give the students more initiative.

A geophysicist of many years experience with staff and consulting men in his profession finds them baffled by field conditions. This he ascribes to an "inadequate philosophical approach to the whole subject of geophysics." He asks for more emphasis on actual field conditions, and on the economics of exploration.

One vigorous letter queries, "Shall the course or the man be placed first?" It also stresses the impossibility of separating geophysics and geology. The writer strongly expresses the opinion that in the majority of schools the "average faculty cannot see out of its trench" and is unwilling to come to grips with reality. He contends that it is the duty of every educational institution to develop the man rather than the science, and to give the student a broad, basic training in fundamentals, so that he can adapt himself to the greatest number of conditions.

In closing the report, the Chairman of the Committee expressed the great debt owed to the many people that cooperated to make it possible, and especially to the Committee members, who gave time and effort so generously in the gathering of the material. He also mentioned the invaluable work of two nonmembers of the Institute, his wife and their secretary, Miss Haley, who had been responsible for sorting and analyzing the data and preparing the tabulations, for both the 1939 and 1940 reports.

Allied Papers and Discussion

Following the presentation of the Committee's report, papers were presented by Shelley Krasnow, W. T. Thom, Jr., and Norman Keevil.

In his exposition of "Some Strong Sentiments on Geophysics Education," Shelley Krasnow rebutted several previously expounded ideas. The contention that we are reaching the period of diminishing returns in geophysics, and so should curtail education in that direction, is contradicted by the many and difficult problems facing the profession. Their solution calls for more, not fewer, men. Many fields of geophysical exploration, notably in mining and in the study of underwater areas, remain inadequately exploited, and large expanses of the earth are still unexplored; new methods are to develop, and there is no predicting what the future will bring. Judicious education of the nongeophysicist in the capabilities of the techniques, and the ultimate reduction of the cost of geophysical exploration, should produce an increasing and elastic demand for these methods. While the speaker wholeheartedly agreed to the necessity for thorough grounding in the basic, theoretical subjects, he felt the student should be given some field work and experience on at least

a few typical (but not necessarily modern) instruments, that he might develop "the intuition that is acquired only by actual work." He also made a plea for some instruction in geophysics for geologists and most engineers, and advanced the thesis that the subject should be given as a cultural course to the liberal arts students, to teach them more about the earth we inhabit.

In discussing "Certain Trends in Geophysical Research and Their Bearing on Geophysical Education," W. T. Thom, Jr. showed that it is more confusing than helpful to lump exploratory geophysics, general geophysics, and structural geology together. General geophysics is concerned primarily with conditions pertaining to the physics of the earth as a whole. So, if classic methods of mathematics and physics enable the earth physicists to predict earth conditions 99.5 per cent correctly, the residue is insignificant. To them the earth is virtually lacking in anomalies. To the exploration geophysicists, however, it is just this insignificant, residual anomaly that is important, and the regional effects in which the earth physicist is interested must here be excluded. In structural geology the struggle has been to pass from the stage of a purely descriptive science to that of an engineering one, where something of a quantitative basis can be attained. Only through the developments in geophysical methods is this beginning to be possible.

These three fields started independently, but have spread and overlap, so that there is a coherent field of earth science—but they are not the same thing, and require different training. In addition to the points already covered, the speaker expressed a belief that the training for a man going into the field of earth science should be a blend of cultural and engineering subjects. He should have some work in all three phases of the sciences mentioned. Field work is imperative, and could advantageously involve collecting additional data on critical points by project studies, compile the extensive, nonconfidential information now available, and subject all this material to analysis, using young recruits and students for much of this latter part of the program.

Norman Keevil presented some thoughts on "Geophysics as the Fundamental Basis for Modern Geological Instruction and Research." He showed the interdependence of geology and geophysics in the interpretation of earth processes and in exploring the earth's crust. Geology, trom collections of minerals (mineralogy and crystallography), of rocks (petrography), and of fossils (paleontology and historical geology), proceeds to the observation of earth features, as in areal geology, physiography, structural geology. These steps synthesize into petrology, ore and oil genesis, sedimentation, diastrophism, and earth history. In geophysics the procedure is from physical and chemical studies of rocks and minerals, geothermodynamics and geophysical chemistry, to electrical, magnetic, radioactive, gravitational, and other studies of the earth.

These synthesize into the physical chemistry of magmatic processes, and the physical properties of the crust and interior of the earth, and geochronology. These geological and geophysical steps then combine to culminate in the interpretation of earth processes, and in the exploration of the earth's crust. To improve the instructional preparation for such work, the speaker suggested: retain the fundamental courses and add general engineering; eliminate some descriptive material and whatever can be assimilated by outside reading; introduce courses in geothermodynamics, geophysical chemistry of the earth, a fundamental course in geophysics, and courses in geological engineering. He then outlined a specific curriculum.

A very animated discussion took place, during which so many pertinent points were made that it is impossible to do justice to the subject matter in the space available.

The necessity for agreement as to just what geophysics is was stressed by M. King Hubbert. He pointed out that geology really means the science of the earth, but that subjects such as meteorology have split off and become separate. As a consequence, geology is no longer completely the science of the earth. Geologists, geophysicists and geochemists are all dealing with one problem but talking about it in different languages. He predicted that if the geology departments in our universities are to survive they must mend their ways and adopt the methods of physics and chemistry.

J. B. Macelwane commented that geophysics is neither physics nor geology, if the fields are defined by the methods and points of view used in those fields. The geologist attacks problems from the standpoint of multiple hypotheses, gathers facts, and then breaks down those hypotheses to see whether they will work. The physicist in the laboratory is studying a closed system wherein the apparatus and the factors are all under his control. The geophysicist, however, takes the methods and instruments of the physicist into the field, where his system is not a controlled, closed one. He is dealing with the same system as the geologist, but from the standpoint of physics, and to do so effectively he must have the geologist's outlook too.

L. W. Blau mentioned the advisability of including chemistry, biology, engineering, and economics in a geophysics course, and expressed his agreement with President Hutchins, of the University of Chicago, who has said that the most theoretical education is the most practical one. He said it was a reflection on the geology departments of our universities that those who were trying to effect improvements in the teaching of geology should have to worry about renaming the subject geological engineering. Either geology will have to adopt the methods of measurement of physics and chemistry, and the applications of mathematics, or it will disappear, and geophysics, or geochemistry, or earth science will

grow in its place, based on those methods of measurement. More pertinent, important, and valuable data have been fed into geology from geophysics in the last 16 years than came from geology itself.

Hans Lundberg pointed to the fact that applied geophysics, like the medical art, is an art still, and must be learned by practice. Teach the students the facts, the science of the various branches, in the university, and then let them choose their branches and learn the art in the field. This speaker also commented on the poor attendance of the geologists at these meetings, and compared this situation unfavorably with that obtaining in many countries abroad, where geophysics has been organized in a manner similar to that of the geological surveys, and geophysicists and geologists work together.

Apropos of the poor attendance of geologists at these meetings, Richard M. Field cited, as a contrary example, the American Association of Petroleum Geologists, wherein is "illustrated perfectly the proper relationship of geophysics and physics, chemistry, paleontology, seismology, . . . to the production of one of the greatest industries in this country. Half of this discussion . . . today you would not hear before that Society, and it is because they appreciate, I repeat, the engineering point of view." Because of the impossibility of one man's covering the whole realm of these sciences, we must have the cooperation of experts. Many of the questions discussed at these geophysics sessions of the A.I.M.E. are not on the educational side alone, but are "examples of the interrelation of geophysics to geology, oceanography, meteorology, by the engineering method."

G. P. Woollard emphasized the desirability of expanding the field of geophysics, through education, to provide a greater absorptive capacity for graduates. Even engineers and geologists that have no intention of becoming geophysicists themselves should have courses designed to show them the possibilities and capabilities of the science.

W. R. Chedsey commented from the chair that the growing sentiment among educators is toward a simplification of the educational processes, concentrating on a thorough exposition of fundamentals and training the student to do analytical thinking. Then have a few courses to show ways in which the sciences may be applied.

Helmut Landsberg expressed the opinion that the discussions here would make very good and required reading for some deans and university presidents.

Sidney Paige mentioned the wide adoption of geophysical technique by the U. S. Army Engineers, who are using seismic methods in their study of proposed dam sites.

Sherwin F. Kelly advocated a closer liaison between geophysicists and geologists, because geophysics is part and parcel of geology. He further pointed out the unfortunate distinction between so-called "pure"

and "applied" geophysics, made in the questionnaires. The so-called "pure" aspects, such as meteorology, hydrology, oceanography, are in fact as much "applied" science as exploration geophysics. He left the question open for a better choice of terms to distinguish exploration geophysics from "earth physics," or "general geophysics." He called attention to the fact that the United States possesses no institution giving an adequate preparation in all fields of geophysics, and said this was something the Committee on Geophysics Courses, the American Geophysical Union, and the Society of Exploration Geophysicists ought to consider.

A fitting climax to the discussion was a tribute from Thomas Bates, a graduate student in geology, who has been considering geophysics as a career. He said that he would like to express publicly his appreciation of the work of this Committee. He had been trying for two years to dig out the information from geological meetings, geophysicists, and geologists, that he and other students faced with similar problems finally found given to them at this meeting.

Research Needed in Economic Geology

By T. S. Lovering,* Member A.I.M.E.

(New York Meeting, February, 1940)

An economic geologist is concerned primarily with finding deposits of economic value, estimating their tenor and quantity, their shape and position. Thus the primary problems can largely be lumped under three headings: position, character and quantity; and all three are phases of the broader problem of genesis. Distribution, tenor and quantity are of immediate practical importance and are studied constantly by all who exploit mineral deposits. Although the emphasis changes from year to year, the problems of genesis seem more remote and rarely get the serious consideration from the field man that the other three do. Our understanding of the problems of the genesis of mineral deposits has come largely as a by-product of more evidently utilitarian work in the past, and probably will continue to grow in the same manner for many years to come. However, it might be economically fruitful for the producers to have many competent investigators spending their time chiefly on the problems of genesis of mineral deposits.

Many years ago, when first engaged in work for the United States Geological Survey in Colorado, I expressed a conviction which was just then dawning on me by saying that economic geology was turning out to be 80 per cent structural geology, to which Mr. B. S. Butler replied, "Yes, and more than 80 per cent of structural geology is lithology." At the present time there seems to be a widespread feeling among economic geologists that the problems of position or distribution are largely problems of structure and rock types.

Types of Research

Research in any branch of geology can be conveniently divided into two types: field research and laboratory research. Ideas are usually dug up in the field, pruned or developed in the laboratory, and taken back to the field for proving. For either the first or last step the field data should be as complete as possible, and the structural geology of nearly all our mineral deposits still needs more detailed delineation. In general, this means mapping on larger scales. Ideally the structural map should give a factual picture of all visible effects of stress; there is still room for vast improvement in the technique of presentation. Information that is much

Manuscript received at the office of the Institute June 3, 1940.

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needed but seldom recorded on the average structural map includes the relative strength of fissures and faults, and the direction of displacements in three dimensions. Observations on the rock fabric are very revealing and should always be given where a pattern can be discerned. The more general use of structural contours for showing faults, veins, and dikes is very desirable. However, a considerable part of the difficulty is our inability to locate the structures before they are opened up by exploration. It is here that geophysics can be expected to play a role of increasing importance.

Geophysical methods have been largely employed to delineate structure in the field of petroleum geology and to discover reactive mineral deposits *per se* in the field of metalliferous geology. If geophysics were employed primarily to map structural detail in metalliferous districts, it would give information of the highest value and I hope the time will come when this is a routine part of the study of all mining districts. On the other hand, it would seem desirable that more effort be spent in devising geophysical methods that would show some reaction to petroleum-saturated sands in advance of drilling rather than concentrating almost all our attention on the geophysical investigation of structure.

The changes in structure with depth are of primary concern to all fields of economic geology. There is need for the publication of more details of observation on the changes in fracture and fold patterns brought to light in deep mining. It is in this phase of our study that work in the laboratory may prove of great importance in interpretation. Work at high pressures on the physical constants of rock and a study of the reaction of different types of materials to stress under all-sided pressure is very much needed. Geophysical work of the sort carried on at Harvard by David Griggs and by Francis Birch and his colleagues is of great value and should be encouraged. We may hope that such work will be undertaken in other laboratories in the country and that out of this will grow a body of data that will give dependable information on the fracture patterns characteristic of deformation at varying rates and under different loads.

It seems probable that much can be learned of the three-dimensional fracture patterns characteristic of different conditions of the earth's crust through the technique of scaled models, whose theory has been so ably presented by M. King Hubbert. So far, the use of scaled models has been largely confined to the subordinate role of supporting evidence for a theory reached through some other means. Little has been done in the purely objective study of the fracture patterns characteristic of materials deformed under conditions that represent a progressive change in certain factors, such as time and plasticity. Experimental work based on mathematical similitude would yield interesting and valuable results in this neglected field of earth physics.

The technique of petrofabric analysis, so highly developed by Sanders, should be correlated with experimental work at high pressures and elevated temperatures if it is to be used in interpretative work with confidence. When this is done, its application will probably be of value in many fields. Not until its limitations and its field of usefulness are more clearly understood can we hope to have this tool regarded as an essential part of the equipment of the average economic geologist.

One of the chief problems of the petroleum geologist in the future will be that of changes in stratigraphy in sediments that are hidden from the eye. It is to be hoped that geophysical methods can be developed that will help in the interpretation of such changes. I believe, however, that further research in paleontology, with an especial emphasis on ecology and the environment of deposition, may yield valuable information. Data of this sort need to be correlated with detailed studies of paleogeography for much more restricted groups of rocks than ordinarily are considered. The workers in oceanography have thrown much new light on the problems of sedimentation through their studies on micro-organisms recently deposited in sediments. A continuation of this work and that done by Revelle and Shepard[1] in the Gulf of California is very much needed. There is still much to be learned also of the part played by micro-organisms in diagenesis and in the early degradation of organic material deposited with sediments.

Mineral Deposits

The problem of the character and quantity of a mineral deposit is closely tied up with problems of genesis and localization. A larger body of factual information is needed concerning the changes with depth, not only of the obvious constituents for which the deposits are exploited but also for the minor elements that accompany them. In a few localities detailed analyses have been made over a period of many years and a statistical approach to the problem is possible. This is well illustrated by the excellent work done by Broderick in his studies of the copper deposits of the Calumet and Hecla mine, in Michigan. In most places such statistics are not available; but the physicist has provided a method that makes it easily possible to gain this valuable information. Detailed spectroscopic analyses of typical samples of the mineral deposits from various levels would be required. I purposely use the term "mineral deposits," so that it may be extended to include such things as petroleum. The spectroscopic analyses of oils from various levels in fields that are producing from many sands would be of much interest. Such fields as

[1] R. Revelle: Sediments in the Gulf of California (Abs.). *Bull.* Geol. Soc. Amer. (1939) **50**, 1929.

R. Revelle and F. P. Shepard: Current Measurements near the Sea Bottom (Abs.). *Ibid.*

those in the Bighorn Basin, where green oil is produced from the Cretaceous and black oil from the Paleozoic formations, might yield interesting secrets to a capable spectroscopist.

In mining districts the study of the minor constituents should not be confined to the ores. Probably one of the most fruitful sources of information in the field, and one that has been too much neglected in the past, is the study of wall-rock alteration. It seems very probable that a spectroscopic investigation of the alteration along the vein, both horizontally and vertically, will give a much better picture of the changes to be expected with depth in the ore shoots themselves.

The problem of genesis of mineral deposits is largely a problem of concentration by physical, chemical or biological factors. We first face the problem of the source material and the reason for the initial concentration there. Our instinctive approach to this question is through exploration of field data and, although much suggestive information has been assembled, the answers have been inconclusive. In the field of petroleum geology we may hope to get additional information from the work of the oceanographers. I believe there is need for closer cooperation between those interested in research in petroleum geology and expeditions such as that led by Revelle and Sheppard in the Gulf of California.

The general subject of the source materials of the epigenetic deposits has proved even more controversial than that of source materials of petroleum. Work should be done on the spectroscopic analysis of minor constituents of the igneous rocks suspected of being the ultimate source of different types of deposits. Of course such work should not be confined to the rocks as a whole, but should rather take into account the individual minerals that make up the rocks. This study would be even more revealing if it were carried on through a series of rocks that seem to be established as a definite magmatic differentiation sequence. Such work would need to combine excellent laboratory facilities with an experienced field worker.

The purely chemical approach to problems of genesis is instructive but to the impatient economic geologist, who wishes to have the answers *now*, the serenely scientific spirit of a geochemist that deals in a program of work lasting for 200 years is somewhat irksome. The viewpoint of the chemist is well expressed by G. W. Morey,[2] from whom I quote:

> Modern geological thought is concerned largely with the description and interpretation of the past and present processes of nature in the language of physics and chemistry. . . . It is necessary to consider the multicomponent systems, which is the earth, and the relation between composition and the intensive factors, pressure and temperature, which determine the equilibrium of the relations in all systems. . . .

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[2] G. W. Morey: Thermodynamics of Geochemical Processes. Report of the Interdivisional Committee on Borderland Fields between Geology, Physics and Chemistry, 1937, 47–49. National Research Council, 1938.

In its broadest sense, the problem we set out to solve is incomprehensible; it is the complete thermodynamic description of a system of 93 components, under the widest limits of pressure and temperature . . . but we are justified in isolating small problems that are within the limits of our comprehension for detailed consideration, even if they are in part outside our experimental capabilities. . . . It is necessary that we confine our attention largely to those greater parts within our experimental grasp, otherwise we may depart from experimental science into that uncontrolled speculation which is the negation of science. . . . In any consideration of processes in which composition is a variable, and in which properties change with composition, it is necessary to have as a starting point the complete equilibrium relationships of the system of the components in question. This information is so fundamental that little other systematic work can be carried out when that knowledge is lacking. . . . The researches of the Geophysical Laboratory of the Carnegie Institution have dealt with the thermodynamic description of some of the simplest systems entering into the problem. Even the small beginning which has been made has greatly enlarged our horizon and enabled us to view the whole field in a more understanding manner.

Reasoning from Complex to Simple

The view that work, to be of lasting value, must be of a fundamental character, that it must build from the simple to the complex step by step, is almost universally held. It is therefore with trepidation that I suggest the possibility that much might be gained from a different experimental approach. Although it is admitted that the approach is sound when we reason from the simple to the complex, I believe that it is worth taking time to consider the possibility of reversing this reaction and to discuss whether we may not profitably reason from the complex toward the simple. As stated earlier, our problem is primarily one of processes of concentration. Is it not conceivable that much might be learned from an experiment that began with the 93 elements envisaged by Morey as making up the incomprehensible problem whose solution is ideally his goal? The physical and chemical behavior of such a mixture at temperatures and pressures within the reach of laboratory techniques could be regarded as a chemical model of a crystallizing molten earth. Such a geophysical study would have for its aim the preparation of a "flowsheet," to use the terminology of ore dressing. This flowsheet should show the concentration of the various elements in the different compounds stable at different temperatures and pressures. Such information would be decidedly useful in organizing an attack upon the simpler systems to be studied more thoroughly. It is conceivable that some of the moot questions concerning the mechanism by which solutions leave the magma could be studied in some such manner as that suggested above. The perennial question as to whether the metals leave in gas or liquid, and whether the ore-carrying solutions are acid or alkaline when they leave the magma, could also be investigated.

The character of the transporting agent for different types of ore deposits may differ profoundly. A better understanding of their nature will be greatly forwarded when experimental work has demonstrated the

possibility, or let us say the probability, that solutions carrying the metals come off from material similar to source magmas in a certain physical state and with a certain pH at temperatures and pressures corresponding to those that occur in nature. We may then have a somewhat more substantial foundation for speculating on the physical and chemical changes in the transporting agent that take place in response to differences in pressure and temperature. This in turn should prepare us for a study of the mechanism of precipitation. The amazing similarity of certain types of ore deposits throughout the world suggests that similar chemical processes operated. It is to be hoped that eventually we will find criteria that would allow us to say with some confidence whether precipitation in a given type of deposits is due chiefly to falling temperature, to changes in pressure, to changes in pH through reaction with walls, or to reactions of other sorts.

Rock alteration marks the response of a number of different compounds to changing solutions, and must be correlated with the behavior of the early minerals of the ore deposits. The mineralogy of altered wall rock is a difficult study, and this in part explains the small number of detailed reports on the subject. It is to be hoped that the microscope, the spectroscope, and X-ray equipment will be used in conjunction with chemical analyses to give us more information on the changes engendered by solutions that form mineral deposits. Although I have said mineralogy, I do not mean to exclude the morphology of the crystals that have formed. This is a subject that has been little studied by economic geologists and should repay investigation. The physical chemist may also contribute to our understanding of the reasons for changes in crystal forms and their possible genetic significance. The obvious practical benefits that will flow from a better understanding of these changes may well include a more confident recognition of source channels, ore-shoot zones, and the limits of ore deposition.

Fluid Flow

Much still remains to be learned of that broad field of earth physics that belongs to hydrodynamics. The economic geologist would like to know more concerning the physical factors involved in fluid flow through unhomogeneous aggregates. The loss of temperature during the movement of a hot mineralizing solution through vein matter is one that might be investigated by means of properly constructed models. A mathematical attack on the subject is far too difficult and would involve too many simplifying assumptions to be satisfactory. Nevertheless, the loss of temperature in a moving solution is obviously related to the rate of flow, the size of the openings, the surface of contact, the kind of rock, the distance from the source and the length of time that the solutions have been passing a given place. All these factors seem susceptible to experi-

mental attack. Similarly, the problem of the pressures in a moving solution needs experimental investigation. Large differences in pressure are possible in a heterogeneous aggregate. It would be intensely interesting to study the conditions of pressure and temperature drop in hot solutions passing through a model constructed to duplicate certain structural features that apparently localize ore shoots. It is also desirable that more work be done on the permeability of various types of rocks at high temperatures and pressures and varying confining pressures. This could be regarded as a preliminary step in a more systematic attack on the unsolved chemical problem of replacement.

Petroleum Geochemistry

In the field of petroleum geology, studies on origin have been carried forward for many years by Trask and his co-workers. These studies have to a large extent been exploratory and many valuable data have been obtained. The time now seems ripe for starting detailed chemical work that may be expected to yield information to the petroleum geologist comparable to that given the petrologist by the phase-rule studies of the geophysical laboratory. It would be desirable to have competent organic chemists devote many years to the study of the compounds found in the various petroleums and their supposed source rocks. A critical study of the upper limits of temperatures and pressures that may have obtained in sediments is, of course, vital. A study of hydrostatic pressure alone is hardly sufficient, as it seems quite probable that nonuniform pressure may be a more important factor than all-sided pressure. Further work of the type carried on by McCoy and Tragger a number of years ago seems desirable. It would be of interest to use the McCoy technique on some of the recent sediments deposited in the Gulf of California that have such an unusually high organic content, especially the rocks that were reported to carry as much as 20 per cent. A related matter of some interest would be a study of the change of composition of the brines associated with petroleum. Much still remains to be done in this interesting field and here, too, it is possible that spectroscopic analysis may yield significant results.

DISCUSSION

R. B. Sosman,* Kearny, N. J.—Professor Lovering is not the first to ponder the hopelessness of building up a complete understanding of the complex natural multiple-component silicate systems from two-component and three-component systems in the laboratory. Somewhere in the notebooks at the Geophysical Laboratory in Washington is the record of a melt that I made about 25 years ago, of a synthetic rock having the composition of F. W. Clarke's or R. A. Daly's average igneous rock of the earth's crust. In principle, the study of its course of crystallization might shed some light on the differentiation of the crust into the rock sequences found by the petrologist.

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Actually, it did not yield much, mainly because the oxygen pressure could not be controlled, and the relation of ferric to ferrous iron was far from that of the primeval magma. It is not unlikely that oxygen was completely absent from the primitive atmosphere and represents a late product of differentiation. Nevertheless, something might be accomplished by following up this line of approach, utilizing the improved technique now available.

Ultimately we may hope that the chemists' and physicists' collaboration in the study of the structure of compounds, crystals, and solutions will reduce our knowledge of multiple-component systems to a few simple principles concerning atomic nuclei and shells of electrons. That stage of chemical science has not yet been reached. Until that time, I see no substitute for the painstaking investigation, with thermometer and microscope, of the fields of stability of all the phases in the simple systems.

Professor Lovering speaks of the "loss of temperature in a moving solution." Why not a gain of temperature? The fact that the earth is cold at the surface and hotter in the interior is no reason for assuming that outward-moving liquids are always cooling. It must be remembered that various forms of energy, such as chemical energy, kinetic energy, and energy of volume, are easily converted into heat. I suspect that many a lava has been hotter at its outpouring than it was when it started moving. Furthermore, the filtering out of dissolved substances, and the production of high pressures and of large changes of volume, by the simple passage of a liquid through openings of molecular size, are still to be investigated as sources of geologic phenomena.

Geophysical-geological Study of the São Pedro Area, Brazil

By Mark C. Malamphy,* Member A.I.M.E.

(New York Meeting, February, 1936)

The occurrence of outcrops of bituminous schists and sandstones impregnated with heavy asphaltic petroleum first directed attention to the São Pedro area as a possible source of commercial production of petroleum. Since 1920, more than 20 wells have been drilled in this area. The discovery of oil shows and small quantities of natural gas in some of the early wells led to the continuation of drilling in the hope that more important findings would result. During the early part of 1934, detailed magnetometer observations were made in this area, and a considerable number of torsion balance stations were occupied. The results of these geophysical observations help considerably to clarify the structure of the area and the very considerable amount of geologic information available makes possible a much more exact interpretation of geophysical anomalies.

In presenting the results of these observations, and interpretation of the geophysical data, certain actual case problems are included; i.e., geophysical anomalies due to anomalous masses of which the forms and positions are more or less known. Although anomalies due to igneous intrusions are probably more important to geophysicists in Brazil than in other countries, these data have a certain interest for all geophysicists, particularly because they were obtained in the south magnetic hemisphere, where the lines of magnetic force are directed upward, and in a region of moderately low inclination.

This is considered as a geological-geophysical problem, and only passing mention will be given to the petroleum possibilities of the area.

General Geology

The location and general geology of the São Pedro area are shown in the sketch map of Fig. 1, which is adapted from the geologic map of South Brazil compiled by Oppenheim[1],† who gives the geologic column of South Brazil (São Pedro and vicinity) as follows:

Published by permission of the Directors of the National Department of Mineral Production, Ministerio de Agricultura, Rio de Janeiro. Manuscript received at the office of the Institute Nov. 29, 1935. Issued as T. P. 696, February, 1936.

* Consulting Geophysicist, Serviço de Fomento da Producção Mineral, Rio de Janeiro, Brazil.

† References are at the end of the paper.

Rhetic	São Bento series	Basalt flows of the Serra Geral
		Botucatú sandstones
		Local nonconformity
Upper Triassic		Rio do Rasto group: red and variegated sands and clays
		Nonconformity
Triassic	Passa Dois series	Upper Estrada Nova: sandy clay shales, yellow to variegated
		Presumable discordance
Upper		Lower Estrada Nova: gray shales and sandstones
Permian		Limey and bituminous shales of the Iraty group
Lower	Tubarão series	Gray shales and sandstones of the Palermo and Bonito groups
Permian	Itararé series	Glacial sediments of the Itararé
		Nonconformity
Archean		Basement complex of Archean rocks

This area has been subject to relatively intense erosion since Triassic time and is now an eroded rolling peneplain. The average elevation is about 550 meters above sea level but differences in altitude of more than 100 m. are not uncommon. The basaltic eruptives form the scarp of the Serra Geral, which appears a short distance to the west of the São Pedro area. Their average elevation is of the order of 900 to 950 m. above sea level.

Local Geology

This region has been studied in greater or lesser detail by a number of geologists, several of whom have published the results of their observations. C. W. Washburn[2], concurred with the geologists of the state of São Paulo in considering the occurrence of Corumbatahy formations (local São Paulo designation of upper part of Passa Dois series) surrounded by Triassic sandstones of the São Bento series at Pitanga, as indicating anticlinal structure. Washburn considered the supposed structure of Xarqueada as a western extending nose or spur of the Pitanga anticline. Referring particularly to the São Pedro area (the zone where most of the wells have been drilled), he writes: "They (the wells) lie on the south side of the west plunging Xarqueada nose of the Pitanga

anticline. They are located too far down the south limb of this nose to have any chance of producing oil." (At the time Washburn wrote this report, the logs of only seven wells were available. See Table 1.) The Tucum Estadoal and São Pedro 39 (Cyrino) wells had drilled through a

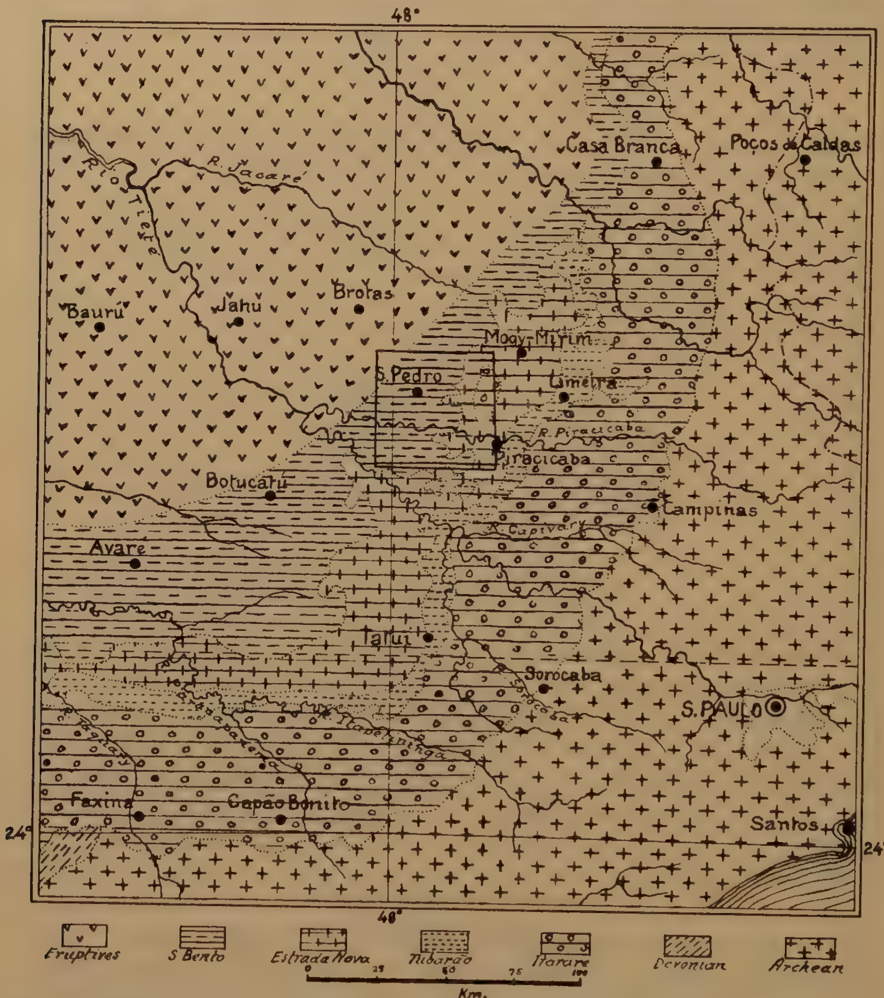


FIG. 1.—SKETCH MAP OF LOCATION AND GENERAL GEOLOGY OF SÃO PEDRO AREA.

considerable thickness of igneous rock, which Washburn considered as representing sills, common to all the formations in the state.

L. F. Moraes Rego[3] studied the geology of this area in considerable detail and made the locations for several of the wells that have been drilled. Utilizing outcrops of "silex" beds, which he considered as indicative of the contact between the Passa Dois and São Bento series, he mapped several domes in the vicinity of the Cascatinha 85, Santo Antonio

90, Xarqueada 81, and Araquá 51 wells. Moraes Rego states that igneous dikes and sills are commonly associated with these domes and admits that some of them may have been formed by intrusive necks, etc. However, apparently he did not consider this as the principal cause of doming and eliminates it from consideration for the larger structures.

Oppenheim studied this area in 1933–34, as well as all South Brazil, and arrived at several interesting conclusions. He explains the tectonic structure of this area as due to the result of the combined actions of faults and igneous intrusions. Some of the faults he considers as older than the intrusions, thereby offering an easy means of entrance to the magma while others are posterior, having resulted from tensional stresses related to the intrusion, or to contraction of the igneous mass through subsequent cooling.

Unfortunately, Washburn did not publish any maps or sections demonstrating his conception of the structure of this area. Moraes Rego speaks of secondary domes but his published report carries only one geologic section applicable to this area (which is reproduced in Fig. 2). Oppenheim, using data obtained by personal observations and the logs of more than 20 wells, published the three sections that are reproduced in Figs. 3, 4 and 5. The locations of the wells of which the logs were used are given in Fig. 6. These logs as classified by Oppenheim are given in Table 1.

Physical Properties of Various Formations

The density and magnetic susceptibility of a number of characteristic samples of the various formations found in this area were determined with what precision available equipment would afford. It is not possible to specify the probable accuracy of these measurements, but probably they are slightly low rather than excessive. The average values determined are given in Table 2.

These data indicate that the greater part of the geophysical anomalies observed will be due to igneous intrusions of diabase, with the relief of the crystalline basement, presumed to underlie the Itararé, playing a subordinate part.

Geophysical Observations

The vertical component of the magnetic field was determined at more than 1000 stations within a total area of approximately 350 sq. km. The horizontal component was measured at more than half of these stations. In addition, magnetic observations at 500-m. to 1000-m. intervals were made along the various roads radiating from this area to distances greater than 100 km. in some directions.

More than 150 torsion balance stations were occupied within the limits of the São Pedro area; at points where the topography was most

favorable rather than at localities where it would be most desirable to know the value of the gravity gradient.

TABLE 1.—*Logs of Wells Drilled in São Pedro Area*

Name of Well	No. of Wells	Elevation of Bases of Formations (Sea Level)[d]							
		T.D.	D.F.	S.B.	E.N.	Iry.	Tub.	Ité.	Diab. Lmts.
Graminha[a]	22	329	490	468	321	284	161[e]		
Kerosene[a]	28	499	536	486	347	312	265	229[e]	
Santa Maria	32	212	540	328[e]					
S. Pedro (Cyrino)[a]	39	478	522	384	242	199	60	44[e]	141 to 74
Tucum	45	147	487	451	341[e]				
Araquá	51	381	492	412	329	241			146 to 111[e]
Graminha	55	469	490	407	279	250	94	21[e]	
Floresta	63	274	505	443	266	231[e]			
Tucum	66	441	488	443	251	235			53 to 47[e]
Floresta	71	372	505	430	262	225	136		136 to 133[e]
Santo Antonio	79	293	528		333	297	235[e]		
Xarqueada	81	768	598	579	392	358	316	−170[e]	
Cascatinha	85	227	510		395	362	322	283[e]	
Santo Antonio	90	473	528		333	301	171	57	57 to 55[e]
Araquá	107	280	492	419	270	212[e]			
Araquá	112	627	492	478	295	223	136	−135[e]	136 to 32
Araquá	115	140	492	418	352[e]				
Tucum Estadoal[a]	1	758	487	432	281	231	66	−271[e]	66 to −8
Pitanga (M. Levi)[a]		460	580					120[e]	
S. Pedro (Balloni II)[b]		1130	543	443	305	272	39	???	76–56 & 39–100 −480 to ???
Araquá C.P.B.[c]		1000?	520	505	375	335	??	??	1000 ? to ???

[a] Logs available to Washburn. [b] Drilling. [c] Data on Araquá C.P.B. is unofficial and incomplete. [d] T.D., total depth; D.F., elevation of derrick floor; S.B., base of São Bento; E.N., base of Estrada Nova; Iry., base of Iraty; Tub., base of Tubarão; Ité., base of Itararé (total depth drilled to in Itararé); Diab. Lmts., elevations of top and bottom of diabase sills, etc., drilled into. [e] Elevation and formation in which well was abandoned.

TABLE 2.—*Density and Magnetic Susceptibility of Rocks of São Pedro Area*

Formation	Density, C.g.s.	Magnetic Susceptibility, C.g.s.
Botucatú	1.95	100×10^{-6}
Estrada Nova	2.10	100
Iraty	2.33	75
Tubarão	2.35	200
Itararé	2.30	200
Diabase (basalt)	2.95	4100
Terra Roxa (decomp. diab.)		1200
Crystalline complex	3.0 ?	2500[a]

[a] Susceptibility of gneiss of crystalline complex measured in Paraná.

Instruments for the photographic recording of the diurnal variation were mounted in an underground chamber near the principal base station

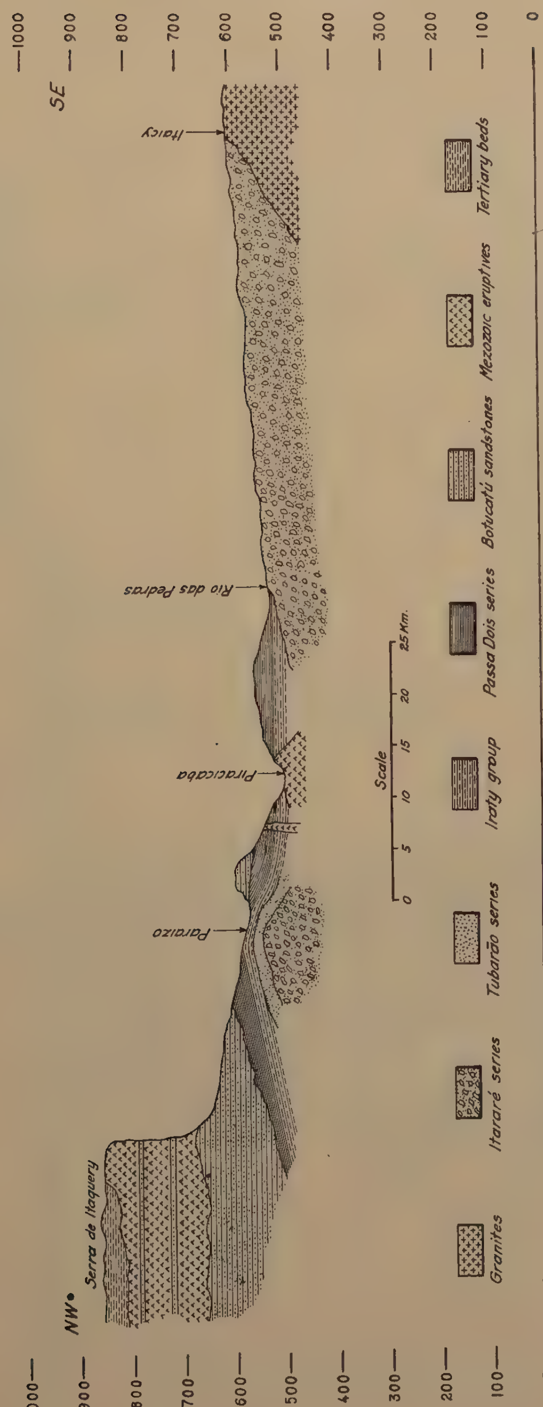


FIG. 2.—GEOLOGIC SECTION OF SANTA CATHARINA SYSTEM, AFTER MORAES REGO[2].

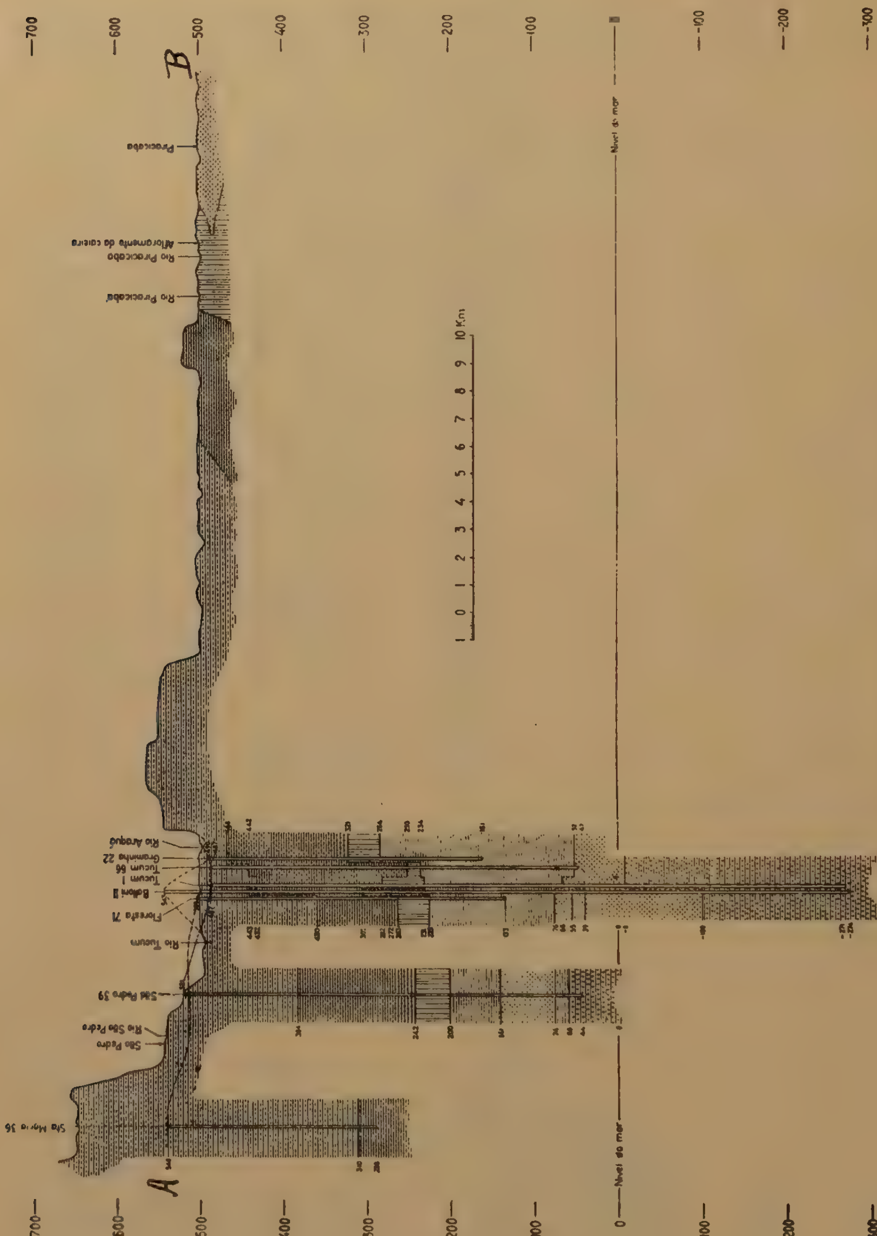


FIG. 3.—GEOLOGIC SECTION AFTER OPPENHEIM[1].

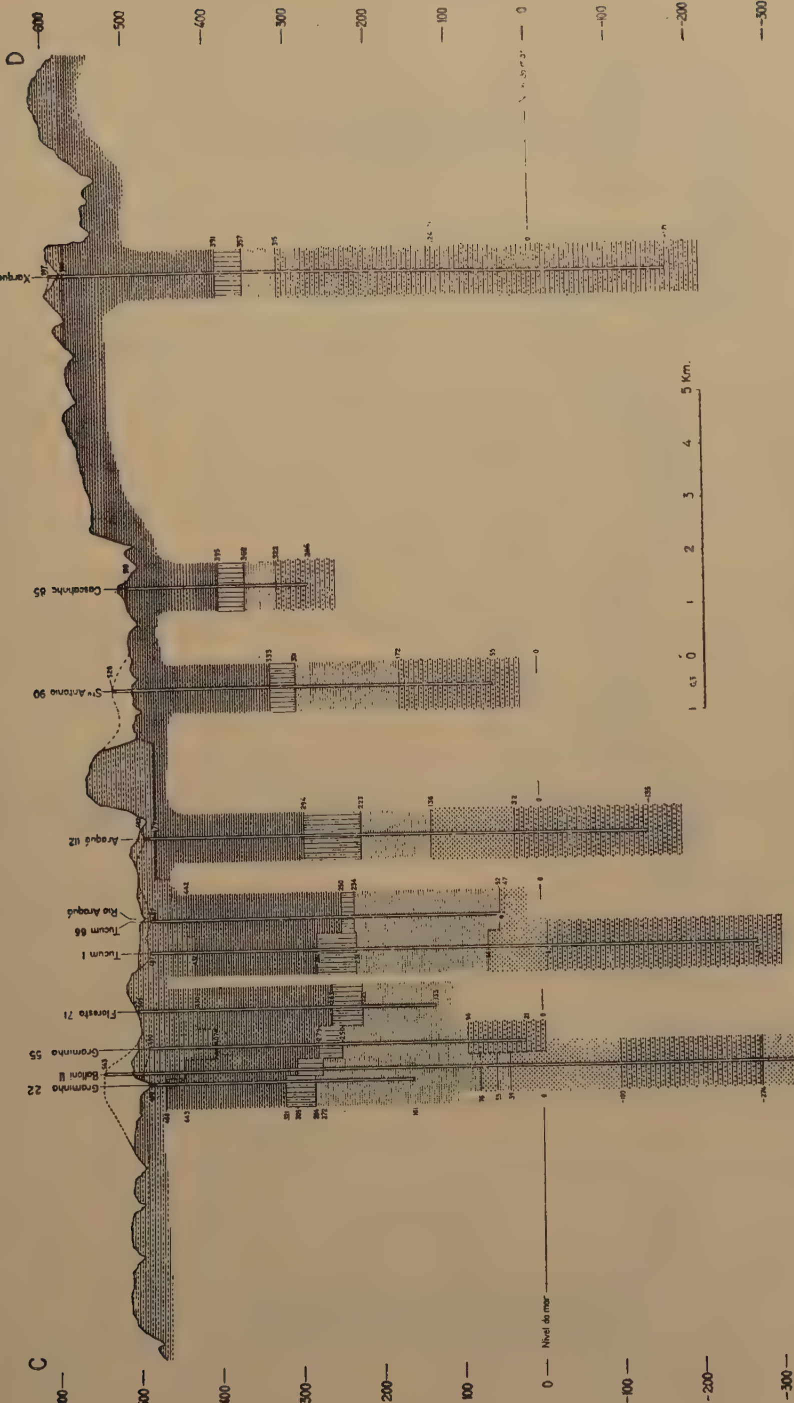


FIG. 4.—GEOLOGIC SECTION AFTER OPPENHEIM[1].

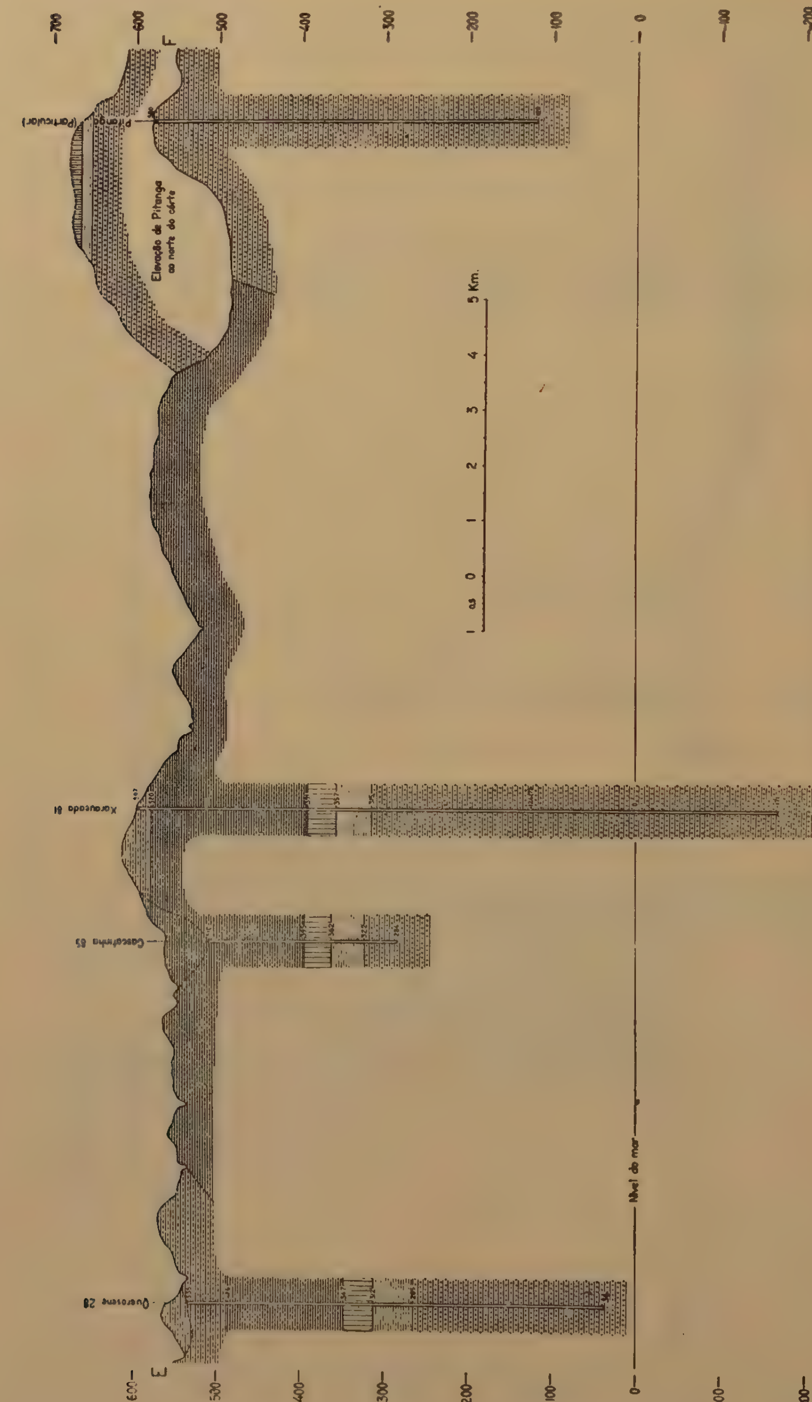


FIG. 5.—GEOLOGIC SECTION AFTER OPPENHEIM[1].

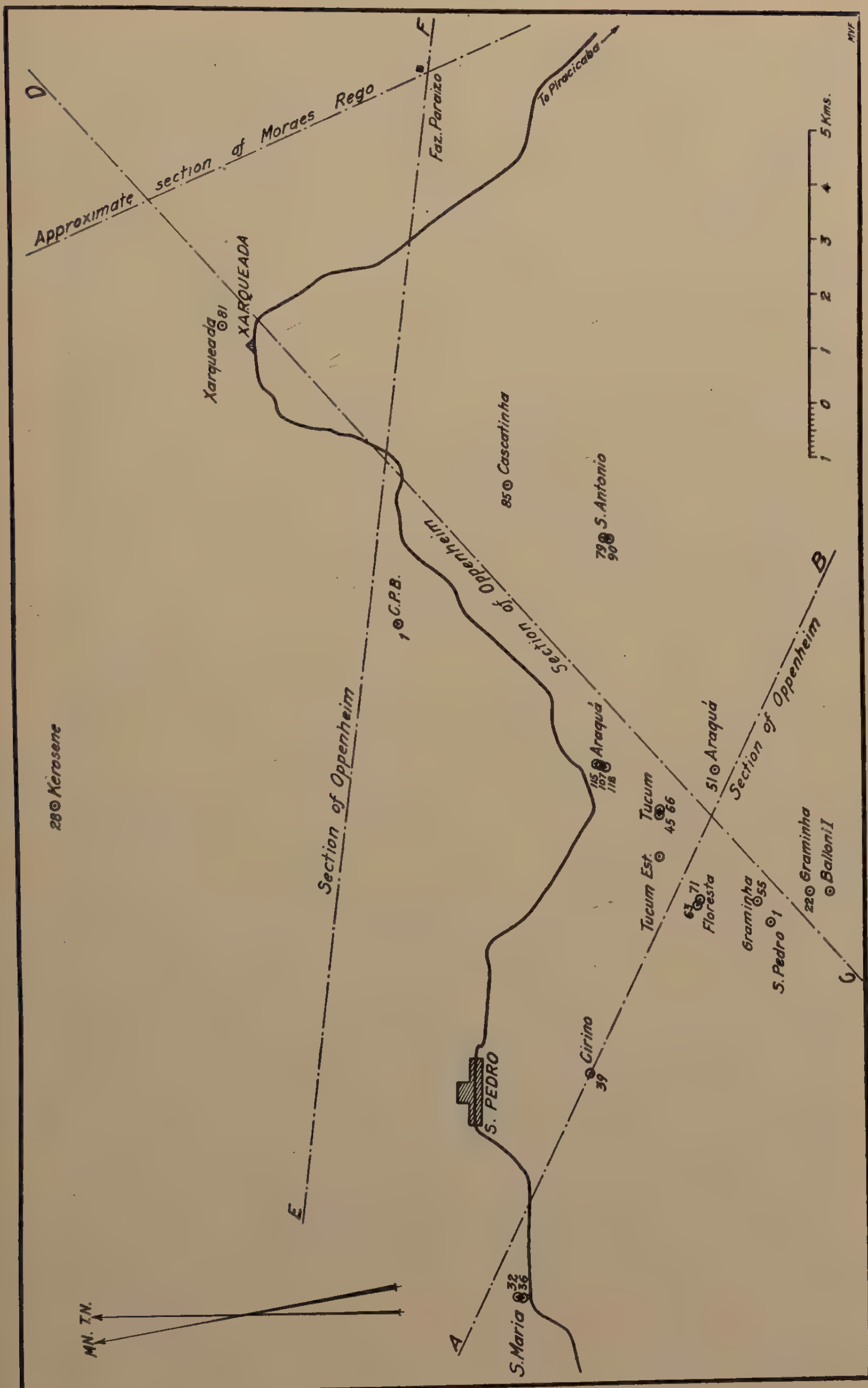


FIG. 6.—LOCATIONS OF WELLS DRILLED IN SÃO PEDRO AREA.

at São Pedro. Temperature coefficients of the field instruments were determined with considerable care and the base station was normally repeated two or more times per day. Agreements obtained on these "repeats" indicate that the values obtained for the vertical intensity are probably correct to within less than 5 gammas, while the horizontal component measurements are seldom in error by as much as 10 gammas.

Terrain corrections for the torsion balance stations were carried out to the 100-m. radius. Normally, 8 azimuths were used, but in unfavorable localities 16 azimuths calculations were sometimes used and in a few instances the correction zone was extended beyond the 100-m. radius. We believe the gradients are accurate to within about 3 Eötvös units, but the error in curvature values is probably much larger.

In this survey, the magnetic method was used as a primary method, the torsion balance data being reserved to confirm the magnetic anomalies and to give an indication of the amount of displacement that might be expected in the magnetic anomalies. Anomalies in the vicinity of wells were first interpreted, free use being made of the logs available. Later, the principles thus established were extended to areas where no wells have been drilled.

Fig. 7 shows iso-anomalies in vertical intensity of the São Pedro area. The field observations are all referred to the base station at São Pedro, which was given an arbitrary value of 100 gammas, and all observations have been corrected for the normal variation in intensity with latitude.

Fig. 8 gives a magnetic profile from Santa Maria to Piracicaba, passing through São Pedro and Xarqueada. To Xarqueada this profile follows an approximately west to east direction, veering slightly to the north. From Xarqueada to Piracicaba, the direction is essentially southeast. A correction for the normal variation in intensity with latitude has been applied to these data. The straight line drawn through the graph represents the normal regional variation in intensity. It is the zero anomaly line for the local anomalies, maxima and minima, due to igneous intrusions. The fact that there is a gradual increase in intensity to the east and north is in itself an anomaly, because correction has already been made for the normal variation with latitude. We believe that this general trend represents the gradual dip of the crystalline basement to the south-southwest.

Fig. 9 shows the gravity gradient and differential curvature observed at a number of stations within the area. Although the station density and accuracy of these gravity observations are not sufficient base for accurate calculations, they are very useful for the interpretation of the magnetic anomalies. Igneous masses may produce both positive and negative magnetic anomalies but can produce only positive gravity anomalies.

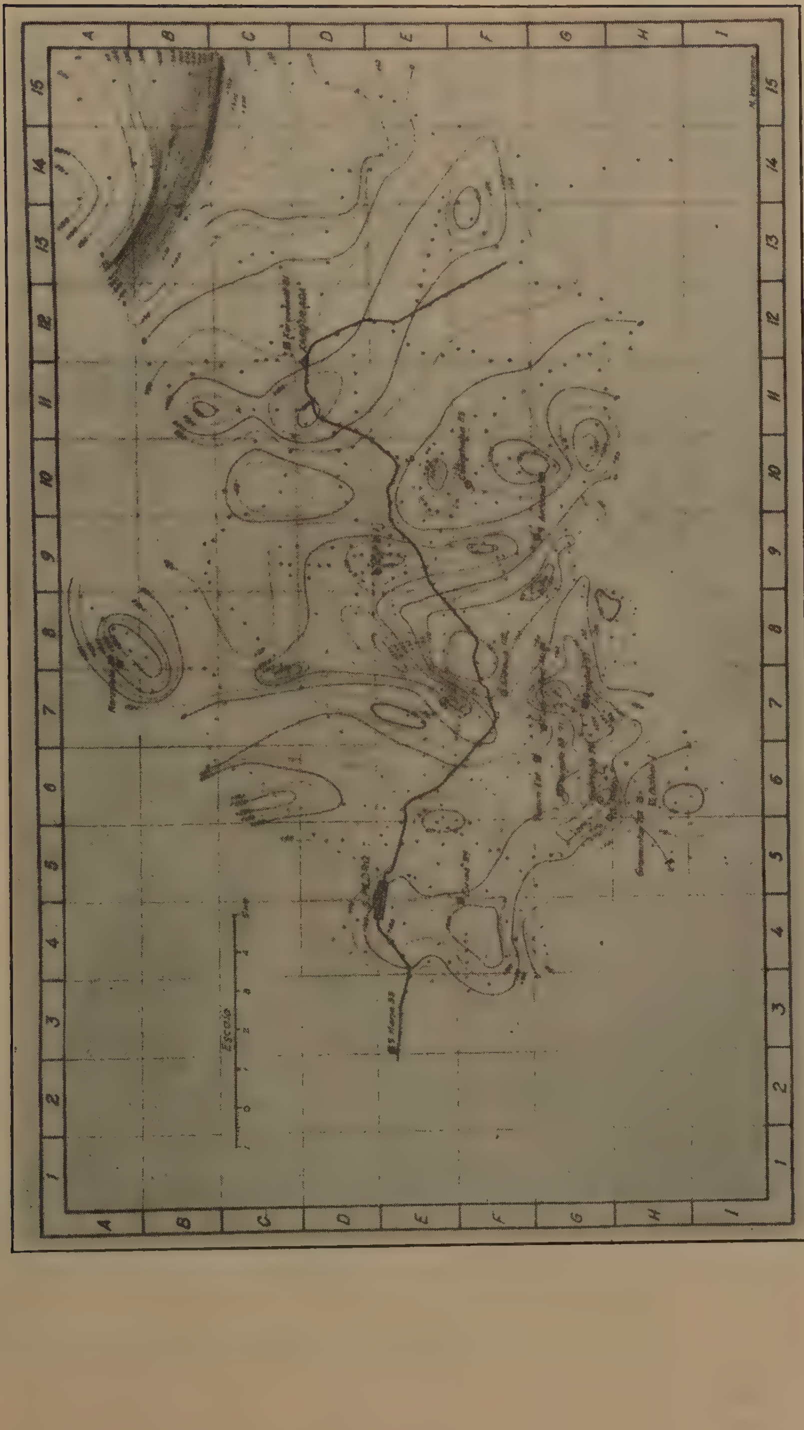


Fig. 7.—Iso-anomaly map of vertical intensity in São Pedro area.

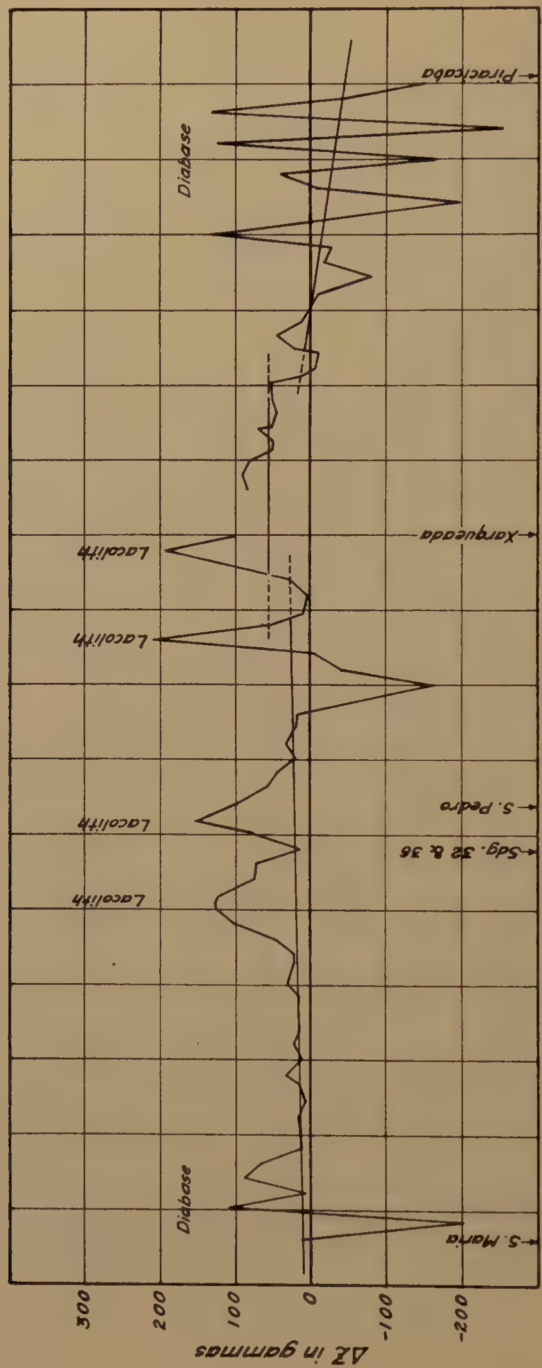
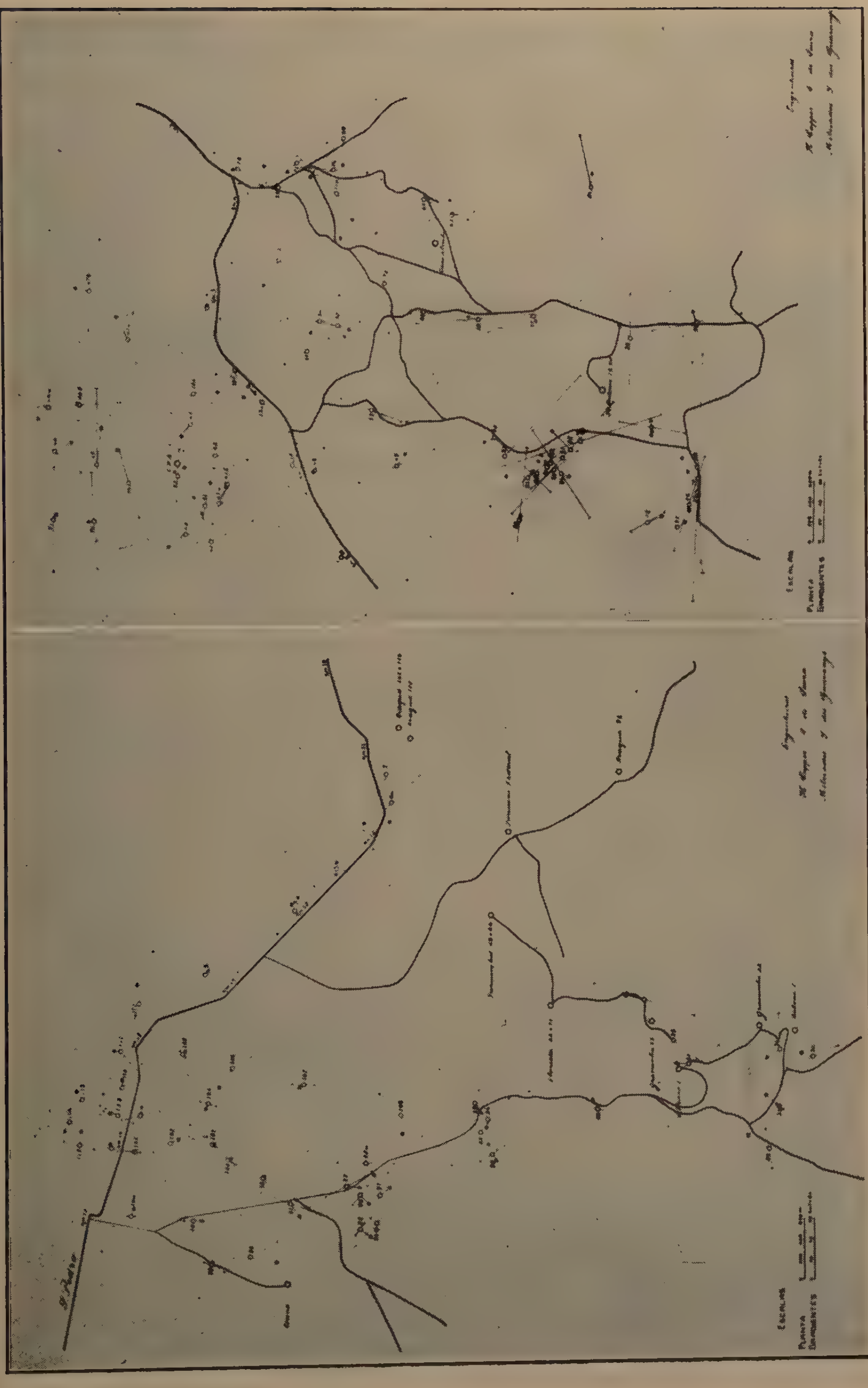


FIG. 8.—MAGNETIC PROFILE FROM SANTA MARIA TO PIRACICABA, PASSING THROUGH SÃO PEDRO AND XARQUEADA.



a b

FIG. 9.—GRADIENT AND CURVATURE VALUES OBSERVED.

Quantitative Calculations

Before interpreting generally the subsurface structure, it is interesting to consider the results of various quantitative calculations intended to show possible distributions of masses that might have produced the observed anomalies, or, at least, the depth to the magnetic poles. For the purpose of these calculations, it was considered preferable to construct intensity curves from the iso-anomaly map of Fig. 7, rather than to attempt to project stations observed at irregular intervals into a line normal to the strike. The equal intensity lines have been adjusted to observed values at a great number of stations and consequently are less liable to accidental errors, either observational or due to minor irregularities in the subsoil. Four curves constructed by this method from the iso-anomaly lines of Fig. 7 are presented in Fig. 10. The sections are taken normal to the strike, and through the centers of the disturbances as indicated by the iso-anomaly curves. The presumed normal value of intensity (zero anomaly) was taken from the curve of Fig. 8, and other similar profiles, projecting or interpolating where necessary. On the basis of these curves, the depths to the magnetic poles were calculated (Table 3), using the various formulas given by Heiland in his discussion of Eve's paper[4]. His rules for calculating the depth of the magnetic pole are all based on the form of the vertical anomaly.

Table 3.—*Comparison of Calculated Depths of Poles with Actual Depth in Well*

Section	Well	Well No.	Depth to Diabase	Depths Calculated by Rules Nos.		
				1	2	3
A-A	Sto. Ant.	90	470	360	400	420
B-B	Araquá	51	346	300	300	385
C-C	Kerosene	28		875	900	980 T.D. 500 m.
D-D	C.P.B.	1	1000 ?	800	750	790

There were considerably more data available in the region of section E-E than elsewhere and our calculations therefore are more elaborate. A number of torsion balance stations in this area made it possible to construct a gradient curve which, although based on an insufficient number of stations, some of which are at some distance from the section, still permits us to attempt to make quantitative calculations. Of still greater importance are the logs of the four wells drilled over this laccolith, two of which penetrated the diabase while the others serve as negative controls in that they give a minimum depth limit. The Balloni II (S. Pedro No. 1) well drilled through three masses of igneous rock having

a total thickness of more than 160 m. It is reported that this well entered into a fourth mass of diabase below 1000 m. The depth to the first mass was 467 m. but, since the well is located on a small hill about

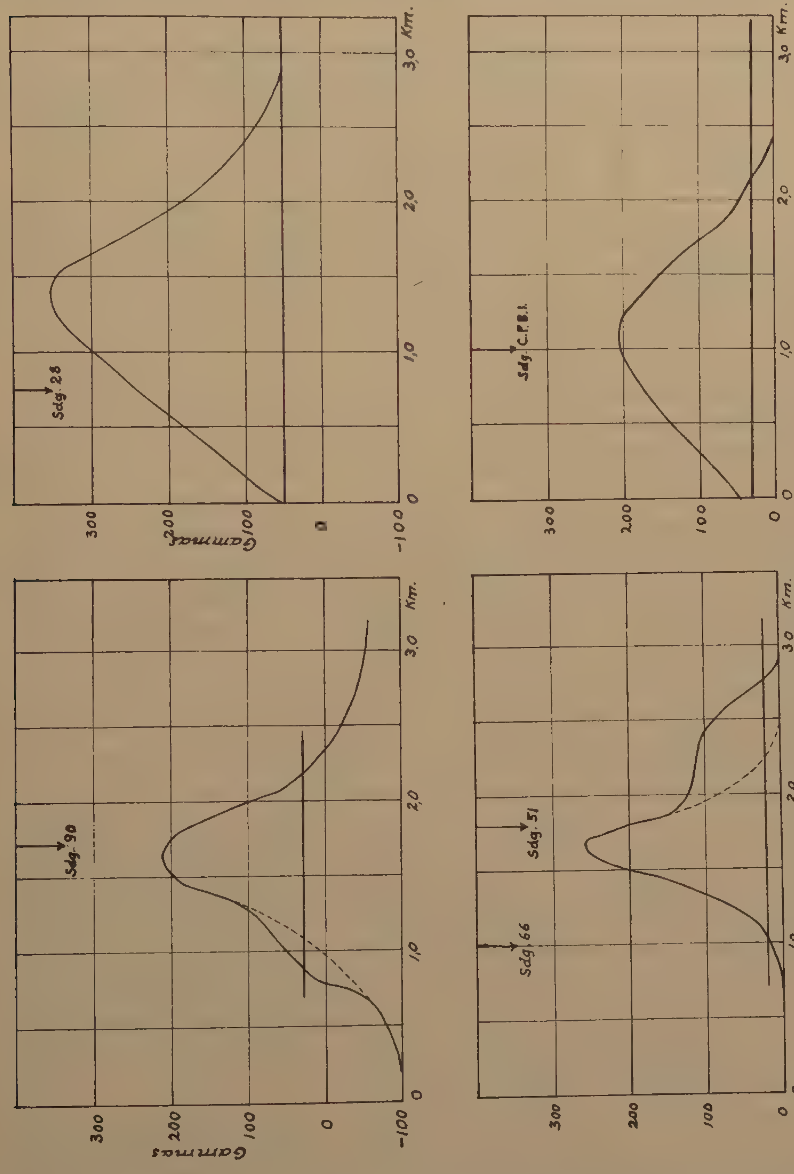


FIG. 10.—RECONSTRUCTED MAGNETIC SECTIONS THROUGH ANOMALIES.

40 m. higher than the average elevation of the surrounding country, the depth to diabase may be assumed to be approximately 425 m.; i.e., 75 m. above sea level. The magnetic and gravimetric anomalies observed

over this intrusive mass are given in Fig. 11, which shows also calculated anomalies and our interpretation of the probable form of the intrusive.

The depths of the magnetic poles were calculated by six of the formulas given by Heiland (Table 4). The depths calculated for the positive

TABLE 4.—*Calculated Depths of Magnetic Poles*

Rule No.	Calculated Depths		Anomaly Used
	Positive Pole	Negative Pole	
1	330	300	Z
2	375	330	Z
3	350	280?	Z
5	400	300	Z and H
6	430	480	H
7	420	540	Z and H (vectors)
Average calculated depth.........	385	375	
Actual depth (corrected)..........	425		

pole are more consistent and more accurate than those for the negative pole, which is to be expected because the positive anomaly is the more important and better studied anomaly. Furthermore, depths calculated from the combined horizontal and vertical anomalies are better than those calculated from the vertical alone, which consistently gives depths that are too shallow, as was also noted in the examples given in Table 3.

On the basis of the positions of the magnetic poles as determined by disturbance vectors (Rule No. 6), and the logs of the various wells, a presumed section through the intrusive was drawn and theoretical anomalies were calculated for this mass distribution. The assumed form is shown in Fig. 9, as are also the theoretical curves calculated.

Graphs for the calculation of the gradient and curvature functions of gravity were constructed in a form similar to those described by Barton[5]. These graphs were for prisms of infinite length, but we do not believe that any great errors were introduced by the finite length of the intrusive. A density difference of 0.7 was used for the diabase and the gradient was calculated by Barton's method. No attempt was made to calculate the curvature because the irregular topography makes the observed values entirely too doubtful for quantitative work.

The magnetic anomalies were calculated from the same gravity graphs, using the procedure described by Haalck[6]. The forms of the resulting curves were similar to the observed anomalies but a considerable discrepancy in the magnitude was at once apparent, the calculated anomalies being only ¼ as large as the observed. The agreement between the observed and calculated gradient curves make it impossible for us to

increase the size of the mass and although our susceptibilities may be low, it is impossible to increase them by a factor of four. It seems more reasonable to assume that the highly magnetic igneous rocks are possessed

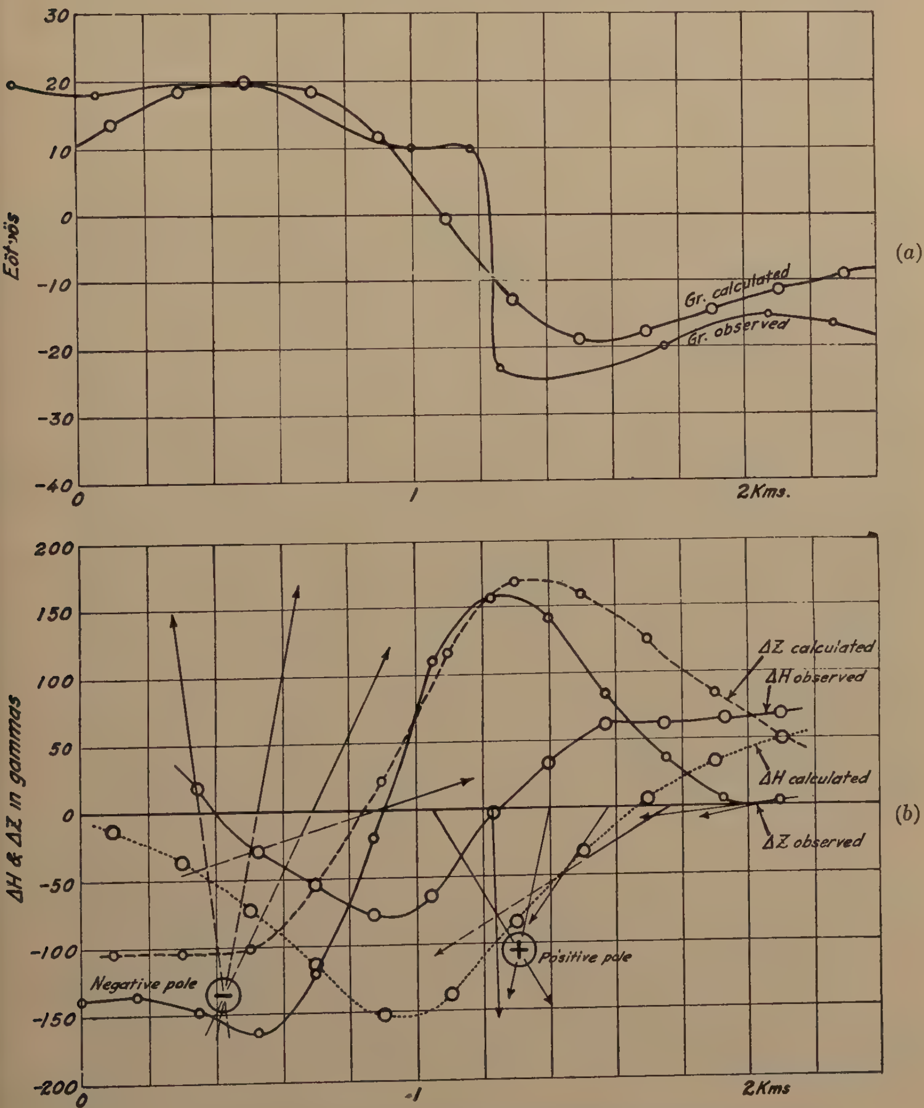


FIG. 11.—OBSERVED AND CALCULATED ANOMALIES OVER INTRUSIVE MASS.

of magnetic properties in excess of those that normally would be induced by the earth's magnetic field alone. These same phenomena have been observed on diabase dikes in Paraná. The calculated anomalies plotted on the figure have been multiplied by four to facilitate comparison.

With patience, we might have arrived at a slightly different form for the intrusive mass, which would have given curves in somewhat better agreement with the observed values. However, there was no point in such procedure because the data presented here are evident proof of the irregular laccolithic form of the intrusives. It is also evident that their form may change considerably in small distances.

The sharp reversal of the gradient may be explained as due to a small dike, which projects from the laccolith and reached the surface. Evidence of the existence of such a dike is found in the presence of "terra roxa" resulting from the decomposition of the diabase. But, since the terrain is under cultivation, it is impossible to locate the dike with precision.

(c)

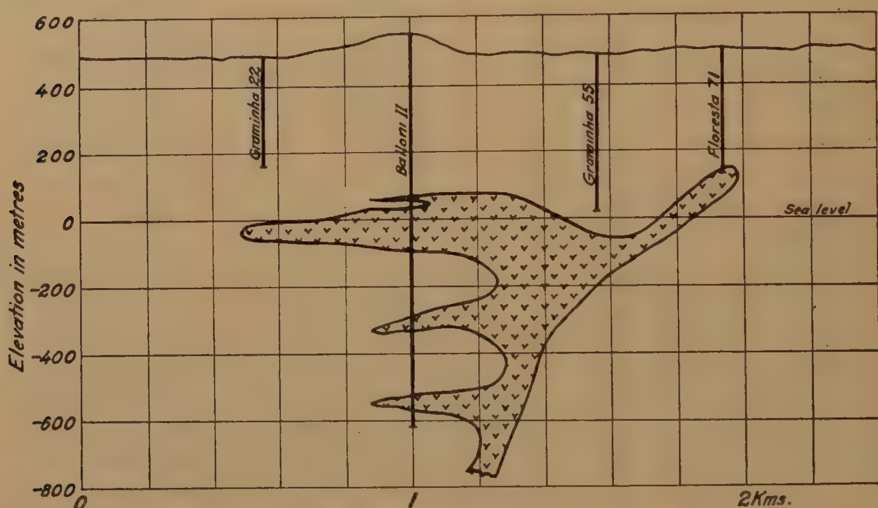


Fig. 11.—Continued.

These quantitative calculations are interesting because they represent a comparison of theoretical and observed anomalies in the south magnetic hemisphere where the inclination is low (20°) and where the masses causing the anomalies are at moderate depths and very irregular in form. All things considered, the agreement between the theoretical and observed anomalies is as good as might be expected under such conditions, and sufficient to allow us to have faith in our qualitative interpretations of similar anomalies.

General Interpretation

Fig. 12 is a combined map showing the results of the magnetic and gravimetric surveys within the central part of the area studied. The closed magnetic anomalies, confirmed by gravity gradients, may be accepted as conclusive evidence of the presence of igneous masses of

laccolithic form. Although it is probable that these masses are connected at great depth, some of the resultant laccoliths are entirely independent from the others at the depth of their greatest development while others are apparently, but not necessarily, connected. These igneous masses are the principal cause of the geophysical anomalies observed in this area. The positive anomalies (magnetic) represent either the northern extremity of the mass or the feeder dike, or both when these are nearly coincident. Some of the negative anomalies represent

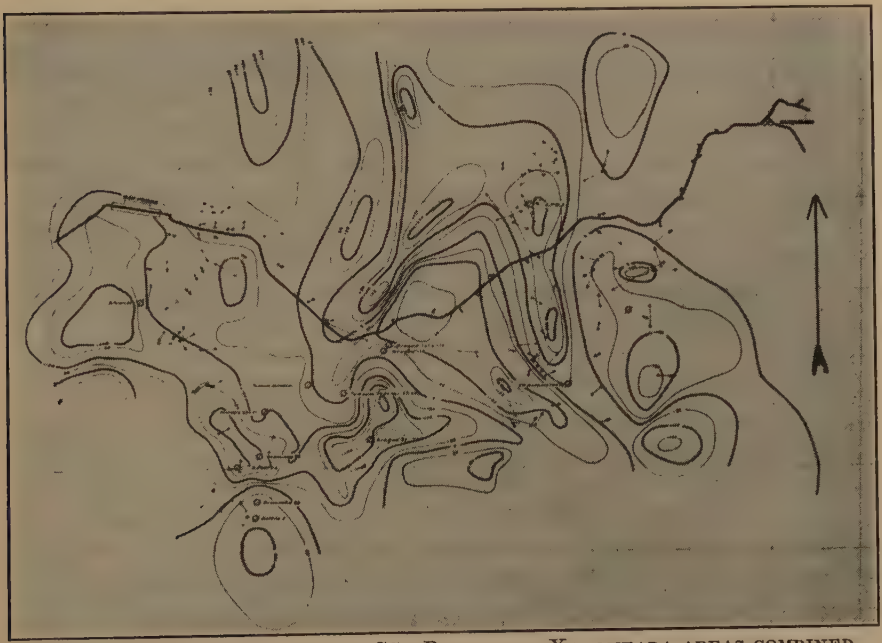


FIG. 12.—GEOPHYSICAL MAP OF SÃO PEDRO AND XARQUEADA AREAS COMBINED.

induced negative polarity on the southern extremities of such masses, but the greater part are due to the absorption of the magnetic lines of force by surrounding magnetically susceptible rocks and therefore are apparent or reflected anomalies.

The laccolithic form presumed for the mass giving rise to the anomalies presented in Fig. 11 may be taken as typical for the area. It is understood that each particular laccolith will have its individual form, depending on the physical resistance of the formations into which it has been intruded, and that these forms will be susceptible to considerable variations. Obviously, the intrusion of a mass of igneous rock of this size will naturally result in a distortion of the overlying sediments, forming a structure that will be at least approximately domal in form. Such structures might well be called pseudo-structures to distinguish them

from somewhat similar structures due to other geological phenomena. Our geophysical data show that most, if not all, of the domes mapped by Moraes Rego are coincident with magnetic anomalies indicating laccoliths that might be taken as adequate explanation of the differences in elevation of the various key beds revealed by the well logs.

However, the Santo Antonio wells are structurally low, compared with the C.P.B. 1 and Cascatinha 85 wells, according to the well logs. Magnetically, the Santo Antonio wells are at the southern extremity of a positive anomaly, which we interpret as a laccolith. The Cascatinha well is in a magnetic depression. In order to correlate these facts it may be presumed either that there has been a fault between these two wells and that the igneous magma has intruded into the sediments on the downthrow side of the fault or that the negative anomaly in the vicinity of the Cascatinha well indicates another mass of igneous rock at a greater depth than was reached in this well, which has been polarized abnormally. Referring to Fig. 12, it will be found that the gravitational gradients offer little help in the determination of this problem. A number of gradients may be interpreted as indicating the presence of a denser mass; also, there are a number of gradients flying in almost directly opposite directions. On the whole, the data obtained in this locality indicate the possibility of a fault and a number of small dikes intruded through the formations on the upthrow side. This interpretation is based on the combined geological and geophysical data.

Small gradients, showing a reversal a short distance to the southwest of the town of Xarqueada, confirm the large magnetic anomaly of this locality. Presumably this represents a large laccolith at considerable depth and is the cause of the structural uplift at Paraizo shown in the geologic section of Moraes Rego (Fig. 2). This laccolith may also be the explanation of what Washburn termed the "westward-projecting spur of the Pitanga anticline."

Tectonics of the Area

Considering all the geological and geophysical data available, the following conclusions may be drawn relative to the structure of the São Pedro-Xarqueada area:

1. Igneous intrusions in the form of laccoliths with associated sills and dikes have been the main cause of the secondary domes mapped by the geologists.

2. The faults presumed to exist by Oppenheim are not clearly evident in the geophysical data, although there is indefinite evidence of a fault between the Santo Antonio and Cascatinha wells. Faults that affected only the Permian and Triassic formations would hardly be expected to produce geophysical anomalies of sufficient magnitude to be recognized over and above the irregular anomalies due to the igneous intrusions.

3. It seems probable that the basement rocks are dipping slightly to the southwest, but it is believed that these rocks will be found only at depths greater than 2000 meters.

Petroleum Geology of the Area

Oppenheim has already discussed the petroleum geology of all South Brazil and there is no need for details here. Suffice to say that the pseudo-structures resulting from the laccolithic intrusions would be entirely adequate to cause the accumulation of petroleum provided adequate source beds existed. A number of the wells drilled have been located advantageously from the structural viewpoint but none has proved productive.

Geophysically, it seems probable that older sediments underlie the Itararé glacial beds. The character of such sediments is an enigma. If they were capable of producing petroleum, it seems at least possible that some migration would have occurred along the contact of the diabase feeder dikes and that those structurally well located wells that drilled through the laccoliths would have had important shows of petroleum. This not being the case, we may presume that the probability of deeper petroliferous beds is very slight.

Since the secondary domes are restricted to the upper formations which are already proved to be nonpetroliferous, surface geological data will be useless in the search for deeper structures. The magnetic and torsion balance methods may be expected to outline only the masses of igneous rock that have caused the secondary structures. It seems barely possible that the reflection seismic method might provide a means of working through the igneous rocks and contouring the underlying formations. However, it is believed that the region does not offer sufficient prospects to warrant the expenditure of additional money in prospecting. It would be far preferable to prospect in an area where it might not be necessary to drill through hundreds of meters of igneous rocks.

Conclusions

These extended geophysical observations have proved that the secondary domes observed by the geologists are due to laccoliths of igneous rocks, some of which were drilled into, or through, in many of the tests drilled in this area. The quantitative depth calculations based on the magnetic anomalies show that depths consistently too shallow were obtained for masses found at moderate depths. On the basis of the geological and geophysical data available, it is concluded that there is little probability of discovering petroleum in commercial quantities in this region.

Acknowledgments

The author wishes to express his appreciation to the Director of the National Department of Mineral Production for permission to publish this report. Special mention must be made to Dr. H. Capper A. de Souza and Dr. Irnack C. do Amaral, respectively chiefs of the torsion balance and magnetometer field parties under the general direction of the author. Thanks are also due their assistants Dr. Decio S. Oddone, Dr. M. Y. Guaranys and Dr. Gabriel M. A. Oliveira, also Dr. L. F. Moraes Rego, Dr. V. E. Oppenheim and Dr. Glycon de Paiva, for helpful information.

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DISCUSSION

J. G. Koenigsberger, Baden, Germany.—The author has found (p. 111) that the susceptibilities he measured in the laboratory are about one-fourth of those necessary to explain the anomalies caused by the same rocks in nature. If the field employed was higher than 10 c.g.s., K in the earth field could only be lower. The method for measuring K is not mentioned.

These (volume) susceptibilities K of the rocks in the São Pedro area given by the author are exceptionally high. Mr. Malamphy has found for the sediments (Botucatu—Itaracé) an average of about 1.5×10^{-4} for K, for gneiss of the crystalline complex, 2.5×10^{-3}; for the diabase, to which the anomalies must be ascribed, 4.1×10^{-3}. G. Grenet[7] measured 16 crystalline schists. The average of the g susceptibility is about 2×10^{-4}, K therefore about 5×10^{-4}. In the central Swiss Alps I have found on about 100 specimens $K = 3 \times 10^{-4}$ (S. Pedro, 2.5×10^{-3}); for diabase basalt, according to Grenet, about 8×10^{-4}; $K = 2.3 \times 10^{-3}$; my own measurement on German diabase was $K = 1 \times 10^{-3}$ (S. Pedro, 4.1×10^{-3}). According to G. Grenet, average of 11 sediments was $K = 5 \times 10^{-5}$; according to E. Rothé, the average of about 30 sediments was $K = 3 \times 10^{-5}$ (S. Pedro area, 1.5×10^{-4}). Therefore it is not probable that the values found by Mr. Malamphy are too high, and I agree with his conclusion (p. 127) "that the highly magnetic igneous rocks are possessed of magnetic properties in excess of those that would normally be induced

[7] G. Grenet: *Ann. Phys.* (1930).

by the earth's magnetic field alone." It may be mentioned that the residual natural magnetism ζrn of relatively young igneous rock is often much larger, sometimes fivefold and more, than the inductive force $KH = \zeta\kappa$. It would be interesting to know approximately the geologic age of these diabase rocks. It is easy to measure approximately with any magnetic variometer used for the field work the natural residual magnetism by the magnetic moment of a piece of rock.

Magnetic Anomalies and Igneous Rocks

By Mark C. Malamphy,* Member A.I.M.E., Irnack C. do Amaral† and Decio S. Oddone,† Junior Members

(New York Meeting, February, 1936)

Most igneous rocks, and particularly those of the basic type, contain relatively high percentages of magnetite and other iron oxides, which give them moderately high magnetic susceptibilities and make them responsible for an important percentage of the magnetic anomalies observed during the course of geophysical surveys by the magnetic method.

The irregular form and high susceptibility of the igneous rocks make them readily polarizable and the position and strength of the magnetic poles formed are not always explicable by theoretical calculations of anomalies due to certain assumed forms in the normal magnetic field of the area under study.

During the last four years our field work has permitted us to make detailed observations of anomalies due to igneous rocks in all the states of south Brazil. The form and position of the igneous rock at some places were fairly accurately known, which made it possible to calculate theoretical anomalies and compare them with the observed data. Some of the data we obtained have been published and additional cases are now awaiting publication. In this paper, we will present a few specific cases where the geology is moderately well known and which we believe to be of considerable value as representing typical magnetic anomalies in the south magnetic hemisphere, where the inclination of the magnetic field is very low.

Although the vertical intensity is negative in the Southern Hemisphere, we have adopted the convention of plotting increase in intensity (negative) as a positive anomaly; i.e., above the x axis. This convention facilitates comparison with anomalies in the north.

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Magnetic observations have been made over tabular masses in the form of dikes and sills, and over massive intrusions in the form of laccoliths. We also have data referring to intrusive masses of syenite of batholithic proportions.

The particular examples to be presented were selected from a great number of anomalies because of certain interesting features that will be explained in the text. Admitting that there is a great variation among the magnetic anomalies recorded over apparently similar masses, we believe that these examples can be considered as typical for the various forms of intrusives to which they are related.

Normal Magnetic Field in South Brazil

The form of a magnetic anomaly due to induced magnetism in a susceptible body is dependent on the form and position of the body and the direction and magnitude of the earth's magnetic field. The examples to be presented were observed in the states of São Paulo, Paraná and Santa Catharina. The normal magnetic field in this area may be expressed by the following limits:

Horizontal intensity	23,500 to 24,500 gammas
Vertical intensity	−9,000 to −6,700 gammas
Inclination	21° S. to 15° S.
Declination	8° W. to 11° W.

When quantitative calculations were made, the values assumed for the normal field are given.

These normal intensity values have been deduced by interpolation from the observations of the Carnegie Institute of Washington and the National Observatory of Brazil, and confirmed by relative measurements with field magnetometers which had previously been calibrated at the Observatory of Vassouras.

Anomalies Due to Diabase Dikes

The vertical, or near-vertical, dike is the simplest and perhaps the most common form assumed by intrusive rocks, particularly the less viscous basic rocks. Dikes are very common in all southern Brazil and we have observed the magnetic anomalies due to this type of intrusives in numerous localities.

In Fig. 1 a large vertical dike of diabase cutting the property of the Ferraria gold mine, near Curityba, state of Paraná, is clearly indicated. More detailed observations were made along the section *A* where the cut in the road permitted observation of the contact of the dike with the gneissic country rock. Electrical resistivity measurements showed the zone of decomposition to extend to a depth of approxi-

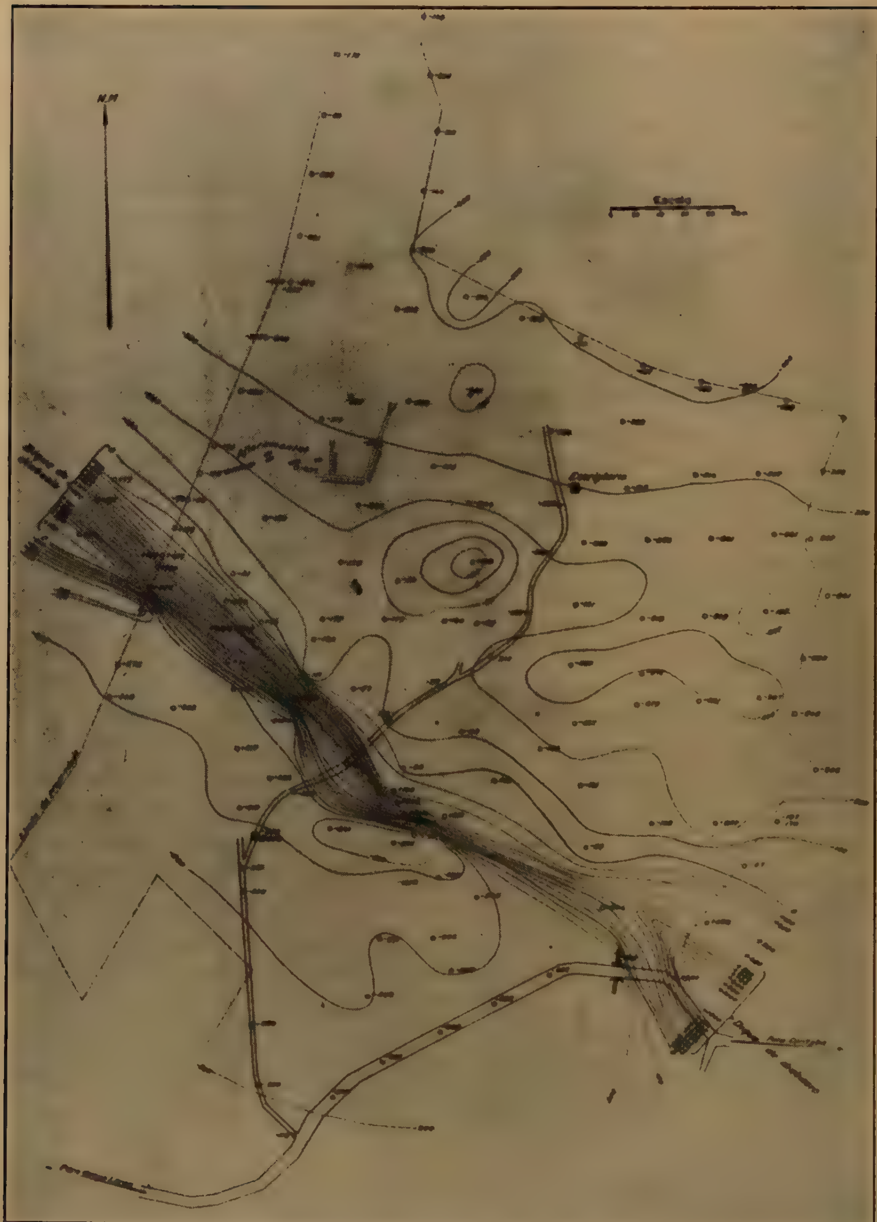

FIG. 1.—ISO-ANOMALY MAP OF THE FERRARIA MINE, CURITYBA, PARANÁ.

mately 30 meters. The contacts of the dike are shown in Figs. 2 and 3. The dike is almost vertical, with a thickness of 20 meters.

Observed and calculated magnetic profiles over this dike are shown in Fig. 4; also another profile observed several kilometers to the southeast and geologic sections for comparison with the observed anomalies.

The theoretical anomaly was calculated by the method described by Haalck[1],* using torsion balance calculation graphs constructed from data published by Barton[2]. The theoretical curve agrees quite well with the observed data as to form of the anomaly and position of the poles. However, in magnitude, the calculated values were only ¼ the observed

FIG. 2.—NORTHERN CONTACT OF DIABASE DIKE AND GNEISS. Note banding in gneiss and form of decomposition of diabase.

values. The magnetic susceptibility of the diabase was determined to be 4200×10^{-6} c.g.s. and might actually be slightly higher. But it is impossible to admit a susceptibility of over $16{,}000 \times 10^{-6}$ c.g.s., which would be necessary to bring our calculated and observed anomalies into complete agreement. For plotting purposes, the calculated anomalies have been multiplied by an arbitrary factor of four.

As shown by the curves presented, a moderately intense negative pole is found on the south face of the dike and a more intense positive pole on the north face. The thickness of the dike is apparently slightly less than the distance between the positive and negative poles; i.e., maximum and minimum in Z.

* References are at the end of the paper.

Fig. 5 presents the results obtained on seven profiles crossing another dike found a few kilometers to the south of the Ferraria dike, which we have named the Timbutuva dike because it was first discovered in the property of the Timbutuva gold mine.

Although diabase boulders are found at several points, and indicate the passage of this dike, we have no definite information as to its thickness. The magnetic anomalies indicate that it is much larger than the Ferraria dike, and an average thickness of more than 50 m. may be presumed. These profiles are distributed over a total length of slightly

Fig. 3.—Southern contact of diabase dike.
Note slight inclination to north.

more than 20 km., and, as shown on Fig. 6, indicate that the dike follows a straight line, perhaps with slight undulations, for at least this distance.

The interesting part of this anomaly is the relative constancy of form over such a great distance. It is also important that it is polarized opposite to what would be expected from theoretical considerations, and opposite to the Ferraria dike. It may be that this dike is connected in depth with the Ferraria dike and that they have been magnetized as a unit, the southmost dike having a negative pole predominating while the other has a positive pole. There is no possibility of explaining this opposite polarity by the inclination of the dike, which presumably remains essentially vertical. Here also, the calculated anomalies are much smaller in magnitude than the observed values.

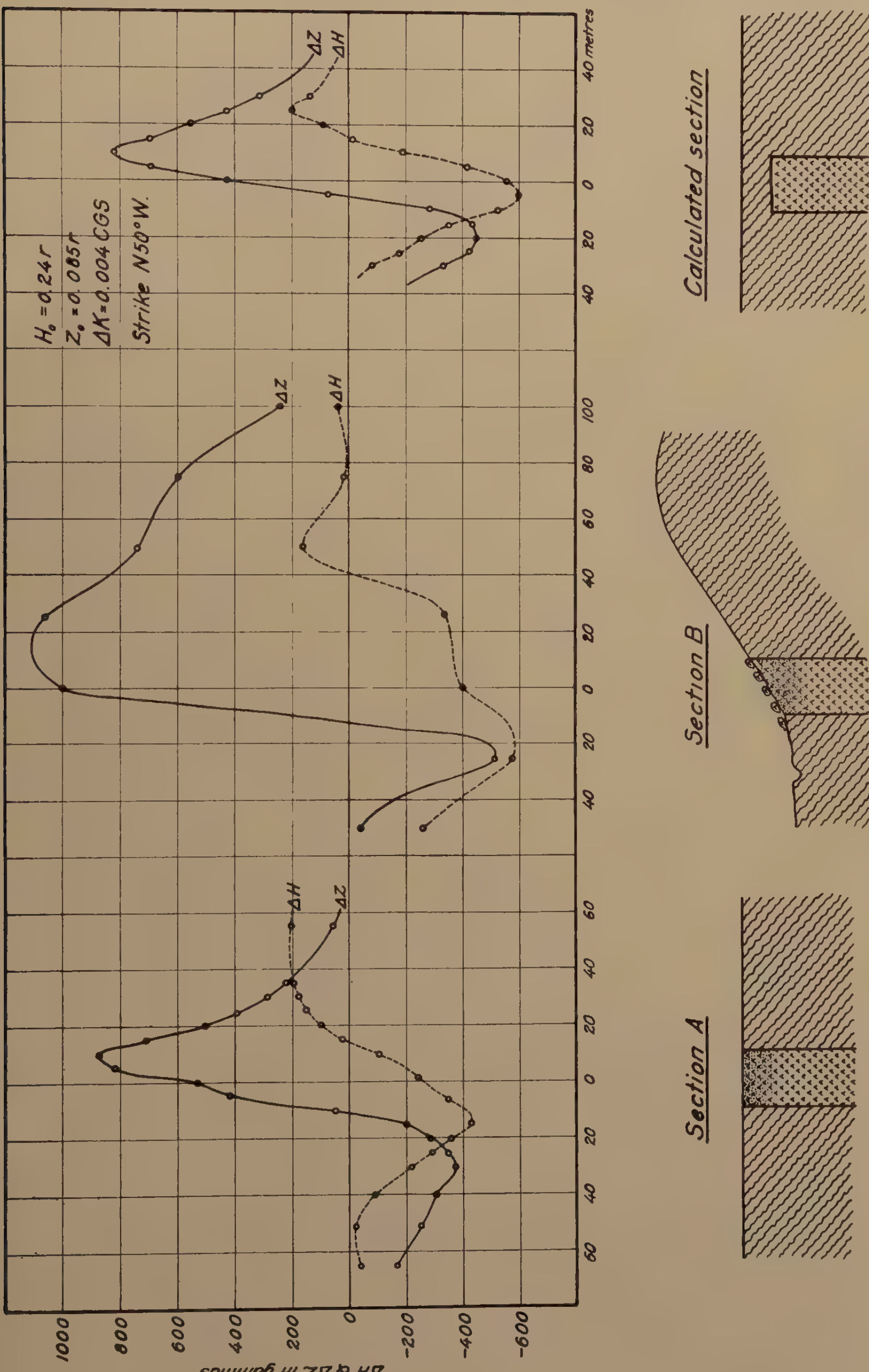

FIG. 4.—OBSERVED AND CALCULATED ANOMALIES OVER THE FERRARIA DIKE.

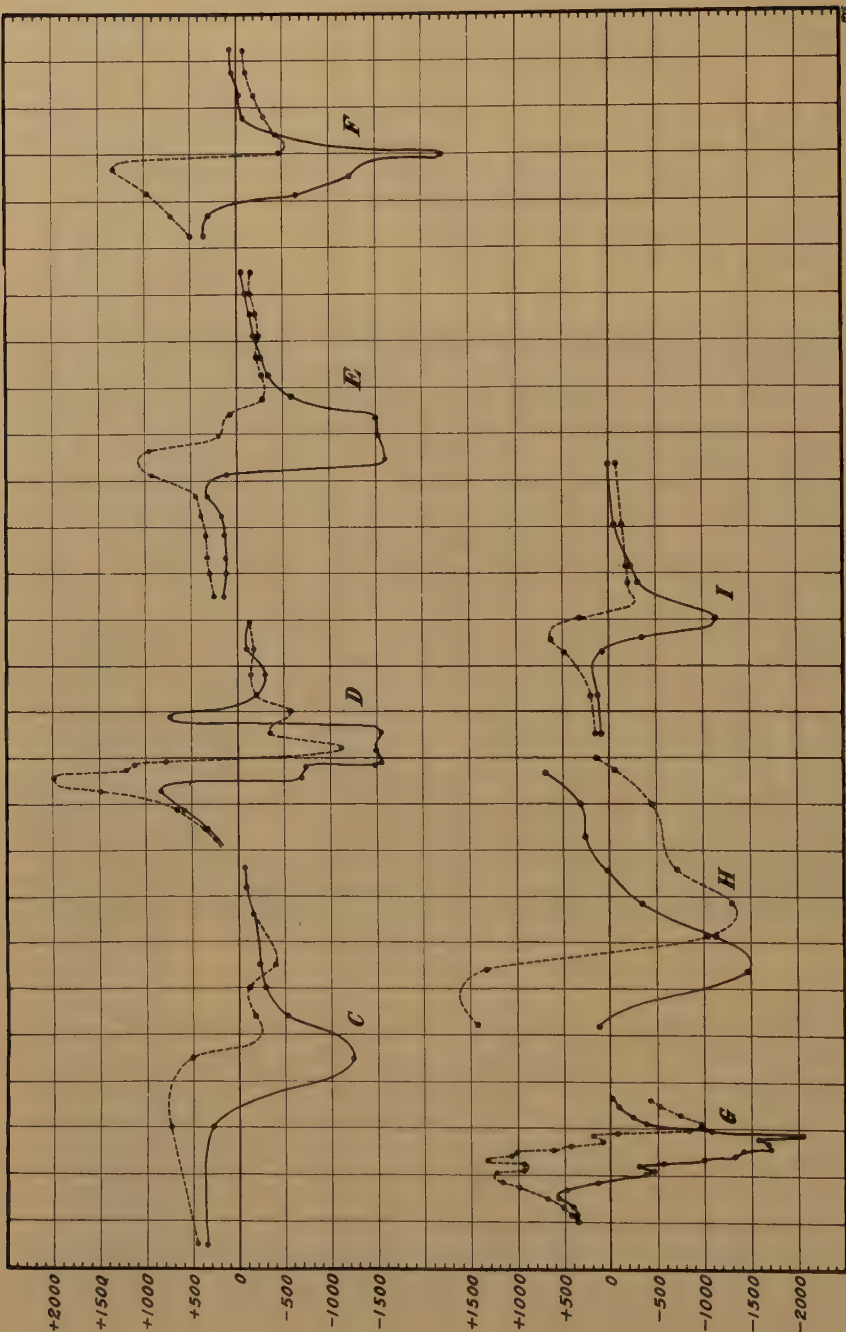

FIG. 5.—SEVEN PROFILES ACROSS THE TIMBUTUVA DIABASE DIKE.

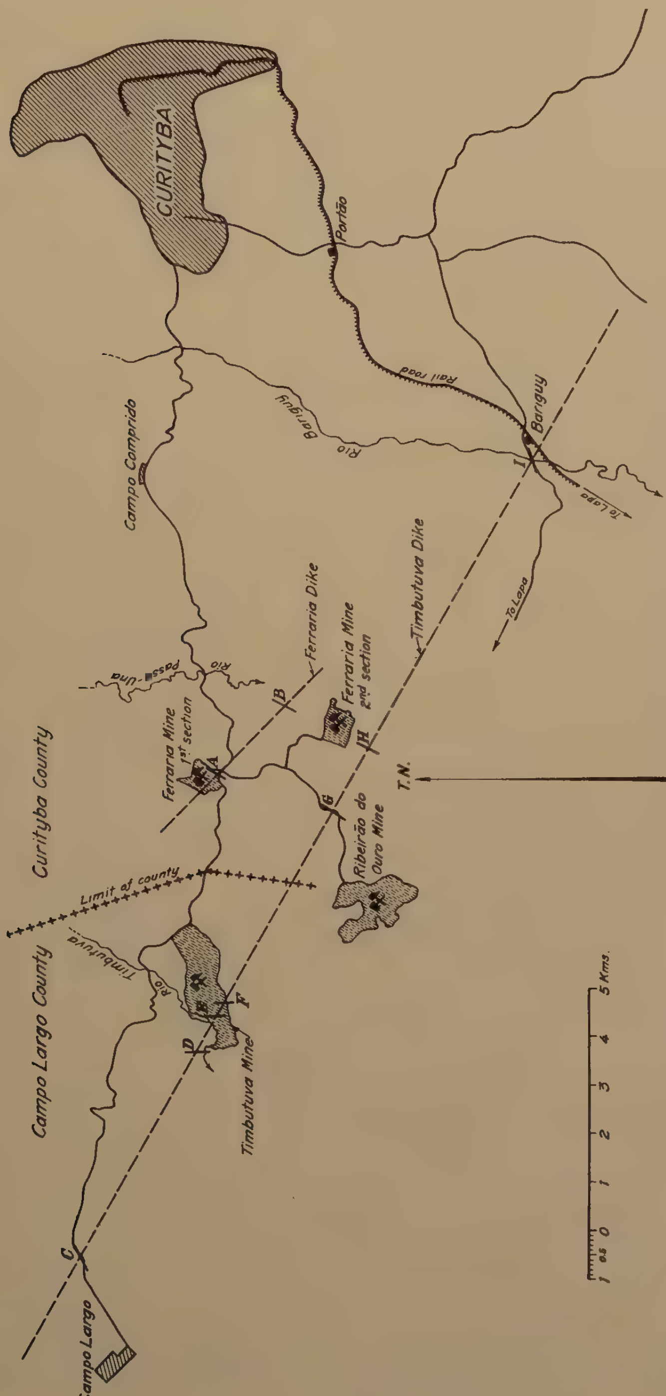

FIG. 6.—SKETCH MAP SHOWING LOCATIONS OF FERRARIA AND TIMBUTUVA DIKES.

We believe that these anomalies may be taken as typical of anomalies due to igneous dikes in the south magnetic hemisphere in an area of low inclination.

Anomalies Due to Sills and Lava Flows

Sills commonly occur in relation with dikes, laccoliths, necks, etc. They are intruded along the bedding planes of sedimentary rocks, are

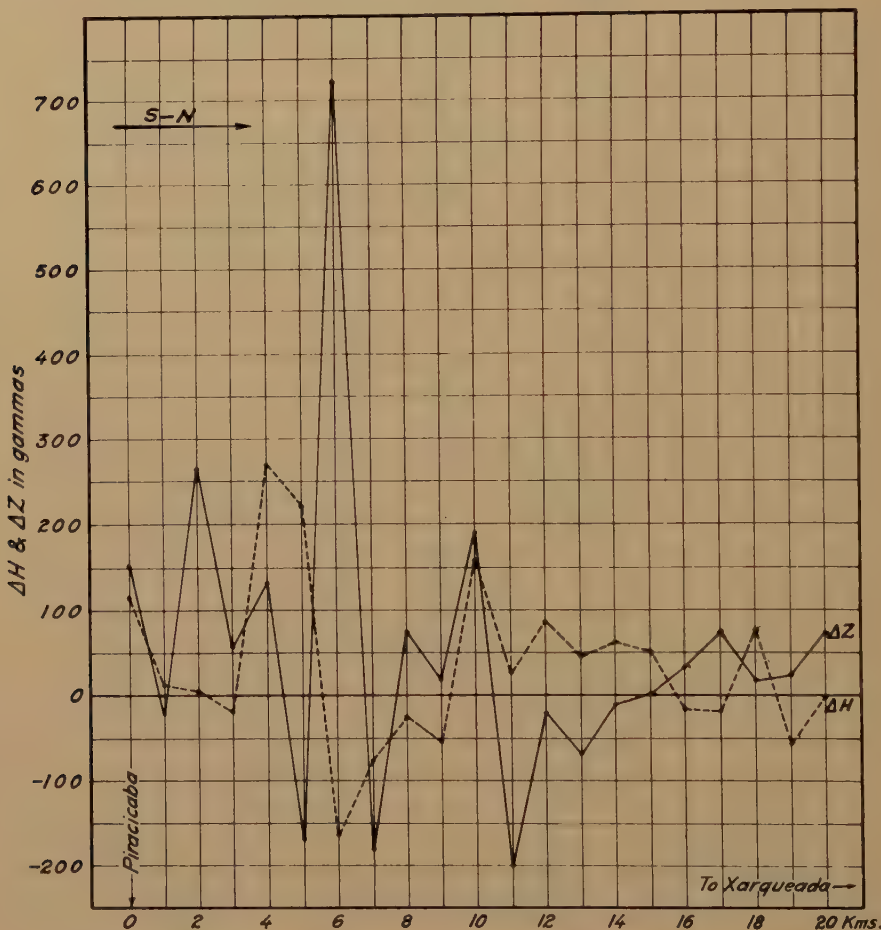

Fig. 7.—Variation in magnetic intensity over sills near Piracicaba.

variable in thickness and may even break over from one bedding plane to another. They may be very extensive in area, or limited to the immediate vicinity of the feeder. Considering the irregular nature of their occurrence, it might be expected that magnetic anomalies due to this cause would also be very irregular.

Fig. 7 shows a curve of the variation in magnetic intensity over a large sill exposed in the beds of the Piracicaba and Corumbatahy rivers in the state of São Paulo. No geologic data are available to aid in explaining the rapid sequence of positive and negative poles on this sill or sills. Fig. 8 shows similar phenomena over sills in the state of Santa Catharina. The existence of these sills is proved by the occurrence of diabase boulders.

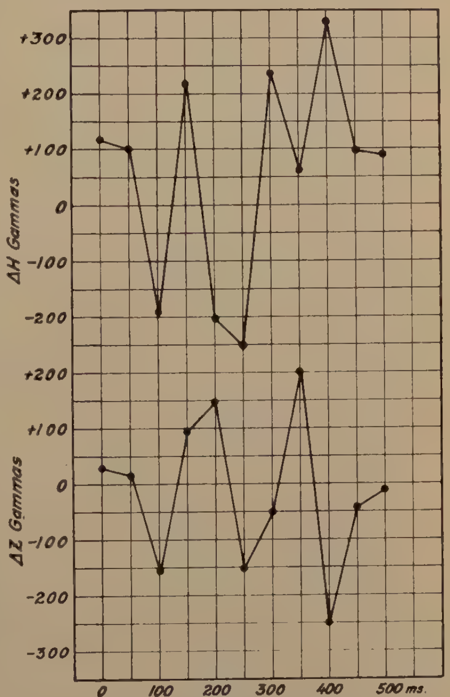

FIG. 8.—VARIATION IN MAGNETIC INTENSITY OVER SILLS IN SANTA CATHARINA.

The data in Fig. 9 are particularly interesting. The Serra Geral is an abrupt scarp of an elevation of about 300 m., which forms the border of the eruptive rocks that cover the western part of South Brazil. The basalt fields extend to the west for hundreds of miles, forming an irregular peneplain, which gradually dips to the basin off the Paraná River. The age of these eruptives is presumed to be Triassic. The Serra Geral is composed of Botucatú sandstones with beds of igneous rocks intercalated. An approximate geologic section observed along the automobile road that climbs from Santa Maria to the city of Torrinhas, on the plateau, is also shown in Fig. 9.

The magnetic data obtained along this road are of great interest because, owing to the climbing of the road, we were enabled to obtain data with the instruments below the level of the sills and flows, and later pass above them. Actually, the scarp is more nearly vertical than is indicated in the figure. The road climbs gradually along the flank and this section was drawn to correspond with the road.

Working on the southern extremities of these tabular masses, we might expect that they would all present negative poles and that when the instrument is below the sill, positive anomalies would be recorded. Actually, the lowest sills *a* and *b* appear to present a single positive pole represented by a negative anomaly in *Z* when the instrument is below the sill and a positive anomaly when it is above the sill. The third sill *c* is of opposite polarity, showing a positive anomaly when the instrument is below and a negative anomaly when the instrument

is above the sill. Sills *d* and *e*, which dip somewhat to the south, are positively polarized. On top of the plateau, there are two large positive anomalies, which may represent either the feeder dikes or laccoliths, the curve later assuming an approximately normal value. Unfortunately, our stations were not spaced close enough on the plateau to provide more definite information.

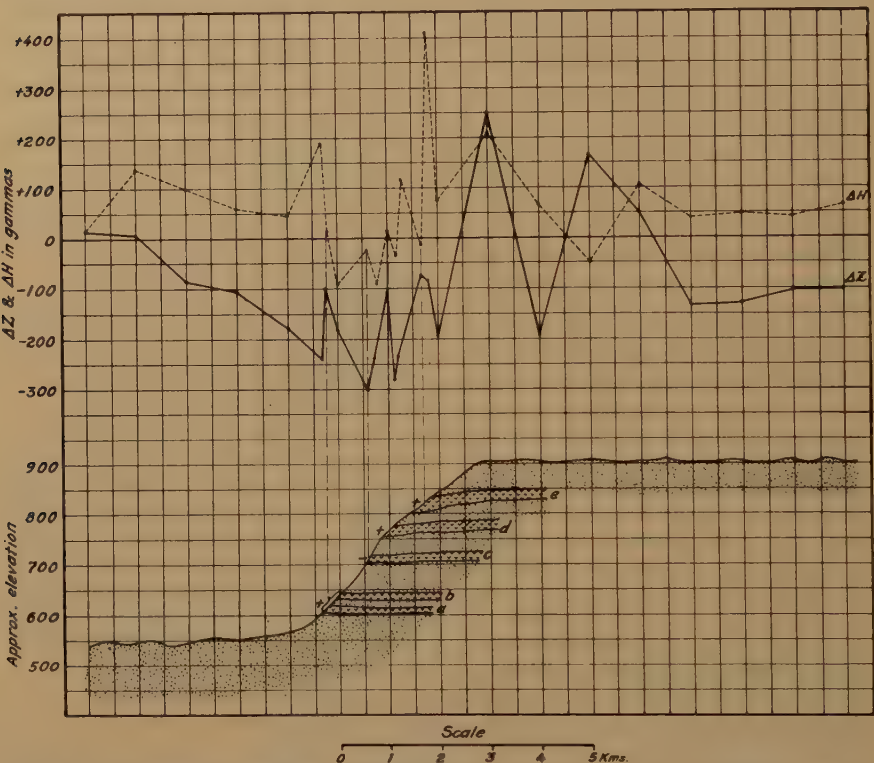

FIG. 9.—MAGNETIC ANOMALIES OVER SILLS AND LAVA FLOWS IN THE SERRA GERAL, SÃO PAULO.

Comparison of these various magnetic anomalies over sills and flows indicates that, although very definite interpretations can be made of anomalies due to dikes, those due to horizontal masses are very complicated and defy exact interpretations. Consecutive poles may be due to changes in thickness of the sill, to breaking across from one bedding plane to another, or to feeder dikes. The result is a multipolar mass, and theoretical calculations can hardly be applied.

ANOMALIES DUE TO LACCOLITHS, STOCKS AND BATHOLITHS

In another paper[3] we have discussed in some detail anomalies due to laccoliths. This paper, therefore, will present only Fig. 10, showing

FIG. 10.—ISO-ANOMALY MAP OF SÃO PEDRO-XARQUEADA AREA. SÃO PAULO.

an iso-anomaly map over an area infested with laccoliths, and Fig. 11, observed and calculated profiles over one of these intrusive masses that has been proved by the drill. The figures are self-explanatory. Here also it was necessary to multiply the calculated values by a factor of four in order to obtain approximate agreement in magnitude with the observed data.

The curves of Fig. 12 represent magnetic observations made along the road from Curityba to Capella da Ribeira in the state of Paraná. This road runs essentially north, the straight-line distance being about 15 per cent less than the distance by road. This road crosses two immense masses of syenite, which may either represent stocks or bosses on a single batholith or twin batholiths. No corrections have been made for diurnal variation, temperature or variation with latitude. The magnitude of the first two corrections are very small in comparison with the observed anomalies. The calculated normal variation with latitude for south Brazil is of the order of 4 to 5 gammas per kilometer south for the vertical intensity and from 1.5 to 2 gammas per kilometer north for the horizontal intensity. An average intensity line for the total 120 km. of this profile indicates that the variation is somewhat greater, as shown on the figure.

These two masses of syenite form important topographic elevations and considerable difficulties were encountered in blasting a way for the new military road. Stations to the north and south as well as between the two masses are at least 200 m. lower than those above the centers of the masses.

These masses do not show any definite evidence of polarization, but rather an increase in vertical intensity, which cannot be considered as very large in view of the enormous size of the intrusives. The minima on either side of the southern intrusive are primarily due to differences in elevation; at some of these stations, the intrusive mass towered above the instruments in almost vertical cliffs.

Variations in Magnetic Intensity over Gneiss of Archean Age

Although gneiss is a metamorphic rather than an igneous rock, its common occurrence along the Atlantic seaboard of Brazil, and its moderately high magnetic susceptibility, make it an important cause of magnetic anomalies in this region. The anomalies that we have observed and not been able to correlate directly with igneous intrusions have been interpreted as representing the accidental relief of the crystalline complex, composed mainly of gneiss. For that reason, some examples of magnetic phenomena observed over these old metamorphic rocks are included.

Fig. 13 shows a profile from Curityba to Campo Largo, in the state of Paraná. (See Fig. 6 for location of road.) Apart from the accentu-

ated anomalies definitely correlated with diabase dike, variations of a hundred or more gammas in a few hundred meters are quite common. Susceptibility determinations on a number of fresh samples of gneiss obtained at various quarries near Curityba gave values averaging greater

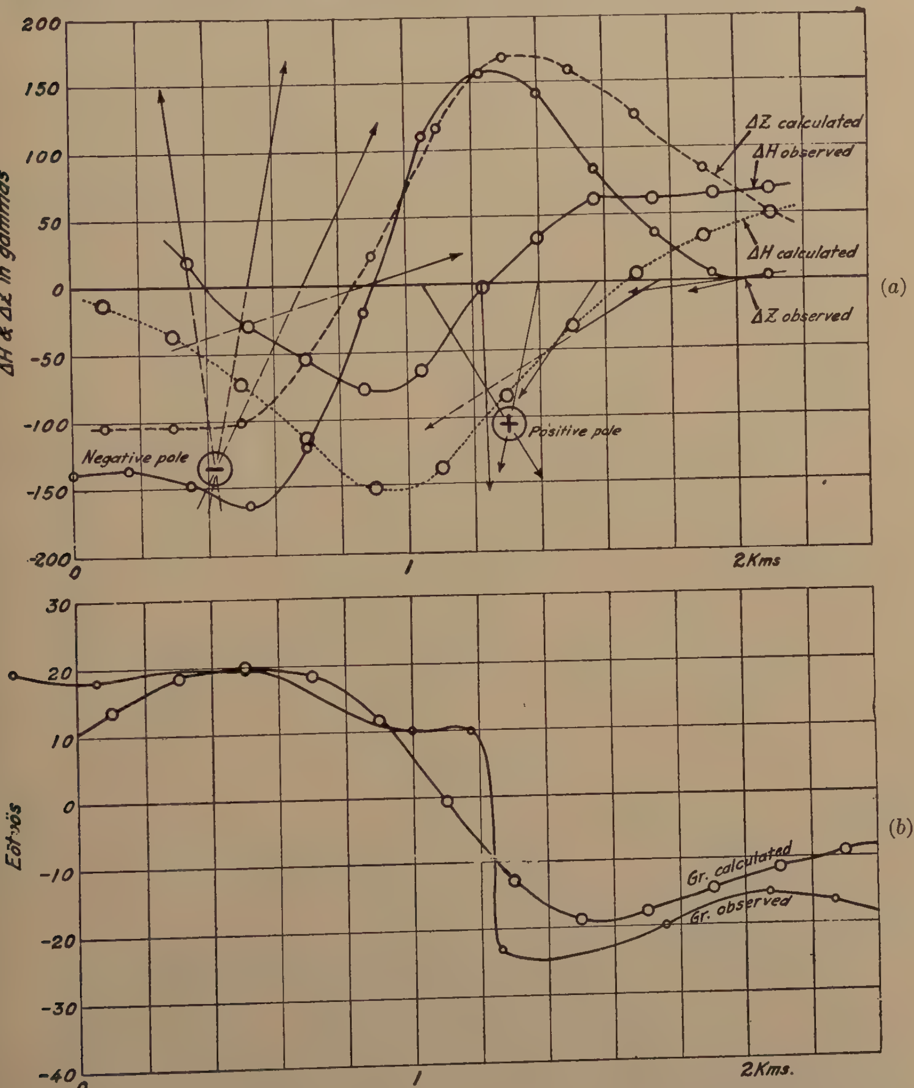

FIG. 11.—DETAILED OBSERVATIONS AND CALCULATED ANOMALIES OVER A LACCOLITH.

than 2000×10^{-6} c.g.s. It is difficult to say whether these variations are due to differences in the magnetic susceptibility of the gneiss or to variations in the depth of decomposition. The first seems to be the best explanation.

Fig. 14 shows another profile, which begins in the Tertiary basin of Curityba (probably very thin sediments) and crosses the Archean gneiss to enter into the Glacial Itararé. This curve shows little variation in vertical intensity over the gneiss (in the scale to which it is drawn) until the Itararé sediments near Lapa are reached. Here, as the road turns to the south, a gradual increase in intensity due to latitude would be expected, but on the contrary, the curve actually shows a decrease, which can best be explained as due to the rapidly sinking crystalline basement. Near the Rio da Varzea, the intensity begins to increase rapidly, at a much higher rate than the normal latitude variation, which must be interpreted as representing an uplift in the crystalline basement towards Mafra. These data are in entire agreement with the interpreta-

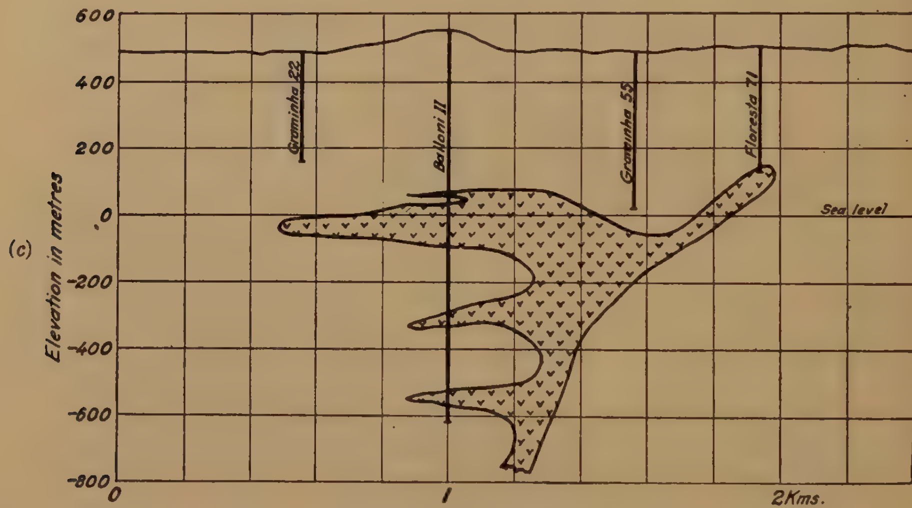

FIG. 11.—CONTINUED.

tion of a magnetic survey carried out in northern Santa Catharina in 1933, and described in a paper now awaiting publication[4].

GENERAL CONCLUSIONS ON DATA PRESENTED

The data presented in this paper, reinforced by other similar data that have been or will be published elsewhere, lead to certain conclusions relative to the application of the magnetic method of prospecting in the south magnetic hemisphere, particularly in regions where the vertical intensity is relatively low as compared to the United States or Central Europe.

1. Although many authors have expressed theoretical opinions that the horizontal intensity anomalies would be larger than the vertical in regions of low magnetic dip, our experience indicates that the vertical

magnetometer is still the more useful instrument. There is no question that the horizontal magnetometer is very useful for detail work, but the interpretation of horizontal intensity profiles of considerable length is much more complicated, and better results will be obtained by giving primary consideration to the vertical intensity variation.

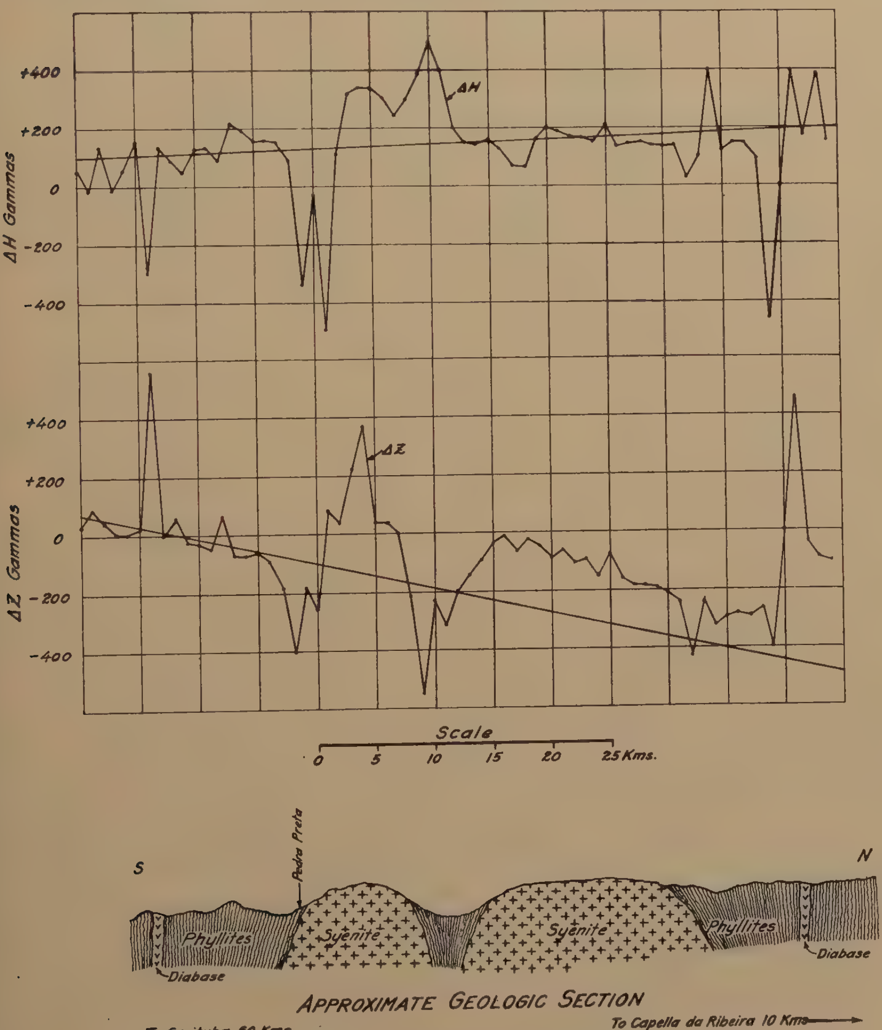

FIG. 12.—MAGNETIC PROFILE OVER SYENITE BATHOLITHS.

2. Theoretical anomalies calculated for highly susceptible rocks of definite forms give curves that are comparable in shape with the observed curves provided they do not have abnormal polarization. However, the magnitude of the calculated anomalies is much too small, and no reasonable assumed magnetic susceptibility will bring the calculated

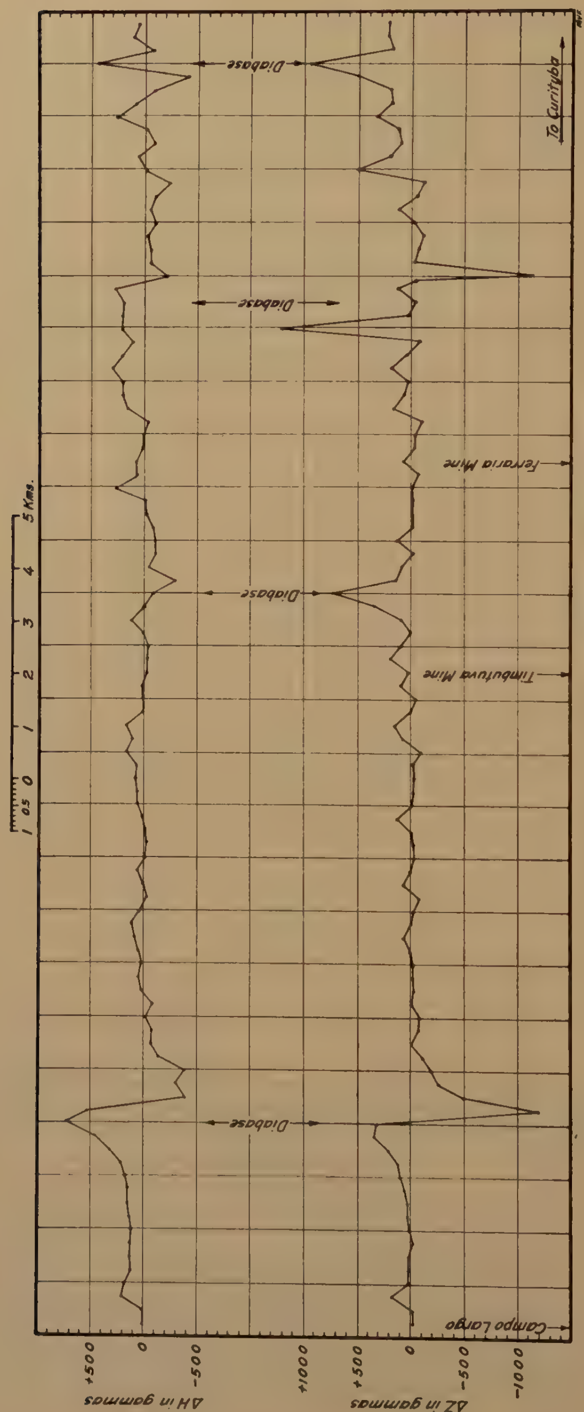

FIG. 13.—VARIATIONS IN MAGNETIC INTENSITY OVER GNEISS.

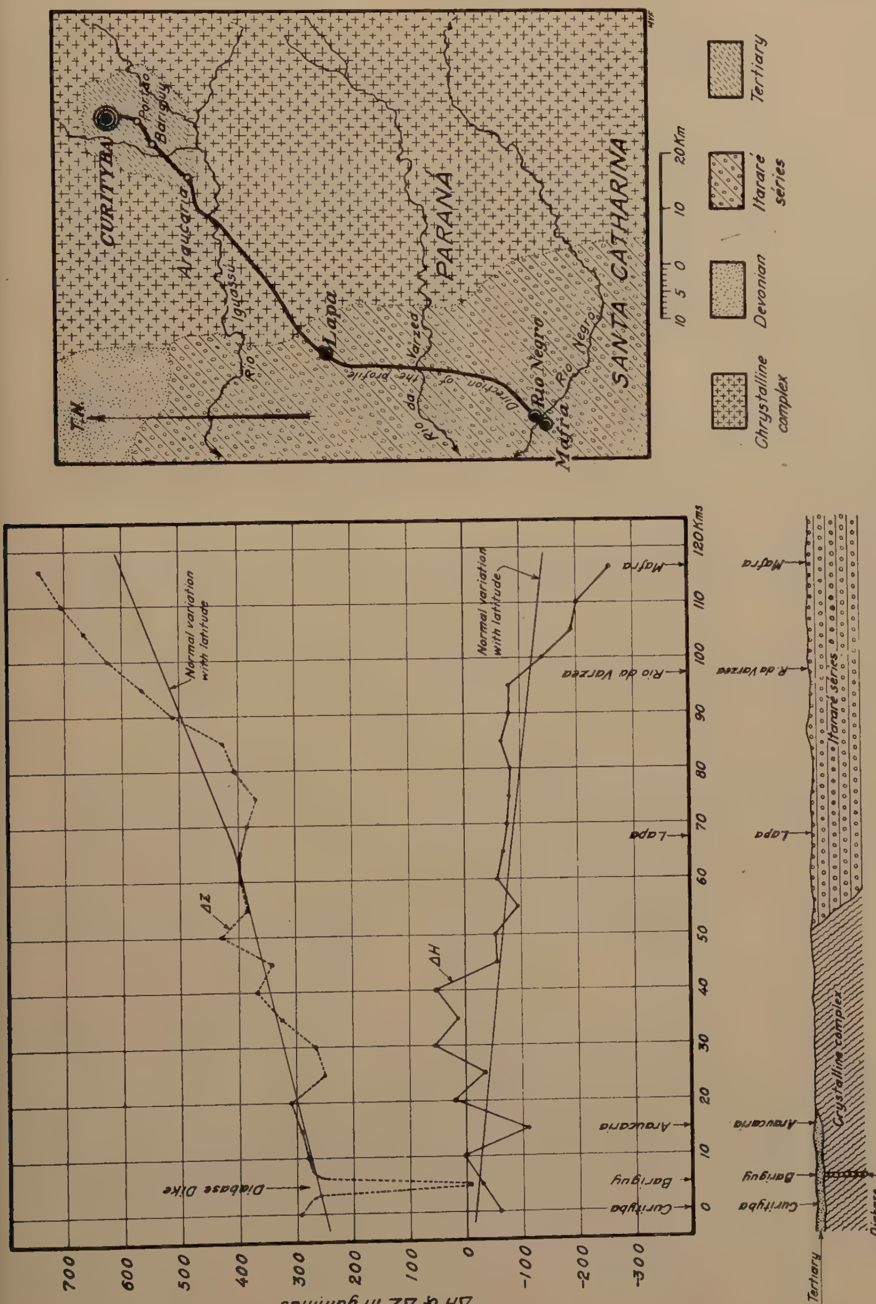

FIG. 14.—MAGNETIC PROFILE FROM CURITYBA, PARANÁ TO MAFRA, SANTA CATHARINA. Insert map shows approximate direction of road along which these observations were made.

and observed values into agreement. In our examples the calculated values have been multiplied by an arbitrary value of four, but although we may admit our determination of the susceptibility to be in error by as much as 25 per cent, we can never admit a susceptibility of 0.016 c.g.s. for diabase or basalt.

3. Dikes give rise to anomalies, which, at least in the case of almost vertical dikes with a strike of 40° or more from north, show definite poles on their faces; the maximum and minimum of the anomaly indicating the approximate position of the vertical faces. Probably, for dikes that do not reach the surface, these points of maxima and minima would be found somewhat away from the points at which these faces would reach the surface if projected.

4. Depths to magnetic poles, calculated by means of the various formulas given by Heiland, show values that are at least of the right order of magnitude, when dealing with bodies at depths of several hundreds of meters.

5. Except in regard to sills, a fair idea of the form of the igneous intrusion can be obtained from a qualitative study of the anomaly.

6. In spite of the greater magnitude of anomalies due to igneous masses, some information relative to the relief of the crystalline basement, when not too deep, can be obtained from an extensive regional survey carefully carried out with all of the normal corrections applied.

Acknowledgments

The authors wish to express their appreciation to the Director of the National Department of Mineral Production, Dr. D. Fleury da Rocha, and the Director of the Serviço de Fomento da Producção Mineral, Dr. Djalma Guimarães, for permission to publish these data. Thanks are due to Dr. Gabriel M. A. Oliveira and Dr. Melciades Y. dos Guaranys for aid in the field work.

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A Magnetic Survey of the Ivry Ilmenite Deposit

By David A. Keys*

(New York Meeting, February, 1936)

The object of this investigation was to determine with a vertical magnetic variometer the extent of the titaniferous ore deposit that occurs not far from Ivry in Terrebonne County, Quebec, about 70 miles north of Montreal. The Ivry mine, on lots 37 and 38 of range V, Beresford township, Quebec, is one of the largest and most accessible to transportation of the deposits of ilmenite in Quebec. The ore consists of an aggregate of ilmenite anhedra serving as host for numerous fine plaques of hematite. No magnetite is known from any occurrence near this lot, and in this way the deposit differs from many others that contain a large percentage of magnetite with the ilmenite. The deposit at Ivry thus has much lower magnetic susceptibility than those containing abundant magnetite, and a sensitive magnetometer was necessary for the work, although a body of magnetite with ilmenite about a mile from the Ivry mine was successfully prospected with a dip needle.

The principal orebody is exposed in a quarry known as the Ivry mine, from which about 16,000 tons of ore have been shipped. Small exposures of ilmenite isolated by drift serve to show the presence at the surface of some of the bodies indicated by the magnetometer, but most of the hill is covered with glacial till and second growth hardwood. At the lower levels the till gives place to fluvioglacial sands and gravels from which the forest has been cleared. The increasing thickness of the sand and gravel as the hill descends makes the tracing outward of the deposits from the rocky knob of anorthosite a difficult matter. Within the orebodies too, some masses of anorthosite persist, which may account for some local irregularities.

Fig. 1 is a photomicrograph, taken by reflected light, of a typical specimen of the ore. The dark patches are hematite and the light parts are ilmenite. No trace of magnetite was found. Fig. 2 is a photomicrograph of a thin section of the country rock showing the texture of the anorthosite. The mean of four analyses quoted by Robinson[1] is: titanium, 19.26 per cent; iron 45.41 per cent.

The ilmenite ore is not sufficiently magnetic to deflect an ordinary compass needle, but the effects of the induced magnetism in the ore by

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Published with the permission of the Director, Quebec Bureau of Mines. Manuscript received at the office of the Institute Jan. 22, 1936. Issued as Contribution 102, July, 1936.

* Department of Physics, McGill University, Montreal, Quebec.

[1] References are at the end of the paper.

the earth's magnetic field may be measured with an Askania variometer. Some specimens of ore and some of the anorthosite show weak permanent polarity. The country rock has a residual magnetic effect on th vertical variometer, which depends upon the thickness of the overburden.

A survey was made in the usual manner with an Askania vertical variometer, the sensitivity of which was found by calibration to be 36.4 γ per scale division. Lines were cut through the underbrush and readings were taken along them at stations approximately 50 ft. apart. The readings were reduced to a common basis by adding or subtracting the appropriate number of scale divisions for the change in position of the auxiliary magnets. The magnetic moments of the auxiliary magnets

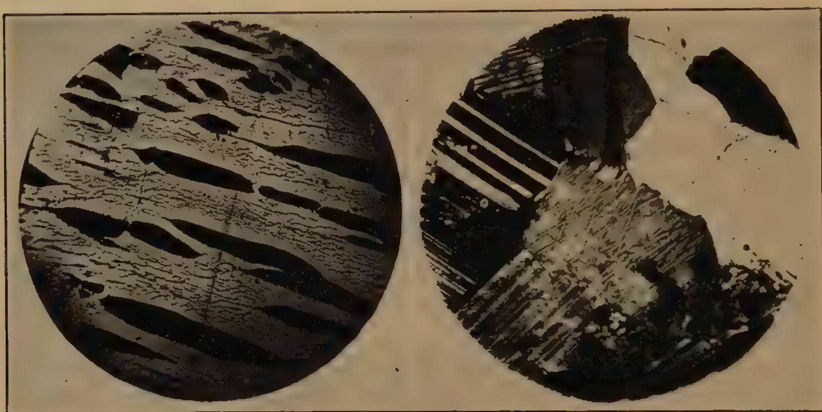

Fig. 1. Fig. 2.

Fig. 1.—Ilmenite ore etched with HCl. Reflected light. Dark patches are hematite.

Fig. 2.—Thin section of country rock. Anorthosite. × 118.

were found in the laboratory, and calibration curves for the variation of deflections with the positions of the magnets determined.

Fig. 3 is a sketch map of the area surveyed, on which the magnetic readings in scale divisions are shown, one scale division being equivalent to 36.4γ. The normal value of the vertical reading over anorthosite in the neighboring region where no ore was present was found to be -21.0 divisions. The actual value of the vertical component of the earth's magnetic field corresponding to the reading -21.0 divisions on the magnetometer was 0.5471 gauss. The vertical magnetic anomalies along the north-south lines are shown in Fig. 4 and the anomalies along the east-west lines in Fig. 5. From the results of experiments on models and from experience in the field[2,3,4], the probable dip of the orebodies and their possible shape are shown in Fig. 4. The strike and limits of the ore may be deduced from the anomalies. In the plan (Fig. 6), outcrops are shown by diagonal ruling and the probable location of the ore as deduced from magnetic readings is shown by vertical ruling.

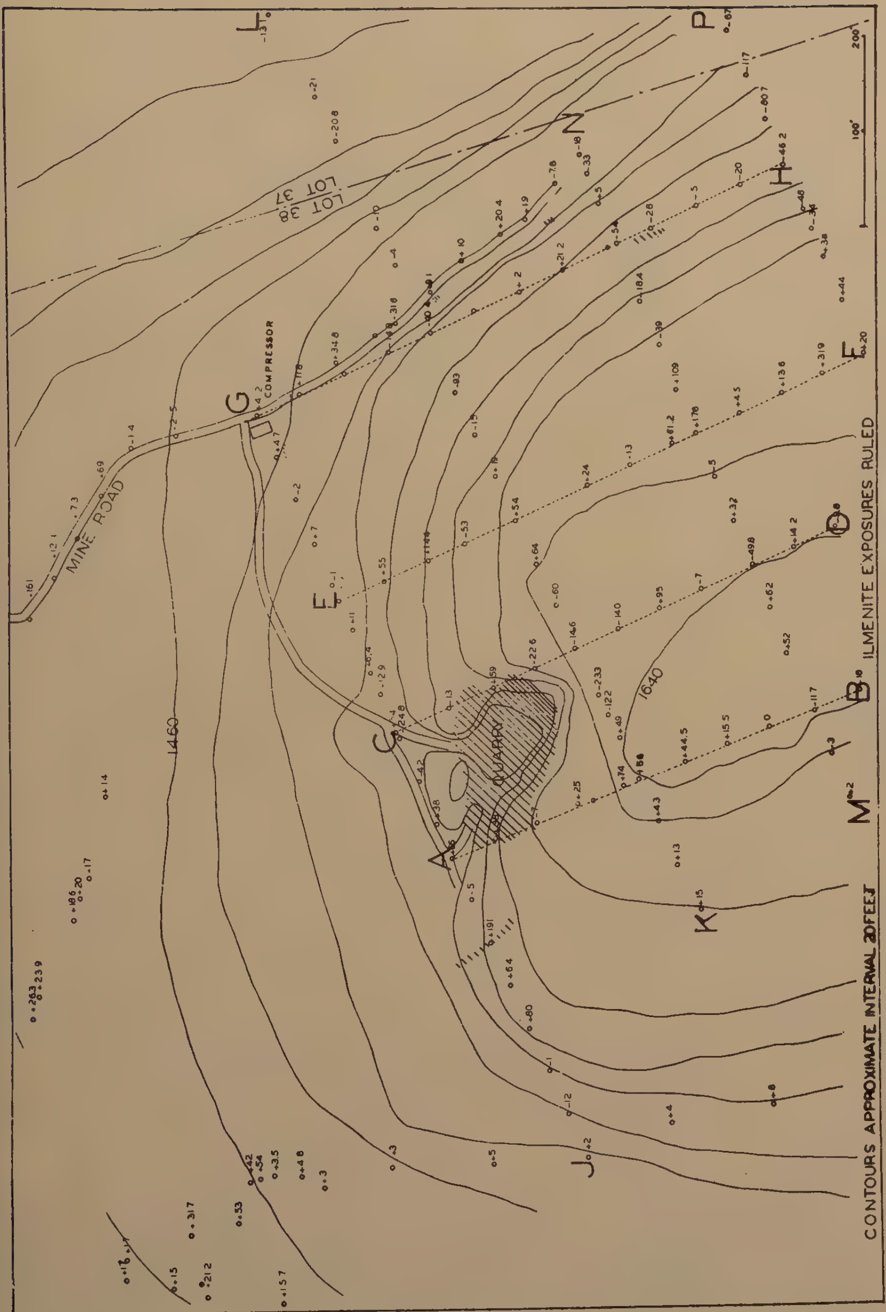

FIG. 3.—SKETCH MAP OF IVRY ILMENITE DEPOSIT. OUTCROPS SHOWN WITH DIAGONAL RULING.

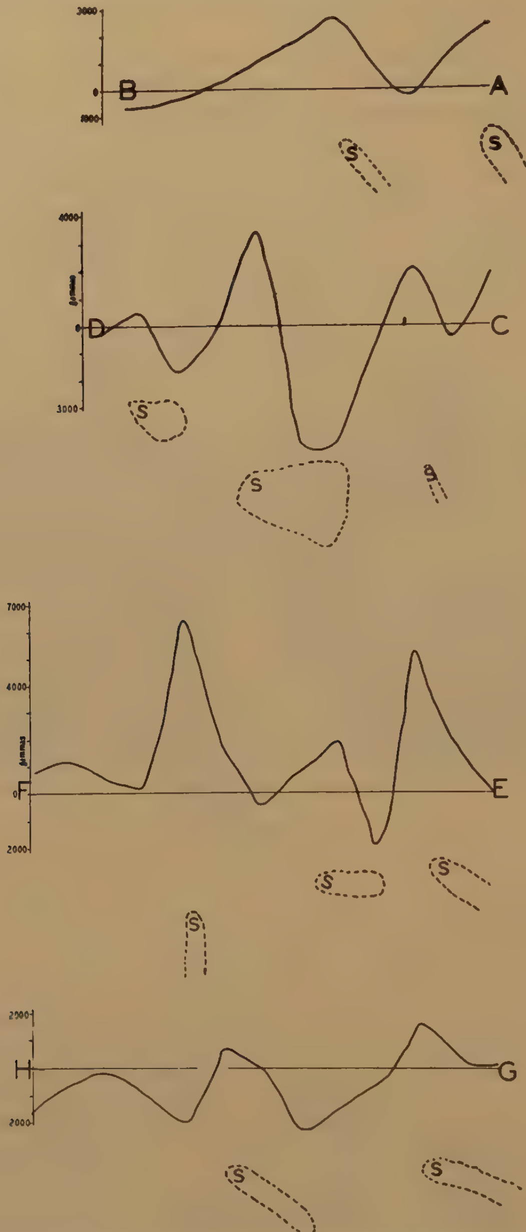

Fig. 4.—Vertical magnetic anomalies along north-south traverses, Ivry mine.

The broken ruling gives the resultant interpretation from the observations made.

An estimate of the probable depth of the deposit in the quarry was made as follows: The orebodies appear to be more in the shape of pods

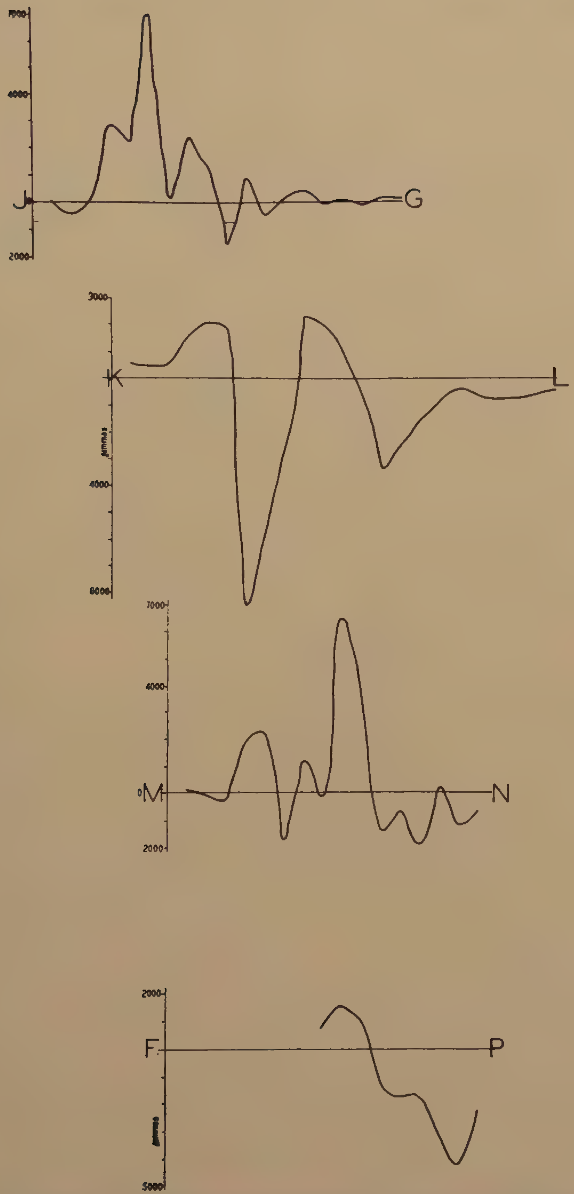

FIG. 5.—VERTICAL MAGNETIC ANOMALIES ALONG EAST-WEST TRAVERSES.

than in veins. This opinion is based on the results of the survey. For such a deposit the approximate depth may be estimated, if certain assumptions are made. In a flat body lying in the earth's magnetic field, the induced south pole will be near the south end of the deposit and the north pole near the north end. In this instance the north pole

FIG. 6.—IVRY MINE AREA. OUTCROPS SHOWN WITH DIAGONAL RULING AND ILMENITE, AS DEDUCED FROM MAGNETIC OBSERVATIONS, WITH VERTICAL RULING.

is probably at some distance from the south pole of the orebody. There is no overburden over the south end of the orebody in the quarry and a reading in the pit was taken with the magnetometer (R_0) and a second reading taken 14.4 ft. vertically above (R_1). When the normal reading is subtracted from these readings, the vertical anomalies in these two positions due to the orebody are V_0 and V_1. The value of $V_0/V_1 = 1.32$.

Assuming as a first approximation, an inverse-square law of force from the single pole, one gets $1.32 = \frac{(x + 14.4)^2}{x^2}$, which gives $x = 93$ ft. where x is the distance to the pole from the variometer. The actual thickness of the deposit would be more than double this value and the geology of the surrounding region suggests that this is not an unreasonable value. Other assumptions as to the positions of the poles will lead to different values for the depth[5,6,7].

The investigation at the Ivry mine shows the value of the vertical magnetic variometer in the exploration of ilmenite deposits in which no magnetite occurs. An interpretation of the results gives the location of the main orebodies in the area and the approximate thickness of one of the deposits is estimated from magnetic measurements. No drilling has been done on the deposit but the results of the survey are in agreement with the known geology of the region.

This survey is part of some magnetic investigations made by the author with the assistance of Dr. F. Fitz Osborne, of the Department of Geology, McGill University. The author is indebted to him for the photomicrographs and the geological discussions.

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Tracing a Basic Dike by Geoelectrical and Geomagnetic Methods

By W. R. Johnson, Jr.,* G. R. MacCarthy,* J. C. McCampbell,* Junior Member A.I.M.E., and H. W. Straley, III,* Member A.I.M.E.

(New York Meeting, February, 1937)

Abstract

In the spring of 1935 the authors undertook to compare the geomagnetic and direct-current earth-resistivity methods of tracing a concealed dike along its strike. An area near Chapel Hill, North Carolina, was selected, where a diabase dike was known. For rapid reconnaissance the magnetic method offers several advantages over the electrical. The resistivity method is useful in determining the position of underground water, a factor of consideration in regions where mining is contemplated. All factors considered, the coincidence between the positions of the dike as determined by the two methods is striking.

The entire paper was mimeographed and issued as A.I.M.E. Contribution 106 in February, 1937.

* University of North Carolina, Chapel Hill, N.C.

An Instance of Abnormal Magnetic Polarization in South Africa, Together with a Graphic Method for Determining Effects of Magnetic Pole Distribution

By F. Bahnemann*

(New York Meeting, February, 1935)

In discussing the problem of abnormal magnetic polarization, C. A. Heiland[1]† emphasized the unfortunate fact that our knowledge of the geological and physical conditions relating to such occurrences is very limited and that a more detailed insight into their physical causes would be extremely valuable for interpreting measured anomalies. He also pointed out that to arrive at quantitative physical formulas that would express the conditions that cause abnormal polarization many more occurrences of the kind would have to be recorded. The first part of this paper is a contribution to such descriptions.

Abnormal Magnetic Polarization in South Africa

The occurrence was mapped in the course of a magnetometric survey carried out about 70 miles to the east of Johannesburg and east of the Witwatersrand gold field, for the purpose of detecting the position of the suboutcrop of certain magnetite-bearing shales that occur in the lower portion of the Witwatersrand system of sediments, shales whose position in section in relation to the ore-bearing members of the same system of sediments are known. In this case the shales are covered by younger strata several hundred feet thick.

Lines of magnetometric traverse were run at right angles to the strike for a strongly magnetic body. This body is a shale zone in the Witwatersrand sediments and has a strike of N.20°E. (magnetic) and dips 15° to the east. The field work was done by a vertical intensity field balance (A. Schmidt principle). The plotting is done in such a way that the sign is reversed from that usually employed in the northern hemisphere, so that a positive profile is obtained in abnormal cases while the same conditions in the northern hemisphere would give a negative one. This method of recording should be kept in mind in reading what follows.

Manuscript received at the office of the Institute Nov. 26, 1934. Issued as Contribution 79, February, 1935.

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† References are at the end of the paper.

Curve A, Fig. 1, is a record of observations made on one such traverse across the bed. Because of the abnormal magnetic conditions revealed by the curve, a borehole was put down to penetrate the bed causing the magnetic disturbance. This borehole was located at a point giving the maximum intensity in one of the traverses.

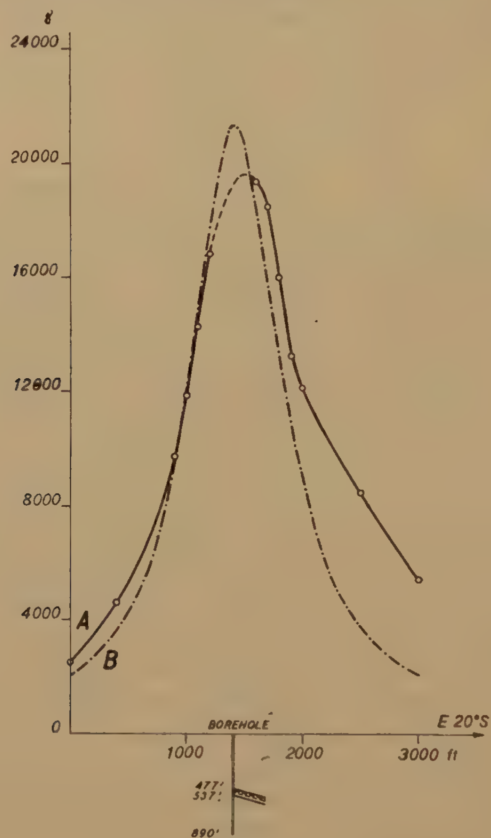

Fig. 1.

At a depth of 477 ft. in this borehole a bed of magnetic shale, 60 ft. in true thickness, was passed through. A basic intrusive sheet 11 ft. thick lies about 11.5 ft. below this bed. Magnetic shales occur in an old borehole 7600 ft. east of this borehole, at a depth of 2600 ft. from the surface. From this information and the strike of the shales as determined by magnetometer, the dip angle is calculated to be 15° to the southeast. This is checked by measurements of the angle between the bedding planes in the cores and a plane at right angles to the core axes, which give the same results.

A purely qualitative investigation of the bore cores with the field balance revealed the presence of strong permanent magnetism, the general

direction of which is at right angles to the axis of the cores. The orientation of the permanent magnetism of the cores is indicated in Fig. 3, which is a cross-section of a piece of core.

In this instance, we know the thickness, depth and altitude of a strongly magnetized body and also that the rocks in which it lies are

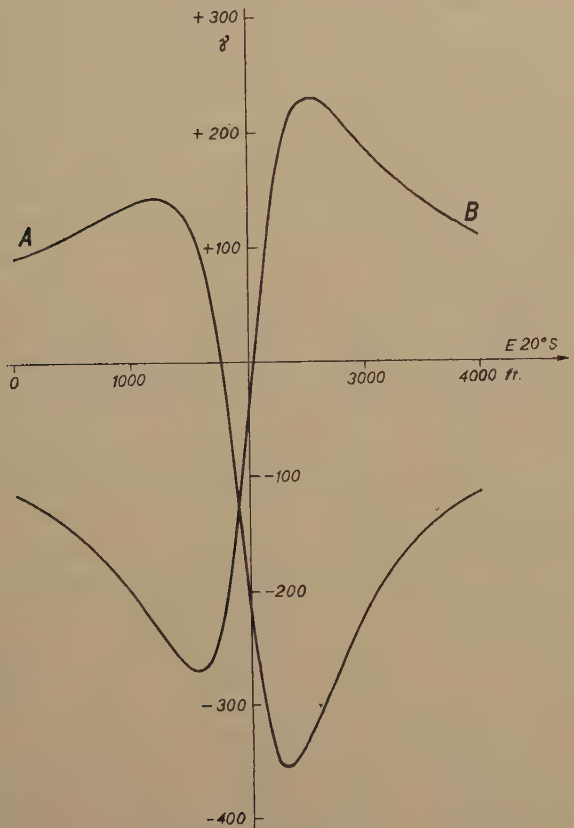

Fig. 2.

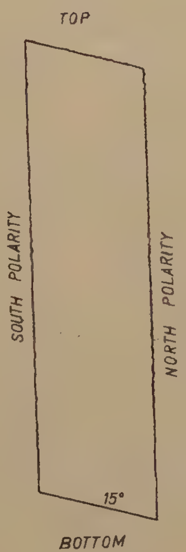

Fig. 3.

much less magnetized. We are, therefore, justified in applying the formulas developed for such cases by Roessiger and Haalck[3,4]. Fig. 2 shows the results of calculations based on the given data; curve A represents a dip towards the southeast; curve B, a dip towards the northwest, the latter curve being included to cover all possible conditions. The susceptibilities of the magnetic Witwatersrand shales range between 0.05 and 0.07. Fig. 2 was calculated for $K = 0.0653$. The difference between these curves and that of the measured values (Fig. 1-A) is apparent.

A glance, moreover, at the general cases compiled by Haalck[2] on the grounds of the induction theory shows, from a purely qualitative stand-

point alone, that the magnetization in this case cannot be made to coincide with any one of the calculated cases. Even if it is assumed that the shales dip towards the northwest, a qualitative resemblance cannot be found.

Induction by the earth's field therefore explains neither the amount nor the direction of this anomaly.

If permanent magnetism is assumed, the shape of the measured curve (A, Fig. 1) leads to the assumption of a strong component of the magnetization parallel to the dip. Conclusions as to the amount of this dip cannot be drawn from the asymmetric shape of the curve, as would be possible in pure induction.

Since in this instance both the depth of the suboutcrop and the true thickness and shape of the disturbing body are known, it is possible to calculate the components of the magnetization at least approximately. In Fig. 1, A shows the measured curve, B a curve calculated from the assumption that the components of the magnetization parallel and perpendicular to the dip are: $I_{\text{par.}} = 85{,}100$ gammas, $I_{\text{per.}} = -3666$ gammas.

The left part of the calculated curve coincides closely with the measured values. The unconformity of calculation and observation in the right part of the curve may be due to the fact that the proportion of the two components $I_{\text{par.}}$ and $I_{\text{per.}}$ does not meet the true conditions, or that the effect of another magnetic band above the shale under consideration makes itself felt. Certain indications point to the greater probability of the second assumption. For our purposes it suffices to show that this calculation also indicates the existence of strong permanent magnetism, the direction of which does not correspond to the present earth's field. The order of magnitude of the magnetization (as shown in Fig. 1-B) is probably correct.

The strong permanent magnetism found in the bore cores transverse to their axis corresponds with the values measured on the surface and also with the values calculated on the assumption of the presence of a permanent magnetism not explained by the present earth's field.

The magnetic conditions encountered in this district today probably have not in most cases attained to a state of equilibrium, and the unconformity of the actual magnetization from that which would be expected as resulting from pure induction through the earth's field seems to increase from Johannesburg towards the east.

Unfortunately, we have not been able to advance a satisfactory explanation for this abnormal occurrence. It may be due in part to the effects of heat and pressure. The bore cores show a narrow band of intrusive igneous rock near the magnetic shales, which clearly indicates that they have been subjected to thermal as well as mechanical influences. All of the Witwatersrand rocks have been subjected to folding and to igneous intrusion at more than one period in their history. The occur-

rences described here differ from those within the Rand area itself, where such abnormalities are rare or absent, in that they lie on the east flank of a major anticline while the major structural axis on the Rand trends southwest. They are completely covered in this neighborhood, however, and other data as to their structure are not available. They have been extensively intruded by dikes and sills of basaltic composition and of late Karroo (Jurassic) age, and such intrusions are uncommon or absent in areas on the West Rand, where similar beds of magnetite have been studied and where they have shown normal magnetization. As far as our scanty data go, therefore, it would seem that this abnormally magnetized bed differs from similar ones that are normally magnetized, in that it was tilted by earth movements so that it dipped nearly at right angles to the direction of the earth's magnetic field, while similar beds on the Rand dipped more nearly in the same direction as the earth's field. The principal difference seems to be that at a certain period in its history the abnormally magnetized bed was subjected through extensive igneous intrusion to intense heat and pressure, which the more normal beds escaped. We must await further data in regard to the geological structure and to the magnetization of the beds in the district for the solution of this problem.

It is probable much may be done by laboratory experiments towards the solution of problems of abnormal polarization. Heiland[1] refers, for instance, to an experiment whereby the direction of polarization in a nickel wire was changed by mechanical stresses applied to the wire. Whether the results of an experiment of this kind can be applied to conditions obtaining in the crust of the earth is open to question at the present stage, and such problems can be solved only with the further accumulation of both field and laboratory information.

Acknowledgment

The writer is indebted to Mr. J. A. Woodburn, Consulting Engineer of Far East Rand Areas Company Limited, for permission to publish this paper and to Dr. A. Noack, geophysicist to the company, for supplying the data.

Graphic Method for Determining Effects of Magnetic Pole Distribution

The so-called "single-pole method" is an attempt to arrive at the form and position of a disturbing body in the earth's magnetic field by a series of calculations in which single unit poles are so placed in relation to the earth's surface that their combined effect will finally approximate that of the observed effects of the disturbing body.

This method, which was developed by Nippoldt[5], has the advantage, in practice, that no assumption as to the causes of the magnetization of the body needs to be made. Nippoldt has shown how to approximate the intensity profiles that most frequently are found by observation by the effects of suitably chosen pole series. Moreover, if it is possible to determine the shape and depth of the disturbing body from geological data, and so to prove that the theoretical calculations are correct, it is possible also to gain some idea in regard to the direction and strength of the magnetization.

Without geological data one cannot be certain that the main axis of the hidden body is defined by the direction of the pole series. With some geological data, however, the pole constellation can be of great assistance in the interpretation of underground conditions. Difficulties in interpretation are caused also by too small a number of poles, by topographic variations or by the fact that the body investigated is irregular in shape and in its magnetization.

Graphic Method to Facilitate Calculations by Method Described by Nippoldt

In Fig. 4, the curved lines are lines of equal vertical intensity of a pole, of unit pole strength 1, which lie in a vertical plane through the pole. The line AB is a horizontal line through a point lying vertically above the pole at unit distance 1. For this point the vertical intensity has the value 1. Distances along AB are measured by the same unit of length. If line AB is taken to represent the earth's surface, the disturbance caused on the earth's surface by the pole of pole strength 1 is obtained by reading on each point where knowledge of the disturbance is desired the value given by the isomagnetic line at that point. Values of points between lines can be obtained by interpolation.

To find the total effect of several poles at definite points on the earth's surface, referred to as "stations," proceed as follows:

Fig. 5 shows four poles, the effect of which is sought at the stations 1 to 8 on the earth's surface, which is taken in this case to be horizontal. Profile 4 is first drawn to the scale of Fig. 4 on tracing paper and the profile is placed over the graph so that pole 1 coincides with the unit pole and the earth's surface with the line AB. The isomagnetic lines at stations 1 to 8 give the values of the vertical intensity caused by pole 1.

Pole 2 is next made to coincide with the unit pole and the effects of pole 2 at the stations 1 to 8 can then be read off along the line on the tracing which represents the earth's surface. The process is repeated until the effects of all the poles at the stations in question are recorded.

The value of all the poles at any one station can then be obtained by addition, as indicated by the following formula, in which Z_i^k is the

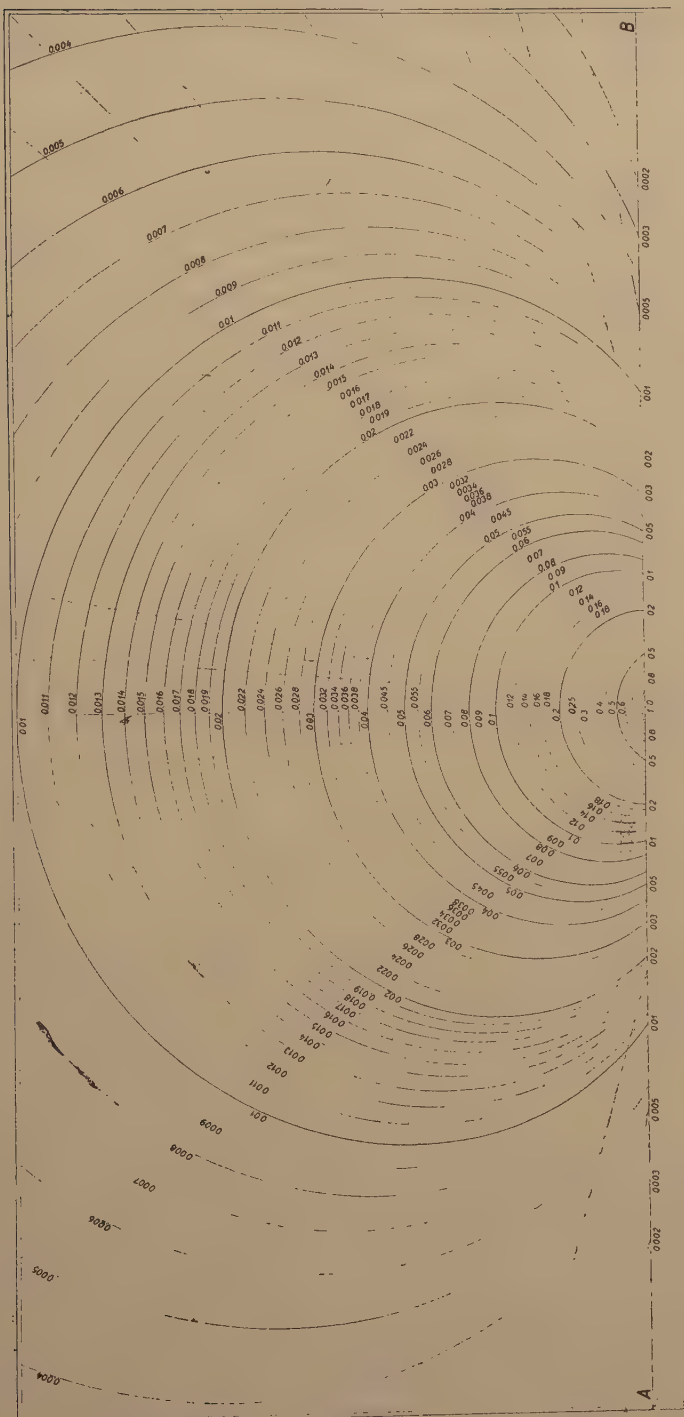

FIG. 4.

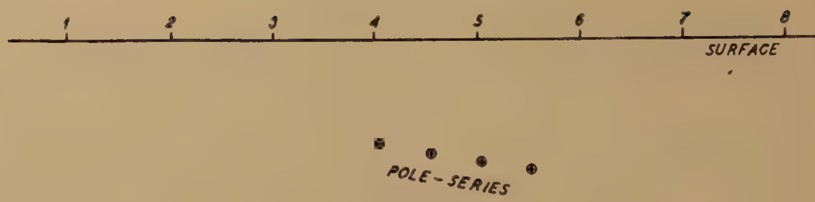

FIG. 5.

FIG. 6.

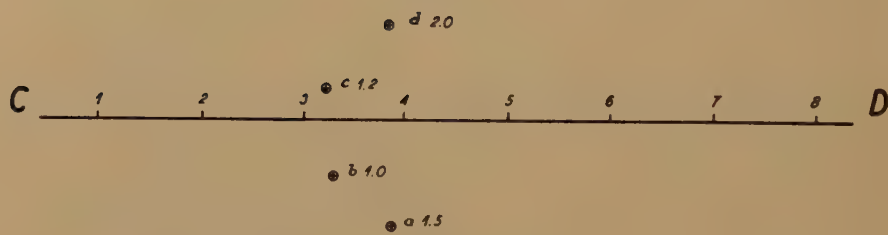

FIG. 7.

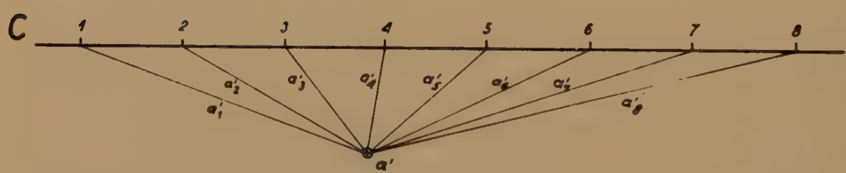

FIG. 8.

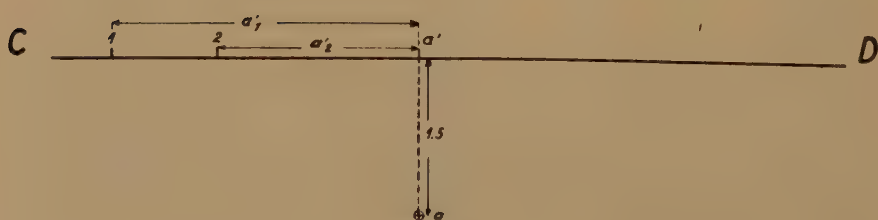

FIG. 9.

effect of pole i at station k, and the total effect of a series of n poles at k is given by:

$$Z = \sum_{1}^{n} i\, Z_i^k = Z_1^k + Z_2^k + \cdots + Z_n^k$$

It is clear that in this manner the effect of a pole series on any possible surface, such a one for instance, as is shown in Fig. 6, can be found without additional difficulty.

If the extension of the disturbing body is about equal in all three dimensions and its depth is of the same order of magnitude as its horizontal dimensions, it may be necessary to distribute the poles not only in a vertical plane but in three dimensions in space. In Fig. 7, CD represents a traverse with stations 1 to 8. A, b, c, d are poles of pole strength 1, the relative depth of which shall be for a, 1.5; b, 1.0; c, 1.2; d, 2.0. These poles do not lie in the same plane.

The effect of pole a with a depth of 1.5 can be found as follows: Fig. 8 is a plan showing position of pole a and traverse stations 1 to 8 on the earth's surface. a lies 1.5 units below the earth's surface (assumed to be flat in this case) but is projected on to it at the point a'. a_1', a_2', a_3', a_4', etc., are distances from the projection of a to the various stations.

On a piece of tracing paper pole a is represented by a point Fig. 9. A horizontal line representing the earth's surface is then drawn above it at a height of 1.5 units and from pole a a line is drawn at right angles to the horizontal line, the intersection of these two lines at a' representing the projection of a on to the earth's surface.

Fig. 9, which is on tracing paper, is now applied to Fig. 8 so that a' in Fig. 9 overlies a' in Fig. 8 and CD lies along the lines a_1', a_2', etc., in turn.

In this way the distances on the earth's surface between a' and each of the traverse stations can be marked on the line CD in Fig. 9.

Fig. 9 can then be applied directly to the graph of Fig. 4, and the intensities read off as in the previous cases. This process is repeated for each pole in turn until the effects of all the poles at the stations are recorded and the figures so obtained are then added to give the resultant effects. By this method every possible pole distribution can be treated accordingly.

An improvement in accuracy of reading will be obtained by increasing the scale (the scale of the table is 5 cm. = 1) or, by using the same curves and changing the isomagnetic 0.1 into 1 and the unit of length from 5 cm. into $5.\sqrt{10}$ cm. (Fig. 10).

On Fig. 11 are shown lines of equal horizontal intensity. To use this figure follow the method described for vertical intensity.

It should be remembered that with three-dimensional distribution of the poles some of the effects will be negative and others positive.

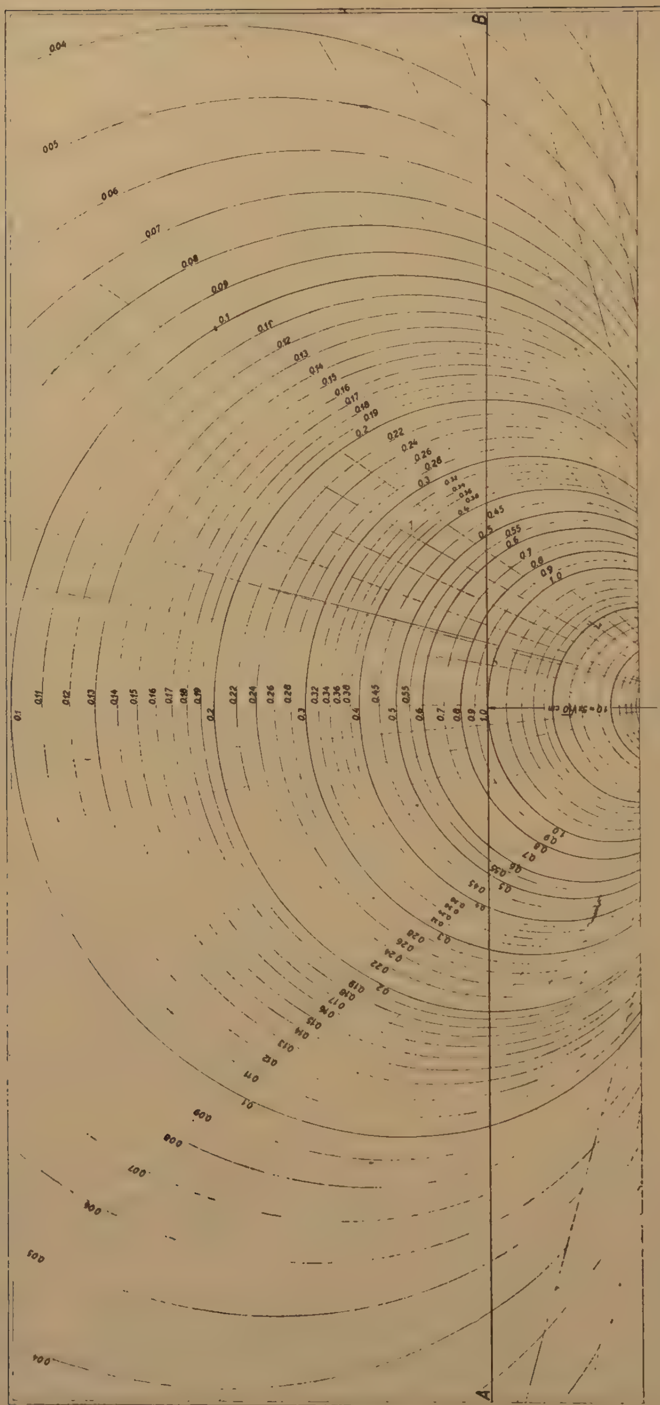

Fig. 10

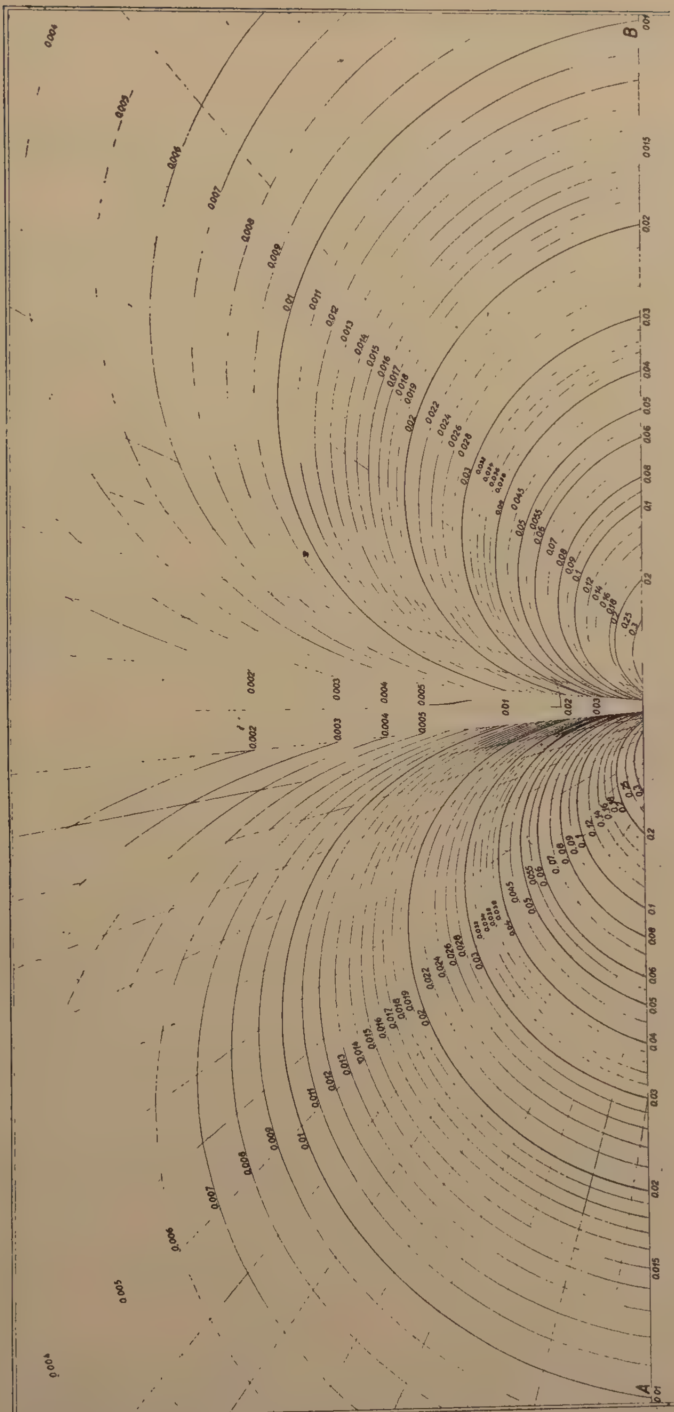

FIG. 11.

For further details in regard to this method and its interpretation, the reader is referred to Nippoldt[5].

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DISCUSSION

(Sherwin F. Kelly presiding)

F. W. Lee,* Washington, D. C.—I am surprised that the author does not use simpler methods of calculation, as there are graphs available that can be employed for working out the inverse square and inverse cube effects.

L. B. Slichter,† Cambridge, Mass.—That is true. In this case, the author drew up a chart for the simplest case of single poles, which seems to me to be a convenient means for the purpose of working out the effect of a single magnetic pole.

F. W. Lee.—We cannot assume any kind of a single pole along a line that is continuously magnetized. We can assume a doublet and get the final results just as easily with the curves I have already mentioned.

L. B. Slichter.—I would be interested to learn upon what the author based his conclusion as to the position of the end of the bed.

F. Bahnemann (author's reply).—Apparently there has occurred a misunderstanding of the charts given in the paper. As Dr. Slichter pointed out, the charts are to be used for cases in which the assumption of one or more single poles as cause of the disturbance is justified. This being so, they give the values for the vertical and horizontal intensities at any distance from the pole without any calculation. It was never meant to treat the case of abnormal magnetization by this method. I agree with Dr. Lee that we cannot assume any kind of a single pole along a line that is continuously magnetized, therefore the case mentioned in the paper was calculated according to formulas derived for a magnetized plate.

As to the question about the position of the end of the bed, there is geological evidence that the bed is practically infinitely long.

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† Associate Professor of Geophysics, Massachusetts Institute of Technology.

Polar Charts for Interpreting Magnetic Anomalies

By Sylvain J. Pirson,* Associate Member A.I.M.E.

(New York Meeting, February, 1936)

The main value of earth magnetic measurements, outside of certain mining problems, resides in the study of deeply buried tectonic phenomena related to regional and local geology. Magnetic surveys are a necessary complement of gravimetric measurements, especially of pendulum surveys. To date, magnetometer prospecting has been used chiefly in the study of sedimentary basins where petroleum structures are associated with the uplifted basement rocks. There are, in the literature, innumerable articles dealing with such surveys as applied to oil geology but few treat of broad geological features related to regional magnetic disturbances of the earth field. An important piece of coordination work has been made by Jean Jung[1] for the magnetic and gravimetric measurements made in France and their geologic significance. A similar attempt at correlation of the known geology with the anomalies of the vertical component of the earth magnetic field has been made for the United States by George B. Somers[2]. Taking the magnetic vector as a whole, W. P. Jenny[3] made a coordination of magnetism and geology for the main producing oil states of the United States. A more detailed study along the same lines is published by A. Van Weelden[4] for southern Oklahoma and northern Texas; in this work magnetic and gravimetric measurements taken at stations as close as one mile apart are compiled and compared to the regional geology of the Ouachita-Amarillo uplifted area. From the literature mentioned, it is evident that reliable interpretation of magnetic anomalies can be arrived at only when the magnetic stations are numerous and cover the area in a close net of measurements, therefore the works of Somers and Jenny, based on sparsely distributed Government measurements, can be regarded only as preliminary programs indicating lines along which further work should be done, whereas the coordination made by Van Weelden is an example of the contribution to the knowledge of regional geology that can be arrived at by the use of applied geophysics.

With the hope that further systematic magnetic measurements will be carried on to solve regional and local geological problems, the author

Manuscript received at the office of the Institute May 14, 1935. Issued as Contribution 91, November, 1935.

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[1] References are at end of paper.

offers the present method of interpretation of magnetic anomalies. Different assumptions (which the author feels are justified in a certain measure) are made for the establishment of the method and the calculation of the interpretation diagrams.

Assumptions

1. The magnetic susceptibility of sedimentary beds is supposed to be negligible with respect to that of the basement rock, igneous or metamorphic. The magnetic anomalies observed regionally are ascribed to the basement-rock topography. Approximate values of the susceptibility coefficient k of rocks are given in Table I. These show that

Table 1.—*Approximate Values of Susceptibility Coefficient of Rocks* k, 10^{-6} *C.G.S.*

Sedimentary rocks	0–400	(J. Koenigsberger)
Acid igneous rocks:		
Granite	1000–2000	(L. B. Slichter)
Basic igneous rocks:		
Serpentine	2000	(L. B. Slichter)
Gabbro	3000	(L. B. Slichter)
Olivine gabbro	6000	(L. B. Slichter)
Basalt	8000	(L. B. Slichter)

igneous rocks, especially those of the basic type, which often constitute the basement rocks, are much more magnetic than sedimentary beds. Hence the majority of magnetic anomalies observed at the surface of the earth are assumed to be due to igneous rocks and principally to the basement topography. It may seem difficult to justify the assumption that the effect of sedimentary rocks will always be negligible compared with the effect of the basement rocks, especially when layers of black sands are present in the geologic column. The effect of such beds of high susceptibility must be eliminated when the study of the basement topography is the object of the magnetic measurements. The charts given herein may serve that purpose.

2. The rocks, sedimentary or igneous, are assumed to be paramagnetic; that is, do not retain their magnetism after the energizing field has disappeared. Consequently, the magnetic anomalies observed are due to an induced magnetism, which has the direction of the magnetizing field; i.e., the earth magnetic field H. This assumption again may not seem to be justified generally, since it is known that certain mineralizations show a polarity of their permanent magnetism different from that of the earth magnetic field. There is at present no method of interpretation that can take into account such unknown anomalous polarity in the basement complex.

3. It is further assumed that *regional* magnetic anomalies are produced by geologic features that have one horizontal dimension much

larger than their depth below the surface. Hence in interpretation of regional anomalies we deal with so-called two-dimensional geologic features, the horizontal dimension being supposed to be infinite in the direction of strike. A vertical diagram has been calculated on this assumption to be used for two-dimensional bodies. Before any attempt at interpretation of magnetic measurements is made one must ascertain the validity of the preceding assumptions in the region under investigation and must discard large irregular anomalies such as those due to volcanic plugs, basalt and lava flows, magnetite masses, direct current power lines, iron pipes, et cetera.

Charts for Two-dimensional Bodies

Let us consider Fig. 1, in which H represents the total earth magnetic field with respect to a system $Oxyz$ of axes where Ox is horizontal and taken in the direction of strike of the magnetic anomaly. Ox makes

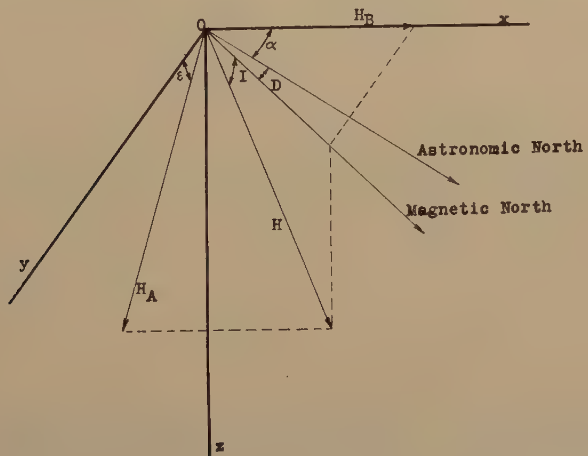

Fig. 1.—Components of the earth magnetic field.

an angle α with the astronomic north. Let D and I be respectively the declination and inclination of the earth magnetic field. For a comparatively small region D and I may be considered constant.

In order to calculate separately the anomalies of the vertical and horizontal components of the earth magnetic field, the vector H is decomposed in two components; one, H_B, along the X axis parallel to the infinite dimension of the geologic feature and the other, H_A, in the plane yOz perpendicular to the same direction. H_A alone is responsible for the vertical and horizontal components of the magnetic anomaly perpendicular to the strike of the geological feature. The values of H_A and H_B are given by

$$H_A = H\sqrt{\cos^2 I \sin^2 (\alpha \pm D) + \sin^2 I} \qquad [1]$$
$$H_B = H \cos I \cos (\alpha \pm D) \qquad [2]$$

which can be computed from Government magnetic maps covering the regions under investigation. It is also necessary to compute the value of the angle ϵ made by the direction of H_A with the horizontal plane; which can be obtained graphically or from the relation $\tan \epsilon = \dfrac{\tan I}{\sin (\alpha \pm D)}$.

Under the influence of the magnetic component H_A, the rocks produce an additional induced magnetic field, which is directly proportional

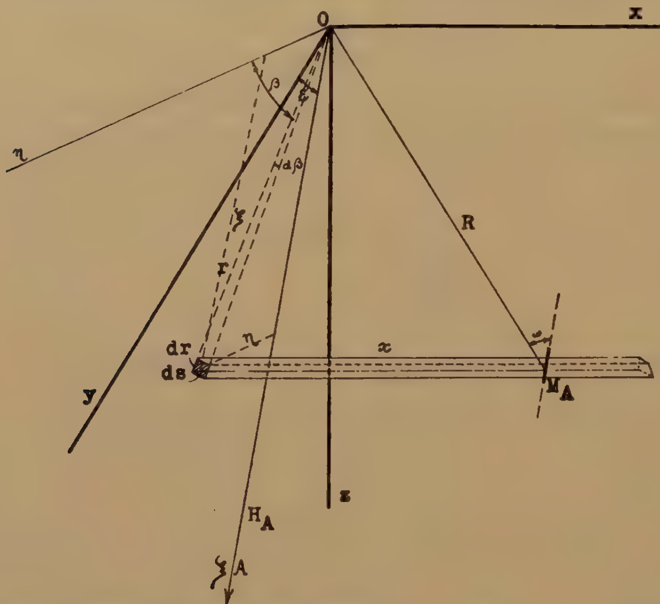

FIG. 2.—TWO-DIMENSIONAL BODIES. EFFECT OF INFINITE CYLINDER OF CROSS SECTION ds FOR TRANSVERSE MAGNETIC POLARIZATION.

to the magnetic susceptibility of the rocks and of the same direction as the inducing magnetic field (assumption 1). Hence, at every point in the substratum, one can consider the presence of an elementary magnet, or dipole, of magnetic moment M_A of which the value is given by $M_A = kH_A$ and which has a direction parallel to H_A (Fig. 2). Every dipole M_A produces at any exterior point O a magnetic potential

$$dV_A = \frac{M_A \cos \omega}{R^2} \qquad [3]$$

where ω is the angle made by the direction of polarization of any dipole M_A with the direction determined by the dipole and the point O. R is the radius vector from the dipole to the point O at which the potential is measured.

Let us consider Fig. 2, where the $O\xi$ axis is taken in the direction of H_A, and calculate the magnetic field produced at a point O of the ground surface due to an infinitely long horizontal distribution of magnetic elements of moment M_A per unit length. Fig. 2 represents half of the infinite magnetic bar of coordinates $\xi\eta$. The magnetic potential due to such an infinite bar is given by:

$$\Delta V = 2kH_A \int_0^{\infty} \frac{\xi dx}{(r^2 + x^2)^{3/2}} \qquad [4]$$

since

$$\cos \omega = \frac{\xi}{(r^2 + x^2)^{1/2}} \qquad [5]$$

Carrying the integration by substitution of $x = r \tan \theta$ [6]

$$dx = r \frac{d\theta}{\cos^2 \theta} \qquad [7]$$

(where θ varies from 0 to $\pi/2$) one obtains

$$\Delta V_A = 2kH_A \frac{\xi}{r^2} \qquad [8]$$

Let us now consider a cross section $ds = rd\beta dr$ in the zoy plane and a cylindrical distribution of magnetic dipoles M_A, the cross section of the cylinder being ds.

The total potential due to a cylinder limited by the angles β_1 and β_2 and the radii r_1 and r_2 is given by

$$V_A = 2kH_A \int_{\beta_1}^{\beta_2} \int_{r_1}^{r_2} \frac{\sin \beta}{r} ds \qquad [9]$$

The component of the anomaly of the magnetic field due to H_A and parallel to H_A is given by:

$$\Delta\xi = -2kH_A \int_{\beta_1}^{\beta_2} \int_{r_1}^{r_2} \left\{ \left[\frac{\partial}{\partial r} \left(\frac{\sin \beta}{r} \right) \right]^2 + \left[\frac{\partial}{r \partial \beta} \left(\frac{\sin \beta}{r} \right) \right]^2 \right\}^{1/2} \cos 2\beta ds \qquad [10]$$

since the elementary magnetic component due to the infinite bar of magnetic elements M_A makes an angle $\pi - 2\beta$ with H_A (Fig. 3).

Carrying the operations indicated in the preceding formula, one obtains

$$\Delta\xi = -kH_A \log_e \frac{r_2}{r_1} (\sin 2\beta_2 - \sin 2\beta_1) \qquad [11]$$

for the expression of the magnetic anomaly in the direction of H_A due to an infinite magnetic cylinder of cross section $ds = r(\beta_2 - \beta_1)(r_2 - r_1)$.

Similarly, the magnetic anomaly in the direction perpendicular to H_A and due to the same cylinder is given by:

$$\begin{aligned}\Delta\eta &= 2kH_A \int_{r_1}^{r_2}\int_{\beta_1}^{\beta_2}\left\{\left[\frac{\partial}{\partial r}\left(\frac{\sin\beta}{r}\right)\right]^2 + \left[\frac{\partial}{r\partial\beta}\left(\frac{\sin\beta}{r}\right)\right]^2\right\}^{1/2} \sin 2\beta ds \\ &= -kH_A \log_e \frac{r_2}{r_1}(\cos 2\beta_2 - \cos 2\beta_1) \qquad [12]\end{aligned}$$

From formulas 11 and 12 is calculated the diagram or reticle of Fig. 4, in which each element represents the cross section of an infinite horizontal

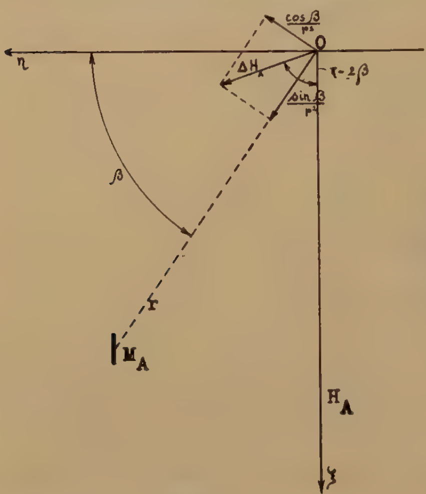

FIG. 3.

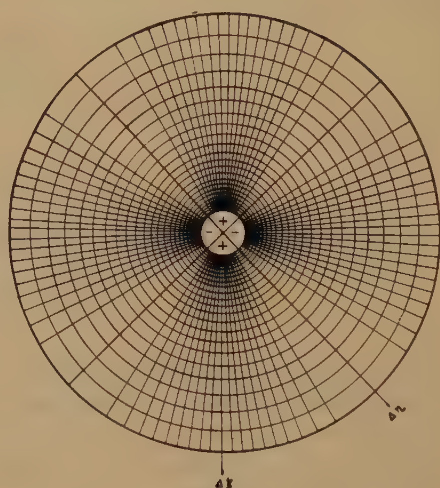

FIG. 4.

FIG. 3.—TWO-DIMENSIONAL BODIES. COMPONENTS OF THE ANOMALY DUE TO AN INFINITE BAR OF MAGNETIC DIPOLES.

FIG. 4.—POLAR CHART FOR THE COMPUTATION OF THE VERTICAL AND HORIZONTAL COMPONENTS OF THE MAGNETIC ANOMALY DUE TO A TWO-DIMENSIONAL BODY. Value of each block: one gamma for $H_A = 0.5$ Gauss and $k = 2000 \times 10^{-6}$ c.g.s.

cylinder of which the effect is to produce an anomaly of the earth magnetic field of 0.00001 Gauss (one gamma) when the intensity of the H_A component of the earth magnetic field is 0.5 Gauss and the magnetic susceptibility of each magnetic element is $k = 2000 \times 10^{-6}$ c.g.s. For other values of H_A and k, the magnetic anomalies are obtained by proportional reduction.

Knowing the anomalies $\Delta\xi$ and $\Delta\eta$, the vertical $\Delta\gamma$ and horizontal $\Delta\mathcal{H}$ components of the magnetic anomalies due to a subsurface geologic feature are calculated by

$$\Delta\gamma = \Delta\xi \cos\epsilon \pm \Delta\eta \sin\epsilon \qquad [13]$$
$$\Delta\mathcal{H} = \Delta\xi \sin\epsilon \pm \Delta\eta \cos\epsilon \qquad [14]$$

the signs $\pm$ being chosen according to the respective directions of $\Delta\xi$ and $\Delta\eta$.

The procedure to be followed in order to obtain the anomalies due to an assumed subsurface geologic structure consist in superposing the diagram Fig. 4 to a cross section of the substratum at right angle to the strike and to compute the number of elements falling in the outline of the geologic feature, signs being taken into account as indicated on the diagram. The components $\Delta\xi$ and $\Delta\eta$ are obtained in laying successively the axes of the diagram marked $\Delta\xi$ and $\Delta\eta$ parallel to H_A. It is worth while to notice that the scale at which the cross section is drawn is immaterial except that horizontal and vertical scales must be equal.

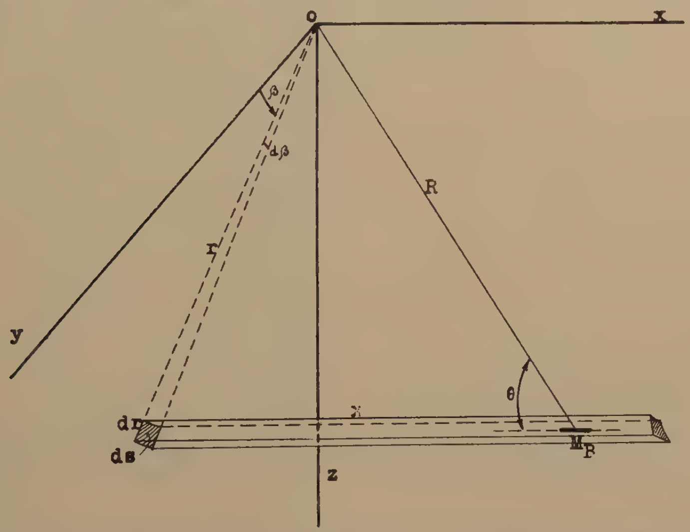

Fig. 5.—Two-dimensional bodies. Effect of infinite cylinder of cross section ds for parallel polarization.

The horizontal component H_B of the earth magnetic field parallel to the strike of the magnetic feature does not contribute to the vertical and horizontal components of the magnetic anomalies, as can be seen from the following considerations. Under the influence of the H_B component (Fig. 1) rocks take an induced magnetism of moment $M_B = kH_B$ per unit of volume. At point O on the surface (Fig. 5) the elementary potential due to M_B is given by $dV_B = \frac{M_B \cos\theta}{R^2}$ where θ is the angle made by the horizontal with the vector R. The component of the anomaly in the Ox direction is obtained by:

$$\frac{\partial V_B}{\partial x} = M_B \log_e \frac{r_2}{r_1} (\beta_2 - \beta_1)(\cos\theta - \cos^3\theta) \qquad [15]$$

for an elementary volume $r(\beta_2 - \beta_1)(r_2 - r_1)dx$. For an infinitely long cylinder, θ varies from O to π. Thus $\frac{\partial V}{\partial x} = O$ and the anomaly

along the strike of a two-dimentional feature is constant. One can verify similarly that the vertical and horizontal components of the anomaly perpendicular to the strike and due to H_B vanish.

Charts for Three-dimensional Bodies

The preceding two-dimensional magnetic charts have been found suitable for the interpretation of regional magnetic anomalies related to broad geologic features connected with the substratum topography. For the interpretation of local anomalies related to the basement relief where the horizontal extent cannot be considered infinite in one direction, the following method and diagrams of interpretation are offered. In this instance the problem is termed "three-dimensional."

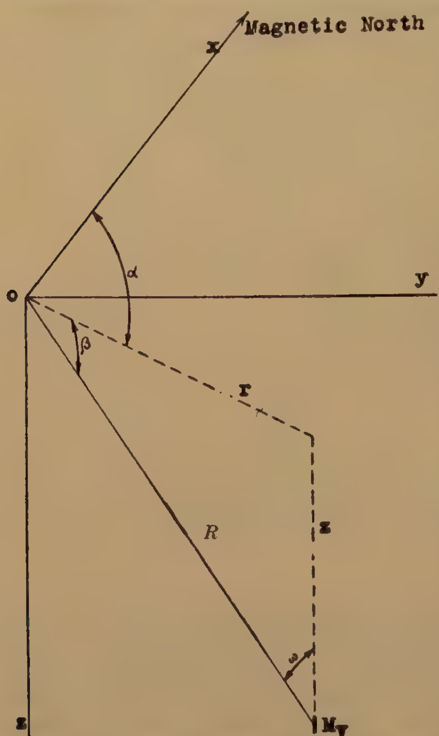

Fig. 6.—Three-dimensional bodies. Effect of magnetic dipole polarized vertically.

Let us consider Fig. 6, where semi-polar coordinates are used for convenience. The magnetic dipole M_v of moment $M_v = kH_v$ induced in the unit volume of the substratum by the vertical component H_v of the earth magnetic field H produces at O an elementary potential

$$dV_v = \frac{M_v \cos \omega}{R^2} \tag{16}$$

where ω is the angle of the radius vector R with the vertical.

Substituting $\cos \omega = \frac{z}{R}$, the potential may be expressed by:

$$dV_v = M_v \frac{z}{(r^2 + z^2)^{3/2}} \tag{17}$$

Considering an element of volume $dv = r d\alpha dr dz$ of the substratum and integrating between the limits z and ∞ for a semi-infinite cylinder of cross section $r(\alpha_2 - \alpha_1)(r_2 - r_1)$ one obtains for the vertical anomaly:

$$\Delta_1\gamma_v = \int_{\text{vol}} \frac{\partial V}{\partial z} dv = M_v \int_{r_1}^{r_2} \int_{\alpha_1}^{\alpha_2} \int_z^{\infty} \frac{(r^2 - 2z^2)}{(r^2 + z^2)^{5/2}} r dr d\alpha dz$$

$$\Delta_1\gamma_v = M_v \log_e \frac{r_2}{r_1} (\alpha_2 - \alpha_1) \sin \beta \cos^2 \beta \tag{18}$$

where β is the angle of radius vector R with the horizontal.

Taking r as an average value of r_1 and r_2 and substituting

$$\tan \beta = \frac{z}{r}$$

in equation 18, one obtains:

$$\Delta_1\gamma_v = M_v \log \frac{r_2}{r_1} (\alpha_2 - \alpha_1) \frac{r^2 z}{(r^2 + z^2)^{5/2}} \qquad [19]$$

Let

$$A = \frac{zr^2}{(r^2 + z^2)^{5/2}} \qquad [20]$$

and the expression of the vertical anomaly becomes:

$$\Delta_1\gamma_v = M_v A \log \frac{r_2}{r_1}(\alpha_2 - \alpha_1) \qquad [21]$$

Assuming that A is a constant equal to 1 and taking $H_v = 0.5$ Gauss and $k = 2000 \times 10^{-6}$ c.g.s. one calculates the polar diagram of Fig. 7 where every element represents an anomaly of a certain number of gammas. This diagram traced on transparent paper is superposed on an assumed topographic map of the substratum and the values of A are evaluated from Fig. 8 according to the average elevation of each element of the substratum. The values of A multiplied by the respective value of each block in γ are added and their sum gives the vertical anomaly at the station considered for the assumed topography. The magnetic effect of the blank circular center part of the diagram of Fig. 7 is obtained by $\pi M_v \frac{r^2}{z^2}$.

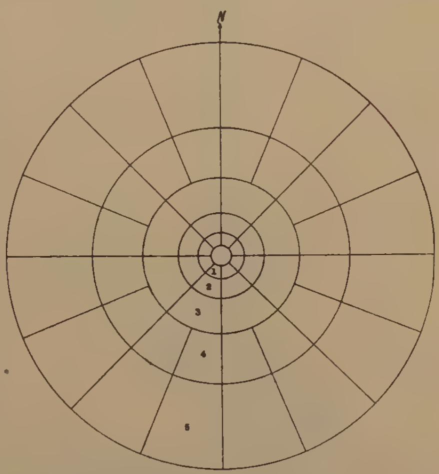

FIG. 7.—POLAR CHARTS FOR THE COMPUTATION OF THE VERTICAL MAGNETIC ANOMALY DUE TO A THREE-DIMENSIONAL BODY VERTICALLY POLARIZED.

Values of individual blocks: Circle 1, 75 gammas; circles 2 and 3, 50 gammas; circles 4 and 5, 20 gammas for $H_v = 0.5$ Gauss and $k = 2000 \times 10^{-6}$ c.g.s.

To evaluate the contribution to the vertical component of the anomaly due to the horizontal component of the earth magnetic field H_h, consider Fig. 9. The induced horizontal dipole M_h produces an elementary potential $dV_h = \frac{M_h \cos\theta}{R^2}$ at point O. θ is the angle made by the direction of polarization of M_h with the radius vector R. Substituting the value of $\cos\theta$ in dV_h as obtained from the relation $R \cos\theta = r \cos\alpha$, we have

$$dV_h = M_h \frac{r \cos \alpha}{(r^2 + z^2)^{3/2}} \qquad [22]$$

The vertical anomaly due to the horizontal component of the earth magnetic field H_h for an infinite vertical block of cross section $r(r_2 - r_1)(\alpha_2 - \alpha_1)$ is obtained by:

$$\Delta_2\gamma_h = \int_{\text{vol}} \frac{\partial V}{\partial z} dv = kH_h \log_e \frac{r_2}{r_1}(\sin \alpha_2 - \sin \alpha_1)\frac{r^6}{(r^2 + z^2)^3} \qquad [23]$$

The diagram computed from this formula is given in Fig. 10, which is to be traced on transparent paper and to be used in the same way as

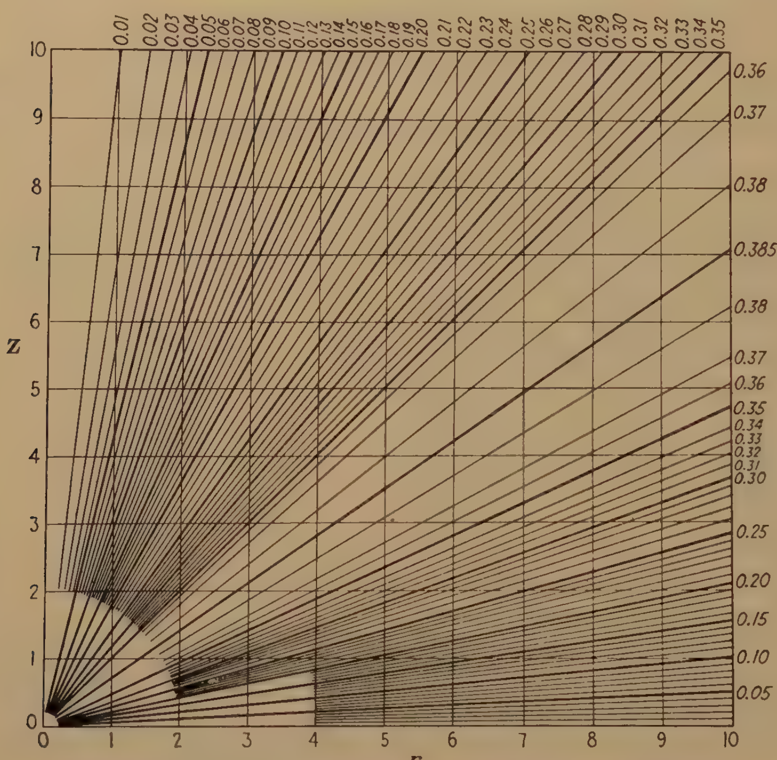

FIG. 8.—VALUES OF COEFFICIENT A AS FUNCTION OF z AND r.

Fig. 7. The arrow N points toward the north. The coefficient $B = \frac{r^6}{(r^2 + z^2)^3}$ is read from Fig. 11 for each elevation z and mean distance r to the block considered. The magnetic effect of the center blank circle of Fig. 10 is negligible here.

The anomalies of the horizontal component of the earth magnetic field due to H may be computed in a similar way for the three-dimensional features. However, only the results of theoretical calculations leading to the establishment of polar diagrams will be given in this paper.

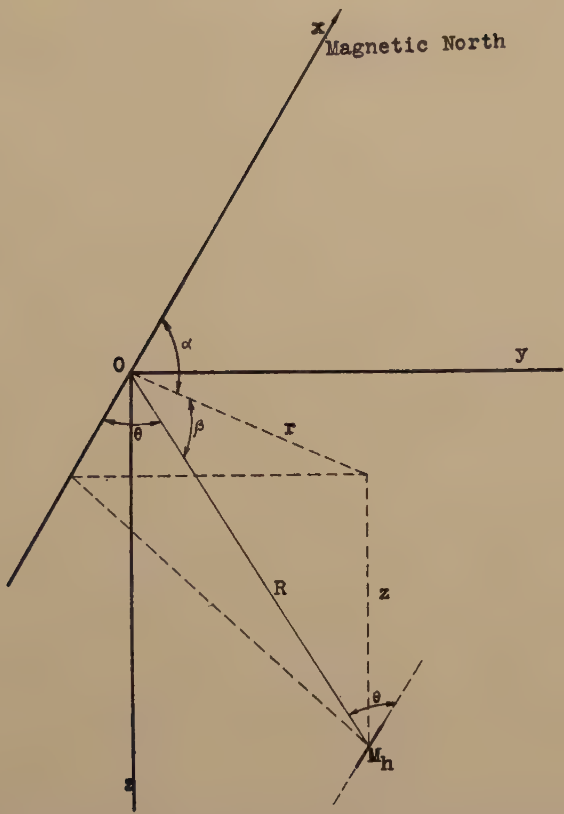

FIG. 9.—THREE-DIMENSIONAL BODIES. EFFECT OF MAGNETIC DIPOLE POLARIZED VERTICALLY.

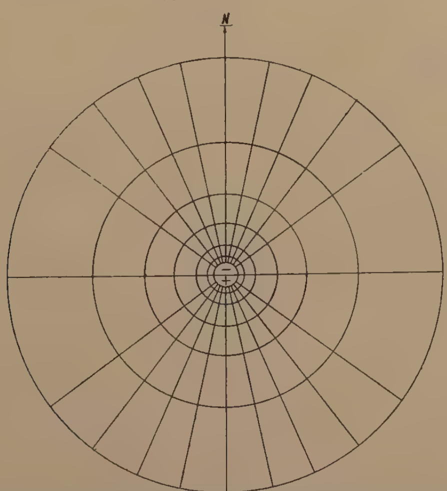

FIG. 10.—POLAR CHART FOR COMPUTATION OF VERTICAL MAGNETIC ANOMALY DUE TO THREE-DIMENSIONAL BODY HORIZONTALLY POLARIZED.

Value of individual blocks: 10 gammas for $H_h = 0.5$ Gauss and $k = 2000 \times 10^{-6}$ c.g.s.

The vertical dipole M_v (Fig. 6) contributes $\frac{\partial V_v}{\partial r}$ sin α to the horizontal east component of the anomaly: Thus:

$$\Delta_1\mathcal{H}_{e_v} = M_v \int_{\text{vol}} \frac{3zr \sin \alpha}{(r^2 + z^2)^{5/2}} r d\alpha dr dz$$

$$\Delta_1\mathcal{H}_{e_v} = kH_v \log_e \frac{r_2}{r_1}(\cos \alpha_2 - \cos \alpha_1)\frac{r^6}{(r^2 + z^2)^3} \qquad [24]$$

Similarly one obtains for the north component of the anomaly:

$$\Delta_1\mathcal{H}_{n_v} = kH_v \log_e \frac{r_2}{r_1}(\sin \alpha_2 - \sin \alpha_1)\frac{r^6}{(r^2 + z^2)^3} \qquad [25]$$

Diagrams given in Figs. 10 and 11 can be used for the anomalies $\Delta_1\mathcal{H}_{ev}$ and $\Delta_1\mathcal{H}_{n_v}$. The magnetic effect of the center blank circle of Fig. 10 can

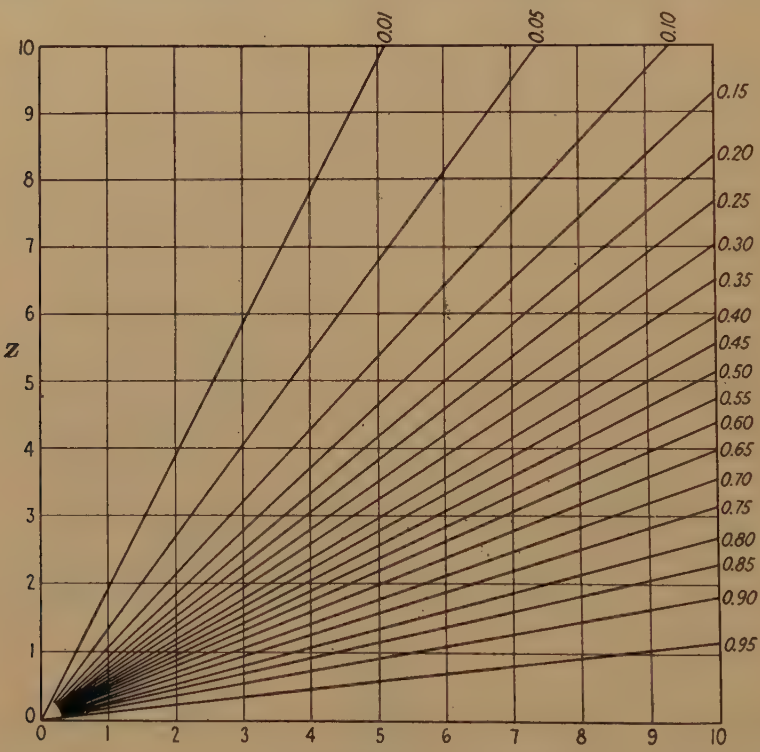

FIG. 11.—VALUES OF COEFFICIENT B AS FUNCTION OF z AND r.

be neglected when the distance from the station to the first circle on the diagram is very small with respect to the depth z.

By similar considerations one obtains the horizontal components of the magnetic anomalies due to the horizontal component H_h of the earth magnetic field. They are given by the following formulas:

$$\Delta_2\mathcal{H}_{n_h} = \int_{\text{vol}} \frac{\partial V_h}{\partial x} dv \doteq kH_h \log_e \frac{r_2}{r_1}\{[(1 - \sin\beta)(\alpha_2 - \alpha_1)] + [2 - (3 - \sin^2\beta)\sin\beta][\tfrac{1}{2}(\alpha_2 - \alpha_1) + \tfrac{1}{4}(\sin 2\alpha_2 - \sin 2\alpha_1)]\} \quad [26]$$

$$\Delta_2\mathcal{H}_{e_h} = \int_{\text{vol}} \frac{\partial V_h}{\partial y} dv = kH_h \log_e \frac{r_2}{r_1}(\sin^2\alpha_2 - \sin^2\alpha_1)\left[\left(\frac{\pi}{2} - \beta\right) - \frac{1}{2}(\cos\beta + 1)\sin 2\beta\right] \quad [27]$$

Formula 26 may be decomposed into two parts which permit graphical computation by polar diagrams:

$$\Delta_2'\mathcal{H}_{n_h} = kH_h \log \frac{r_2}{r_1}(\alpha_2 - \alpha_1)(1 - \sin\beta) \quad [28]$$

$$\Delta_2''\mathcal{H}_{n_h} = kH_h \log \frac{r_2}{r_1}\left[\frac{\alpha_2 - \alpha_1}{2} + \frac{1}{4}(\sin 2\alpha_2 - \sin 2\alpha_1)\right][2 - (3 - \sin^2\beta)\sin\beta] \quad [29]$$

The polar-diagram method of interpretation of magnetic anomalies presented here permits the accurate calculation of magnetic disturbances due to the topography of the basement rocks when the assumptions set forth at the outset of this paper are fulfilled. The assumed basement topography is altered by judicious cuts and tries until the observed and calculated anomalies coincide with a sufficient degree of accuracy. In practice it will be found satisfactory and speedier to calculate and base the interpretation on the vertical components only and to use the horizontal components as a verification. In regions where the inclination of the total earth magnetic field is nearly vertical, a first approximation of the interpretation can readily be obtained by using the vertical component H_v as the total inducing magnetic force of the anomalies. Vertical anomalies will first be calculated for different assumed topographic contours of the substratum by successive cuts and tries. Verifications of the structure will be obtained afterwards by means of the computation of the horizontal magnetic anomalies.

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A Survey of Methods for Determining Depth of Magnetic Ore Bodies

By David A. Keys*

(New York Meeting, February, 1937)

The actual procedure in estimating depth of overburden from magnetic observations made on the surface will vary with the form of the deposit and any theoretical discussion will apply only so far as the form of the deposit conforms to the mathematical premises on which the theory is based. Since the shapes of natural deposits in the field cannot be expressed rigorously in mathematical form, the best that can be done is to assume some simple form and deduce results. Fortunately many small variations in form produce only slight changes in the estimated depth of overburden, therefore the methods have a definite practical value.

For the theoretical treatment, magnetic ore deposits may be divided roughly into three classes, depending upon whether the magnetic anomalies may be considered to arise from (1) a single magnetic pole, (2) a dike or (3) a magnetic doublet. Particular cases, falling under one or other of these classes, have been discussed by different authors.[1–4] In general, it may be stated that when the magnetic effects as measured on the surface are due only to one pole of the body, the other pole being so remote that its effect may be neglected, the resultant anomaly F at any point will vary as the inverse square of the distance r of the point from the pole, which may be expressed mathematically by $F = k/r^2$, where k is a constant. When the deposit occurs in the form of a long, thin dike, the resultant anomaly will be of the form $F = k'/r$ and when the deposit is in the shape of a lens, so that both poles have an appreciable effect on the surface, the result will be similar to that caused by a magnetic doublet, and the form will be $F = k''/r^3$.

The particular class under which a deposit falls may be deduced from the results of the magnetic survey. The second class is readily found, but there may be difficulty in deciding between the first and third. Isodynamic contours of both the vertical and horizontal anomalies will often be a good guide in making a correct decision.

The correctness of the theoretical calculations may be checked in two ways. The first is to make laboratory models of various known shapes and pole distributions, to calculate the anomalies to be expected

Manuscript received at the office of the Institute Feb. 15, 1937. Issued as T.P. 830, August, 1937.

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[1] References are at the end of the paper.

and compare these anomalies with those actually found by experiment. The depth below the level on which the anomalies have been measured can then be calculated by the various methods and the results compared with the known values. If the methods are correct, the experimental results will agree with the true values. Such laboratory experiments are of interest and serve as a check on the validity of the methods, and may also suggest where approximations may be warranted. A second method is to apply the theories to field observations in places where the results may be tested by actual drill records. This method is important, and is the one that has been followed in this paper. The theory and its application in the field will be given for each method, and a numerical check on the calculated depth of overburden from various methods will be given for cases in which the author has personally made the field observations of the magnetic anomalies.

I. When the Magnetic Anomalies Arise from a Single Pole

Such cases occur when the deposit is near the surface and extends nearly vertically to so great a depth that the other pole is remote. The position of the pole is found from the survey. Several methods have been described for finding the overburden in such a case.[1,4]

Method 1.—If ΔZ_m is the maximum value of the vertical anomaly over such a deposit and x the distance measured along the horizontal surface to a point P, where the vertical anomaly $\Delta Z_p = 0.353\Delta Z_m$, the depth of overburden will be equal to x. This follows from the fact that if the line joining the point P to the pole makes an angle θ with the vertical, then $\Delta Z_p/\Delta Z_m = \cos^3 \theta$ and $x = d \tan \theta$ where d is the depth of overburden. If $\theta = 45°$, $\cos^3 45 = 0.353$.

Method 2.—The horizontal ΔH and vertical ΔZ anomalies near the point M of maximum vertical anomaly are determined for various points. These may be combined vectorially at each point and the direction of the resultant field due to the ore body may be found. The resultant vector will be tangential to the line of force at that point and will make an angle φ with the horizontal given by $\tan \varphi = \Delta Z/\Delta H$. From our knowledge of the distribution of lines of force, it follows that the vectors near M will intersect very nearly at the pole of the body. When the vector diagram is drawn to scale, the depth to the pole may be determined.

Method 3.—Suppose ΔZ_m is the value of the maximum vertical anomaly on the surface. Erect a platform and measure the value of the vertical anomaly ΔZ on the platform at a vertical height h above the ground. It follows from the inverse-square law of force from a pole that $\dfrac{\Delta Z_m}{\Delta Z} = \dfrac{(h + d)^2}{d^2}$ where d is the depth to the pole. Hence we obtain the value of d from

$$d = \frac{h}{\sqrt{\Delta Z_m/\Delta Z} - 1}$$

II. When the Deposit Is a Long Narrow Dike

This form of the deposit is revealed both from the vertical and horizontal magnetic anomalies. The strike of the dike may be ascertained, and its magnetic axis. A survey is then made along a line at right angles to the strike of the dike and the vertical and horizontal anomalies are determined at small intervals along this line, so as to obtain a smooth curve of the anomalies along the line.

Method 1.—If the dike is vertical or nearly so and extends to such a great depth that the effect of the other pole may be neglected, it may be shown[5] that the value of the maximum vertical anomaly ΔZ_m at M on the surface will be

$$\Delta Z_m = \frac{2mb}{d}$$

where m is the magnetic pole strength per unit area, b the width of the dike and d the depth from the surface to the top of the dike. Then the vertical anomaly due to the dike at a point P at a distance x from M along the line at right angles to the strike will be given by

$$\Delta Z_p = \frac{2mbd}{r^2} = \frac{2mbd}{x^2 + d^2}$$

where r is the distance from P to a point on the ore body vertically below M. If the point P be so chosen that $\Delta Z_p = \Delta Z_m/2$, then

$$\frac{2mbd}{x^2 + d^2} = \frac{1}{2} \cdot \frac{2mb}{d} = \frac{mb}{d}$$

Hence

$$d = x.$$

Thus the depth of the overburden is found by finding the distance along the line perpendicular to the strike of the dike at which the vertical anomaly is one-half the maximum value.

Method 2.—The depth of overburden above the dike may be determined by drawing the vector diagram of the resultant anomalies near the point of maximum vertical anomaly. This procedure is the same as that given in case I, method 2, and the amount of overburden is determined from the intersection of the vectors.

Method 3.—This method is similar to that described in case I, method 3, but the law of force in connection with a dike is different from that with a single pole. The value of the maximum vertical anomaly ΔZ_m is found on the surface and the value of the vertical anomaly ΔZ at a point h vertically above (determined on a platform). It then follows that

$$\frac{\Delta Z_m}{\Delta Z} = \frac{2mb}{d} \bigg/ \frac{2mb}{d + h} = \frac{d + h}{d}$$

or $$d = \frac{h \cdot \Delta Z}{\Delta Z_m - \Delta Z}$$

where d is the depth.

Method 4.—When the curves for the vertical and horizontal anomalies along the traverse perpendicular to the strike of the dike are examined, a point P may be found on this line at a distance x from the point M of maximum vertical anomaly, at which the horizontal anomaly will be of the same value as the vertical. Since the horizontal anomaly at M, the point above the magnetic axis of the strike of the dike is zero, the point P where the horizontal and vertical anomalies are of equal magnitude must be at the same distance from the point M as M is above the top of the dike. Hence if at P the values of $\Delta H = \Delta Z$, the depth $d = x$.

III. When the Deposit Is in the Shape of a Lens

When the deposit is a lens, both poles of the equivalent magnet may produce appreciable anomalies on the surface. The effect will then be similar to that produced by a magnetic doublet or shell. The depth that can be calculated will be to the center of the equivalent magnet. Both the magnitude and direction of the resultant anomaly at any point on the surface may be deduced from a magnetic survey. Let ΔR_m be the maximum value of the anomaly due to the ore body. Anomalies near this point may be measured on a different level by using a platform 10 or 15 ft. high. If ΔR_h is the magnitude of the maximum anomaly on the platform, the distance to the center of the lens may be deduced by applying the inverse-cube law. The application to field observations of this method will be most reliable when the axis of the equivalent doublet is either vertical or horizontal. Taking the former case as an example, the value of the maximum anomaly on the surface will be the vertical component ΔZ_m. On the platform directly above, the vertical anomaly will be ΔZ. If h is the height of the platform, it follows that

$$\frac{\Delta Z_m}{\Delta Z} = \frac{(d + h)^3}{d^3}$$

or $$d = \frac{h}{\sqrt[3]{\Delta Z_m/\Delta Z} - 1}$$

where d is the depth to the center of the lens.

IV. Field Examples for the Case of a Long Narrow Dike

How well field practice agrees with theory may best be illustrated by applying the different methods to a particular case. For this purpose the results of a survey over the nickel pyrrhotite vein of the Falconbridge nickel mine was used. The depth of the overburden at the place where the survey was made was known from diamond drilling to be 118 ft. As

this deposit is a long dike of considerable vertical depth, the conditions present are those specified in case II. The curves of both vertical and horizontal anomalies along a line perpendicular to the strike were obtained

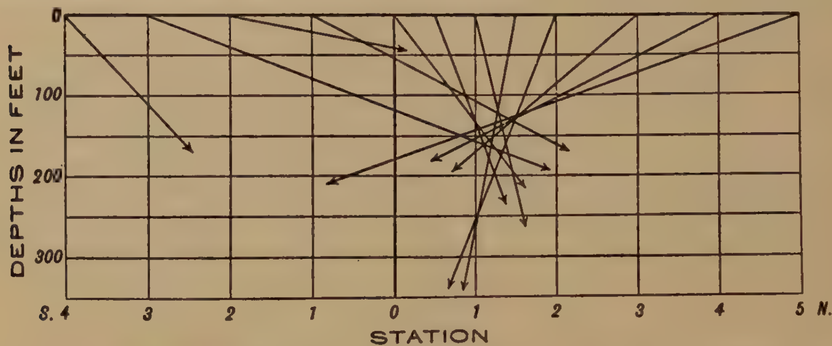

FIG. 1.—VECTORS OF RESULTANT MAGNETIC ANOMALIES DUE TO A MAGNETIC DIKE.

by the author and others,[6] and the platform experiment was carried out. The results of applying the different methods to this case are shown below.

Drill-hole depth of overburden = 118 ft.

Method 1.—$\Delta Z_p = \Delta Z_m/2$. Then $d = x$. Experimental result = 117.5 ft.

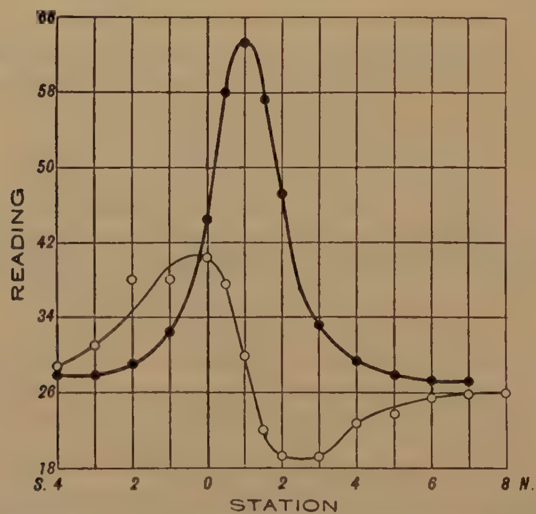

FIG. 2.—HORIZONTAL AND VERTICAL ANOMALIES ACROSS A MAGNETIC DIKE.

Method 2.—Vector method of ΔH and ΔZ. Experimental result = 120 ft.

Method 3.—$d = \dfrac{h \cdot \Delta Z}{\Delta Z_m - \Delta Z}$.

$h = 10$ ft., $\Delta Z_m = 33.2$, $\Delta Z = 30.6$. Experimental result = 117.7 ft.

Method 4.—Distance to point where $\Delta H = \Delta Z$. Experimental result $= 116$ ft.

Mean Result.—The mean result of the four different methods is 117.8 ft., in remarkable agreement with the actual depth. The fact that the different methods give consistent results indicates that such theoretical considerations may be applied successfully to dikes. The experimental results are shown in Figs. 1 and 2.

Conclusion

The methods presented for finding the depth of overburden are those in which the author has faith, providing due regard is given to the deviations that will occur when the conditions are not as simple as those specified. In the experimental case IV, the dike was perhaps a favorable one to use in applying such methods. In connection with the Falconbridge vein, the deviations from the mathematical ideal form on which the methods are based do not produce appreciable errors. The results show that the depth of overburden can be reliably estimated from magnetic observations made on the surface.

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DISCUSSION

(*C. A. Heiland presiding*)

L. Gilchrist,* Toronto, Ont.—Since I was working in the same territory as Professor Keys and also later more extensively over the same territory, I would urge that as many methods as possible be used in obtaining the depth. In this territory new bodies have thus been located.

A great deal of preliminary scouting work is usually done in regions of high latitude with the vertical magnetometer, usually making observations at points somewhat widely separated and at times obtaining evidence of intense magnetic force at isolated points. Since the mathematical formulative representation of the distribution of force is an expression of continuity, there are only two conclusions. Either the source of the force is near the surface and the points of observation are not sufficiently close or numerous to give adequate results for interpretation, or errors have been made in the observations. To locate an isolated center of the magnetic force, the vertical component, the horizontal component, and the declination should be obtained and a simple vector method may then be applied.

* University of Toronto.

A. C. LANE,* Cambridge, Mass.—Keys' interesting paper reminds me that T. B. Brooks was one of the earliest to call attention to the variation of magnetic anomalies† with elevation. He did not go into the mathematics of it, but gave data and a curve to which Keys' formula might be applied.

Personally, I have used only the theory of case 1, and called attention in teaching to the anomalies when a compass is laid on a rock as compared with those given when it is held at waist height or at eye height, as when the Brunton is used in sighting.

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* Tufts College.

† Geological Survey of Michigan (1872) **1**, pt. 1, chap. 8, sec. 5, 231–236. See also *Proc.* Amer. Phil. Soc.

Reference Datum for Magnetometer Surveys

By F. C. Farnham*

(New York Meeting, February, 1939)

Abstract

In this paper it is shown that the vertical component of the earth's magnetic field for the area of the United States can be very closely approximated by assuming it to be the sum of the vertical components of two magnetic fields. The first of these is the field of the centered dipole described by J. Bartels,[1] the axis of this dipole intersecting the earth's surface at two antipodal points, the one in the Northern Hemisphere being at latitude 78.5°N. and longitude 69°W., and the dipole having such strength that the vertical component of its field at its axis pole is 0.63 oersted. The second field is that due to a second dipole situated halfway between the center of the earth and a point on the earth's surface at latitude 38.5°N., longitude 91.5°W., the axis of this dipole being along this radius, the south pole being toward the surface, and the dipole being of such strength that its field at the point on the surface mentioned above is 0.069 oersted. Using the field computed on this basis, tables of latitude and longitude correction factors were computed and are included in the paper. It is also shown that the secular variation in the earth's magnetic field may be roughly accounted for by assuming it to be due to motion of the second dipole described above.

Although there is an error in the equation used for computing the strength of the field due to the second dipole, the distribution of the field computed is very nearly that of the assumed arrangement of dipoles. Subsequent work by the author reveals that minor changes can be made in the location of the second dipole, which will make the computed field fit the observed field even more closely. It has also been found[2] that the vertical component of the field may be represented equally well by a purely empiric relationship.

Manuscript received at the office of the Institute Feb. 16, 1939. Entire paper issued as A.I.M.E. Tech. Pub. 1077, April, 1939.

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[1] J. Bartels: The Eccentric Dipole Approximating the Earth's Magnetic Field. *Terrestrial Magnetism and Atmospheric Electricity* (Sept. 1936) **41**, 225–250.

[2] F. C. Farnham: An Empiric Approximation of the Vertical Component of the Earth's Magnetic Field for the United States. *Amer. Jnl. of Physics* (1940) **8** (1), 69.

Observations on Compensated Magnetometer Systems

By Mark C. Malamphy,* Member A.I.M.E., and Irnack C. do Amaral†

(New York Meeting, February, 1935)

Abstract

Four magnetic field balances and several spare magnetic systems were purchased by the Brazilian National Department of Mineral Production from the Askania Werke A. G., of Berlin, in 1932. Preliminary field tests on this new equipment showed one of the vertical component magnetic systems to be defective. A series of measurements designed to investigate the functioning of these instruments, and particularly the defective system, showed it to have an abnormally low magnetic moment, as compared to the satisfactory systems, and a high-temperature correction that was opposite in sign to that which would be expected for a noncompensated magnetic system.

Since these systems were of the "temperature compensated" type, this condition indicated an accidental overcompensation. Attempts to correct for this overcompensation by readjustment of the compensating masses on the aluminum and Invar spindles failed because the distance by which the position of the masses could be shifted was insufficient. The temperature effect was reduced by this readjustment but was not entirely eliminated, as was to be desired.

Theoretical deductions indicated that the defective condition of the system could be explained as due to three possible causes: (1) considerable loss of magnetic moment of the system; (2) increase in the magnetic temperature coefficient of the laminas; (3) dislocation of the component parts of the system caused by a shock experienced after it left the factory and before it was tested in Brazil.

Later information received from the factory after examination of the returned system indicated that the third cause probably played the most important role.

Temperature calibrations of the defective system for various positions of the counterweights brought out an important point, which it is believed

Published by permission of the Director of the Departmento Nacional dé Producção Mineral, Ministerio de Agricultura, Brazil. Manuscript received at the office of the Institute Nov. 30, 1934. Entire paper issued as A.I.M.E. Contribution 80, February 1935.

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can be applied to all magnetic systems of the "temperature compensated" type:

1. If the system is in good adjustment and therefore essentially compensated for temperature, there is very little time lag between the temperature changes and corresponding magnetic changes.

2. If the system is imperfectly adjusted and therefore not adequately compensated for temperature changes, the time lag will be appreciable and there will be a hysteresis effect.

3. If the temperature gradient is more or less constant in magnitude and direction, a temperature correction may be safely applied to such a system. However, if the temperature changes are fluctuating in both magnitude and direction the time lag and hysteresis effect will result in erroneous readings that cannot be corrected for by means of a temperature correction factor.

Gravimeters: Their Relation to Seismometers, Astatization and Calibration

By C. A. Heiland,* Member A.I.M.E.

(New York Meeting, February, 1939)

Measurements of gravity with gravimeters have come into increased use in this country and abroad in the past five years. Probably 100 to 125 gravimeter parties are working in the United States alone. In oil exploration, gravimeter methods have relegated the gravity pendulum definitely to second place, if not third, and now closely rival and possibly surpass the torsion balance in many applications. Gravimeters have lately been applied in mining, although it is doubtful whether they will ever replace electrical or magnetic methods in this work.

Notwithstanding the widespread application of gravimeters, there has been no article covering the entire field in the domestic or foreign literature. The purpose of this paper is to meet the demand for comprehensive engineering information on the subject.

The working principles of a number of gravimeters will be illustrated and treated in relation to the action of corresponding seismometers and astatization methods. The mathematical discussion of theory has been confined to the most necessary fundamentals and been given in as elementary form as possible. A description of calibration methods constitutes the last part of the paper.

The symbols shown at the top of page 197 will be used in equations throughout the paper.

Dynamic and Static Gravity Methods

Methods of measuring gravity fall into two groups: dynamic and static methods. The first group involves the measurement of time. Examples are: determinations of the period of oscillation of a pendulum, measurements involving the free fall of a body, and comparisons of gravity with the centrifugal force of a rotating body.

Measurements in the second group of gravity methods are concerned with *deflections* of certain indicators when the force of gravity is in equilibrium with another comparison force.

Manuscript received at the office of the Institute Nov. 29, 1938. Issued as T.P. 1049, April, 1939.

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Symbols

a, distance.
A, degree of astatization.
c, spring constant.
d, deflection or vertical distance.
D, vertical distance.
e, distance.
E, potential difference
g, gravity.
h, restituting force.
H, horizontal force.
k, labilizing force.
K, moment of inertia.
l, distance.
L, reduced pendulum length.
m, mass.
M, total mass.
n, scale reading.
p, pressure.
q, resulting restituting force.
Q, force.
r, distance; radius.
R, distance.
$\left.\begin{matrix} s \\ S \end{matrix}\right\}$ sections; surfaces.
T, period of oscillation.
v, volume.
V, geometric magnification.
w, weight.
W, weight.
z, reading on barometer column.
α, angle of torsion.
β, angle of deflection.
δ, density.
θ, angle of rotation or deflection.
τ, torsion coefficient.
φ, angle of deflection or rotation.
ϕ, angle of rotation.
ψ, angle.
ω_0, natural frequency.

Classification of Gravimeters

Only two comparison forces have been used in the construction of gravimeters to date: (1) gas pressure, and (2) spring forces. Thus gravimeters fall into two logical groups: (I) "pneumatometric" and (II) "mechanical" gravimeters. In the first group the pressure of atmospheric air (barometric method) or that of a confined volume of gas (volumetric method) may be utilized. Gravimeters in the second group may be divided into instruments without and with astatization; the latter may utilize the principle of the horizontal or of the vertical seismometer. The better known types of gravimeters fall therefore into the following groups of this classification:

I. *Pneumatometric Gravity Methods*
   1. Barometric gravimeters or gravity methods (Hecker-Mohn)
   2. Volumetric gravimeters (Haalck)

II. *Mechanical Gravimeters*
   1. Unastatized gravimeters (Threlfall and Pollock, Wright, Lindblad-Malmquist, Hartley, Graf-Askania, Gulf)
   2. Astatic gravimeters
      a. Horizontal seismogravimeters (Ising)
      b. Vertical seismogravimeters
         (1) Bifilar gravimeters
         (2) Trifilar gravimeters
         (3) Truman gravimeter
         (4) Mott-Smith gravimeter
         (5) Thyssen gravimeter

Past experience indicates that much greater accuracy may be obtained with mechanical gravimeters. The greater portion of this review will be devoted to them after methods of measuring gravity in the first group have been briefly discussed.

Barometric Gravity Method

If atmospheric pressure is measured at the same locality with an aneroid and with a mercury barometer, the same values would result were it not for the fact that the mercury barometer readings are affected by gravity. At another location the difference between the two values will change in proportion with the change in gravity. For even moderate requirements of accuracy the aneroid was not sufficient, and was soon replaced by measuring the boiling point of water. The barometric method was the first to make possible determinations of gravity on board ship and established the fact that isostatic compensation prevailed not only for most continental but oceanic areas as well. It was perfected principally by Hecker during several voyages. The required precision is considerable: the boiling point of water must be determined with an accuracy of $1/1000$ of a degree Centigrade and the air pressure must be read on the barometer with an accuracy of 0.01 mm. The mean error of this method is as large as ± 40 milligals; the main difficulty is to read the menisci accurately enough on the barometer on moving support. What accuracy this method would furnish in a land survey has probably not been determined; at any rate, it is not likely to even approach the accuracy of a modern gravimeter.

Volumetric Gravity Method

Instead of using the variable atmospheric pressure as comparison force, it is more advantageous to use an inclosed gas volume, kept as constant as possible. In comparing this with a mercury barometer, two volumes are employed, so that the position of the mercury column will indicate their difference in pressure. If the pressure difference is kept sufficiently constant, changes in the position of the menisci of the mercury barometer will indicate changes in gravity. The Haalck gravimeter (Fig. 1) is based upon this principle.[1] In this figure, v and v' are the two volumes, and z and z' are the positions of the mercury menisci, with reference to an arbitrary zero point. If p is the pressure in volume v and p' the pressure in volume v', the difference in pressure Δp must be equal to the weight of the mercury column, so that

$$\Delta p = \Delta z g \delta \qquad [1]$$

where δ is the specific gravity of the mercury.

For differential changes of gravity, reading and pressure:

[1] References are at the end of the paper.

$$dp' - dp = g\delta(dz - dz') + (z' - z)\delta g d \qquad [2]$$

As, according to the gas law,

$$dp' = -\frac{p'}{v'} \cdot dv' \qquad \text{and} \qquad dp = -\frac{p}{v} \cdot dv$$

the changes in gas volume may be put equal to the products of tube section S and change in reading:

$$dv' = -Sdz' \qquad \text{and} \qquad dv = -Sdz$$

and by substitution into equation 2:

$$dg = \frac{g}{p' - p}\left\{dz\left(S\frac{p}{v} + g\delta\right) - dz'\left(S\frac{p'}{v'} + g\delta\right)\right\} \qquad [3]$$

To obtain the required accuracy in gravity it is necessary to increase the sensitivity in reading changes in height of the menisci. This is done by

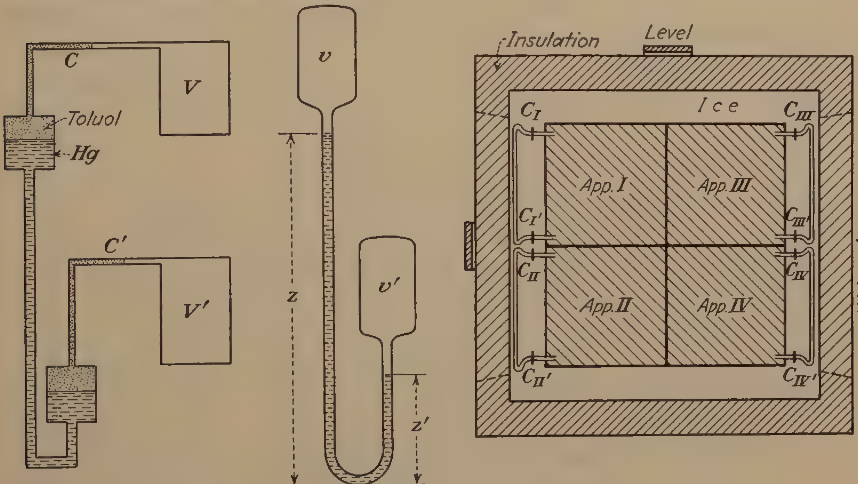


FIG. 1.—HAALCK GRAVIMETER.[1]

connecting the U-tube to vessels of greatly increased section and covering the mercury with a lighter liquid (toluol). Connection between the larger vessels and the gas volumes is made by small capillary tubes C and C', respectively (Fig. 1), on which the readings are taken. The increase in the accuracy of the latter is proportional to the ratio of the large section S to that of the capillary s. In the Haalck apparatus, this ratio is about 10,000; the ratio of the two volumes is more than 2:1 and the distance of the two mercury surfaces about 1.3 meters. With dx and dx' as the change in the readings on the horizontal capillary tubes, formula 3 becomes, with s as the section of the capillary, S the section of the large vessel, and δ' the density of toluol:

$$dg = \frac{gs}{p' - p}\left\{dx'\left(\frac{p'}{v'} + \frac{(\delta - \delta')g}{S}\right) - dx\left(\frac{p}{v} + \frac{(\delta - \delta')g}{S}\right)\right\} \qquad [4]$$

To this, an effect due to inclination of the capillaries must be added. By substituting for the terms in brackets two "scale values" C_1 and C_2, respectively, the change in gravity as measured by the difference in the capillary readings becomes:

$$dg = C_1 dx' - C_2 dx$$

From the dimensions of the instrument, the scale values can be calculated or determined experimentally by tilting the instrument (as the gravity changes from g to $g \cdot \cos \varphi$). In an instrument with the dimensions given above, a change of the menisci relative to one another by about 1 mm. corresponds to a change in gravity of 1 milligal.

In the first experimental model, the mean error was ± 10 mgal. The latest model is a quadruple apparatus (Fig. 1) with a number of improvements; the accuracy attainable with it is estimated to be about 1 mgal. One advantage of the instrument is that it may be suspended in gimbals and be used in a truck without firm support, or on board ship. Haalck[1] has published the results of a number of gravity surveys made with this instrument on the Elbe River and in the Baltic Sea.

Mechanical Gravimeters

In mechanical gravimeters, the elastic forces of springs or wire suspensions are used for a comparison with gravity. Some of the principles involved here have been applied in other lines of physical measurements. For instance, it is well known that the restituting force of a bifilar suspension is directly proportional to gravity.

In its simplest form a mechanical gravimeter consists of a helical spring from which a mass is suspended. Another simple form is a horizontal leaf spring, clamped on one end and loaded with a mass on the other. Changes in the position of the mass in either form of gravimeter between locations indicate changes in gravity, provided all other effects producing such changes are eliminated. The difficulties of keeping these effects (temperature, elastic hysteresis) as low as possible have been responsible for the length of time that has elapsed since the first suggestion of a spring gravimeter by William Herschel in 1833. Recent accomplishments in the construction of gravimeters have largely been due to progress made in the field of metallurgy. The design of a field-worthy gravimeter is largely a matter of suitable selection of construction (spring, wire, beam and case) materials.

That the restituting force of a bifilar suspension is proportional to gravity has been known since C. F. Gauss (1837) introduced such suspension of magnetometers. A. Schmidt, in 1898, replaced the restituting moment of the earth magnetic field by a helical spring and created the trifilar gravimeter, but pointed out that the torsional deflections of a disk suspended by a helical spring had been used as early as 1862 by M. Perrot

to record minute variations in gravity. Since then, bifilar and trifilar gravimeters have been perfected, chiefly by Schweydar, Berroth, Tomaschek and Schaffernicht, and Ising.

In most mechanical gravimeters, in the unastatized instruments in particular, the deflections of the mechanical systems are so small that they have to be magnified. This is done optically, by mirrors or microscopes, or electrically by displacement meters (condensor-ultramicrometer).

The sensitivity of gravimeters is increased by these optical and electrical arrangements, but the determining factor is, first of all, their mechanical construction. A comparison of gravimeters with the corresponding seismometers is useful because it shows that their mechanical sensitivity is controlled by the same laws. This comparison indicates also that "*astatization*" mechanisms used in seismometers are equally useful in increasing the sensitivity of gravimeters.

Comparison of Gravimeters with Seismometers

Credit for first pointing out the relationship between seismometers and gravimeters should probably go to A. Schmidt (1899). Two earlier findings probably were responsible for his observations: (1) bifilar arrangements acting as sensitive seismometers in magnetic observatories; (2) application of horizontal seismometers for recording time variations in the direction of gravity.

Briefly, the relationship between seismometers and gravimeters may be condensed into the statement that the sensitivity of both instruments is inversely proportional to the square of their natural frequency. This is readily shown for a loaded spring whose angular frequency ω_0 is given by $\omega_0^2 = \frac{c}{m}$ with m as mass and c as spring constant, or force per unit deflection. As $c = \frac{mg}{d}$ (d = deflection), $\omega_0^2 = \frac{g}{d}$ or

$$\Delta d = \frac{\Delta g}{\omega_0^2} \tag{5}$$

and by substituting $\omega_0^2 = \frac{g}{L}$ with L as reduced length of the equivalent mathematical pendulum,

$$\frac{\Delta d}{L} = \frac{\Delta g}{g} \tag{6}$$

With V as geometric magnification of the optical or electrical arrangement used with gravimeter or seismometer, the reading $n = d \cdot V$ so that

$$\Delta n = V \cdot L\frac{\Delta g}{g} \quad \text{and} \quad \Delta n = V \cdot \frac{\Delta g}{\omega_0^2} \tag{7}$$

It was recognized in seismology at an early date that the lower the natural frequency of a seismometer, the more closely would its record

approach true ground motion. Lowering the natural frequency has been accomplished by various means, ranging from the simple inversion of a regular pendulum (as in the Wiechert astatic seismograph) to more intricate combinations of lever and spring systems. It is evident from the last equations that a low natural frequency increases the sensitivity of a gravimeter. "Astatizing" mechanisms employed in seismometers and gravimeters are discussed later in this paper. Introduction of astatic systems has also been useful in dynamic gravity methods (Lejay-Holweck pendulum).

The majority of gravimeters are modified vertical seismometers. In station seismology, the simplest types of vertical seismometer (vertical loaded helical spring or horizontal loaded leaf spring) have seldom been used because of their high natural frequency. A decrease of frequency was effected in the vertical helical spring seismograph by combining it with a lever arm (Gray seismometer). If l is the length of the lever and a the distance of the point to which the spring is attached, from the axis of rotation (Fig. 13, ref. 2), the natural frequency $\omega_0 = \frac{a}{l}\sqrt{\frac{c}{m}}$ and the sensitivity of the corresponding gravimeter

$$\Delta n = \frac{Vm}{c} \cdot \Delta g \cdot \left(\frac{l}{a}\right)^2 \qquad [8]$$

Several gravimeters have been constructed along the lines of the Gray seismometer. A seismograph of this kind may be astatized by attaching the spring below the lever axis. Before the details of astatizing mechanisms are discussed, a few of the unastatized gravimeters will be described.

Unastatized Gravimeters

Threlfall-Pollock Gravimeter

The Threlfall-Pollock[3] gravimeter (Fig. 2) is one of the earliest examples of a gravimeter without astatization. It consists of a torsion wire to which is attached a thin quartz bar supported at either end by two studs. The twist of the wire is so adjusted that the bar is in horizontal position. Changes in the position of the bar are read by a microscope.

Assume that the position in which the bar hangs vertical is the initial position, and that it takes a twist of $\theta°$ of one stud and of $\phi°$ of the other to make the bar nearly horizontal, or at the angle φ with the vertical. Then if τ is the torsion coefficient of one-half of the wire, the torsional moment will be $\tau(\theta - \varphi)$ due to one and $\tau(\phi - \varphi)$ due to the other side. This is equal to the moment of gravity of the bar. If l is the distance of its center of gravity from the thread and m its mass, the general equation for this gravimeter is

$$mgl \sin \varphi = \tau(\theta - \varphi) + \tau(\phi - \varphi) \qquad [9]$$

The suspended bar will be in stable equilibrium as long as $\frac{d\varphi}{d\theta}$ remains finite; the lever will upset when $\cot \varphi = \frac{-2}{(\theta - \varphi) + (\phi + \varphi)}$. In this particular instrument this took place for three revolutions of each of the

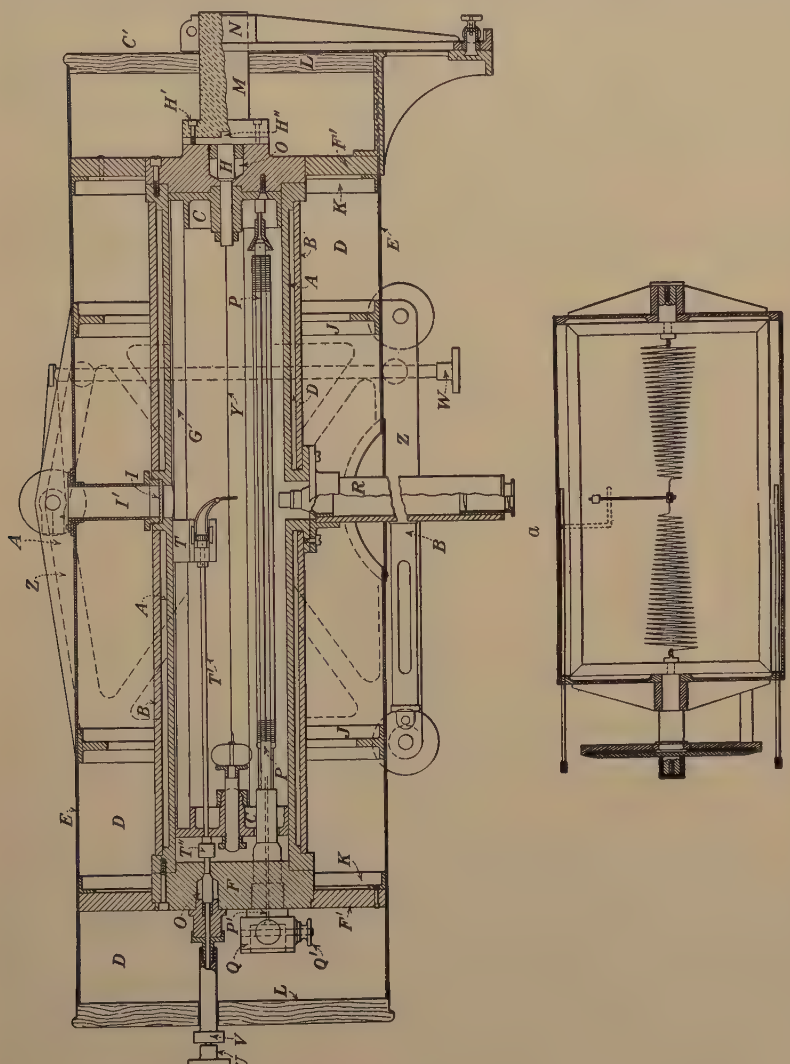


FIG. 2.—THRELFALL AND POLLOCK GRAVIMETER (*a*)[3] AND WRIGHT GRAVIMETER (*b*).[4]

studs; i.e., when $\cot \varphi = -\frac{1}{6\pi}$. The deflection of the lever was not read directly but compensated by a rotation of one of the studs until the end of the bar was coincident with a reference line in the microscope. The sensitivity of the instrument—the change in adjustment of the stud with changes in gravity—is given by:

$$\frac{d\theta}{dg} = \frac{ml \sin \varphi}{\tau} \qquad [10]$$

Wright Gravimeter (Fig. 2)[4]

This instrument is similar in principle to the Threlfall-Pollock gravimeter. Two cone-shaped helical springs take the place of the torsion wire in the latter instrument. Between them is a small bar with a mirror, held in horizontal position by twisting the studs supporting the springs. Work on this instrument was begun more than 10 years ago, but no details on results have been published.

Lindblad-Malmquist Gravimeter

This instrument consists of a light mass suspended by two springs,[5] as shown in Fig. 3. The necessary sensitivity is attained by measuring the

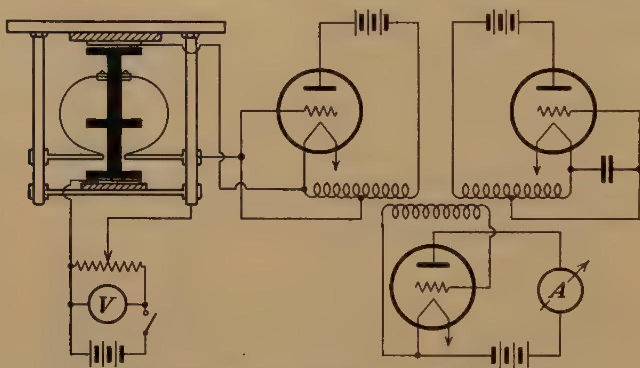


FIG. 3.—LINDBLAD-MALMQUIST GRAVIMETER.[5]

displacement of the mass in respect to a fixed plate by one of the ultramicrometer circuits. The circuit shown in the figure employs the heterodyne method of Whiddington in which the change in the spacing of the condensor plates produces variations in the beat frequency of two coupled oscillators. The condensor-microphone arrangement has been used also where the plates are in the grid circuit of an amplifier tube. With a plate spacing of 2.10^{-3} cm. Lindblad and Malmquist claim to have been able to detect variations in spacing of the order of 3.5×10^{-9} cm.; in their apparatus, a change in gravity of one milligal corresponded to a change in plate spacing of about 5.5×10^{-7} cm., so that theoretically changes of the order of $1/100$ mgal. would have been detectable. However, owing to various sources of error, chiefly due to temperature and hysteresis effects, the mean error in the field for a single observation was of the order of 0.1 to 0.2 mgal. for a single observation; for 5 to 10 repeat observation, it was 0.05 to 0.1 mgal. The ultramicrometer arrangement, in this apparatus, acts merely as an indicator and the deflections are compensated by electrostatic deflection (Fig. 3) of a pair of auxiliary plates. The distance

between the lower plates was so adjusted that a potential difference of 10 volts corresponded to a gravity variation of 1 milligal.

Changes in the position of the plates due to expansion of the suspension were compensated by suitable selection of the frame material. With this compensation, a temperature change of 1° C. corresponded to an apparent gravity change of 1 mgal. To reduce the temperature effect further, the apparatus was enclosed in a thermos bottle and the tempera-

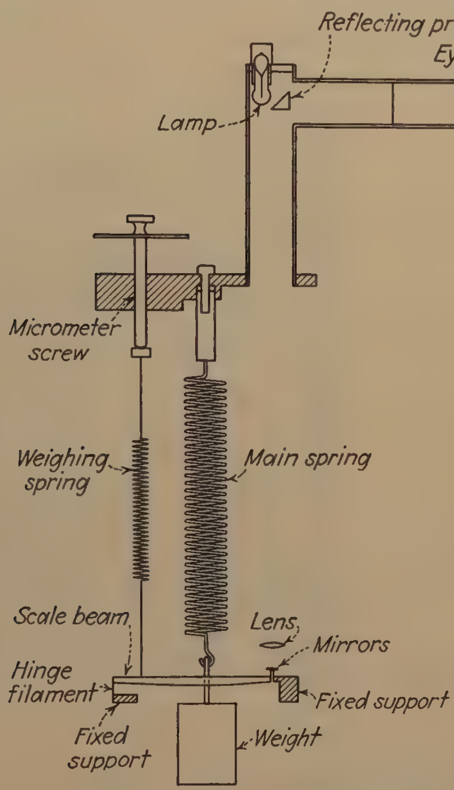


FIG. 4.—HARTLEY GRAVIMETER.[6]

ture kept constant electrically with a thermostat to about $2/100$ of a degree Centigrade. The apparatus proved sensitive and constant enough to register variations of the order of fractions of milligals produced by near-surface ore bodies. The theory of this instrument follows directly from equations 5 and 7.

Hartley Gravimeter[6]

The Hartley gravimeter is shown in Fig. 4. The mass is supported approximately from the center of a beam, which is hinged on the left and whose movement is transferred to two rocking mirrors by flexible filaments in such manner that they rotate in opposite directions, for a movement of

the beam in one direction. Hence, two reflected images of the lamp filament will be seen in the projection eyepiece whose displacement with respect to one another is measured. The beam is hung from two springs. The main spring is made of an alloy of tungsten and tantalum, carries 99.9 per cent of the total load, is wound with high initial tension and in extended position is about 10 cm. long. A small additional spring is provided for compensating the beam deflections; i.e., by bringing any deflection back to zero by a rotation of a micrometer screw. The frame carrying the spring assembly is temperature-compensated by the use of aluminum and Invar steel rods arranged in the manner of a compensated

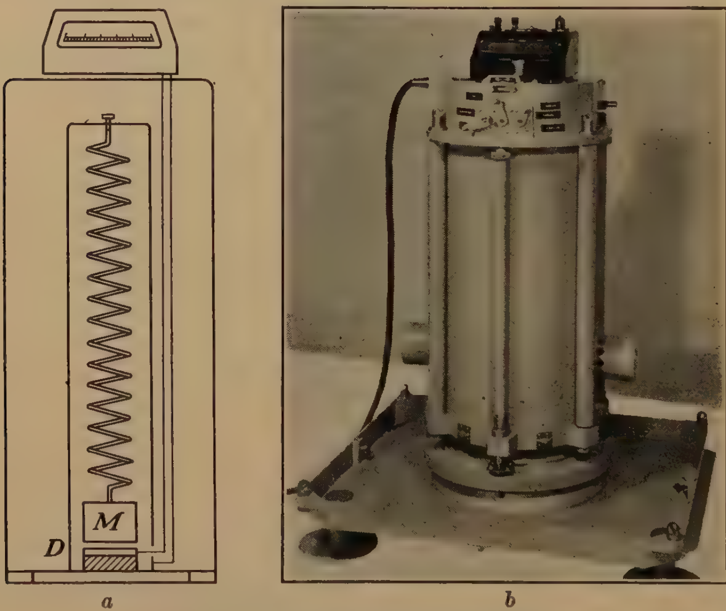


FIG. 5.—ASKANIA-GRAF GRAVIMETER.[7]

grid pendulum. The entire instrument is housed in an airtight cylindrical case of which the temperature is kept constant to one-ten-thousandth (?) of a degree by a thermostat operating from a 6-volt storage battery. Vibrations are suppressed by air damping. The mean error, at the time the article[6] was written, was reported to be ± one milligal but the author indicated that he was about to reduce it to one-fourth or one-fifth of this amount.

Askania-Graf Gravimeter

The Askania-Graf gravimeter[7] (Fig. 5) consists of a mass M suspended freely from a helical spring. The changes in the position of the mass are measured with an electrical "displacement" meter D (capacitive?), which operates without an amplifier but which nevertheless has a magnification

of about 4×10^4. Deflections of the mass are read on a galvanometer. Temperature compensation is provided; in addition, a double battery-operated thermostat is employed. The instrument is tightly sealed to eliminate effects of humidity and temperature. The accuracy is of the order 0.1 milligal.

The theory follows directly from equations 5 and 7. As the natural frequency is 1 sec., it is seen that a deflection of $2.5 \times 10^{-1}\mu$ corresponds to a gravity anomaly of 1 mgal., which with a magnification of 4×10^4 gives a galvanometer deflection of 10 mm. The scale value is linear for the entire range of the instrument (1600 mgal.) and may be checked at intervals by placing a small weight on the main mass (without opening the instrument).

Gulf Gravimeter

The Gulf gravimeter is reported to be almost identical with the original Perrot instrument. A mass is suspended from a helical spring, its vertical displacements being accompanied by rotations of the disk, which are magnified by multiple reflections between two mirrors. A battery-operated thermostat is provided.

Astatization of Gravimeters

It was shown before that the sensitivity of a gravimeter may be increased by lowering its natural frequency, and that a means of doing so was to fasten a helical spring not to a mass directly but to a lever arm supporting the mass. A further decrease in frequency may be made by "astatization." This consists of the application of a restituting force nearly equal (to maintain stability) and opposite in sign to the elastic restituting force, so that for this counter force the name "restituting force" loses its meaning, as its purpose is to drive the mass away from its rest position and to *aid* any deflecting force. The resulting natural frequency may be considered as being proportional to the square root of the difference between two spring constants; the force driving the system toward its rest position may as before be called the restituting or stabilizing force (or moment) while the opposing force (or moment) may be called the "astatizing"[8,9] or "labilizing" force (or moment).

Most gravimeters are vertical seismographs. However, the principle of astatization makes it possible to utilize also horizontal seismographs for gravity measurements; this applies principally to inverted pendulums, as shown below.

Several astatization arrangements are illustrated in Fig. 6 for vertical and horizontal seismogravimeters. Some of these are in use on present gravimeters; others have been used on seismographs only. It is evident that the common principle of all these arrangements is to produce, upon

deflection from the rest position, a moment that assists in increasing the deflection. The commonest astatizing method in vertical seismographs is to attach the suspension spring below the horizontal axis of the lever arm (Fig. 6, Ewing seismograph). This has the effect of moving the point of suspension closer to the axis of rotation when the lever moves downward; while the distance of the mass from the axis remains constant, the effective lever arm of the spring is shortened; as the natural period is

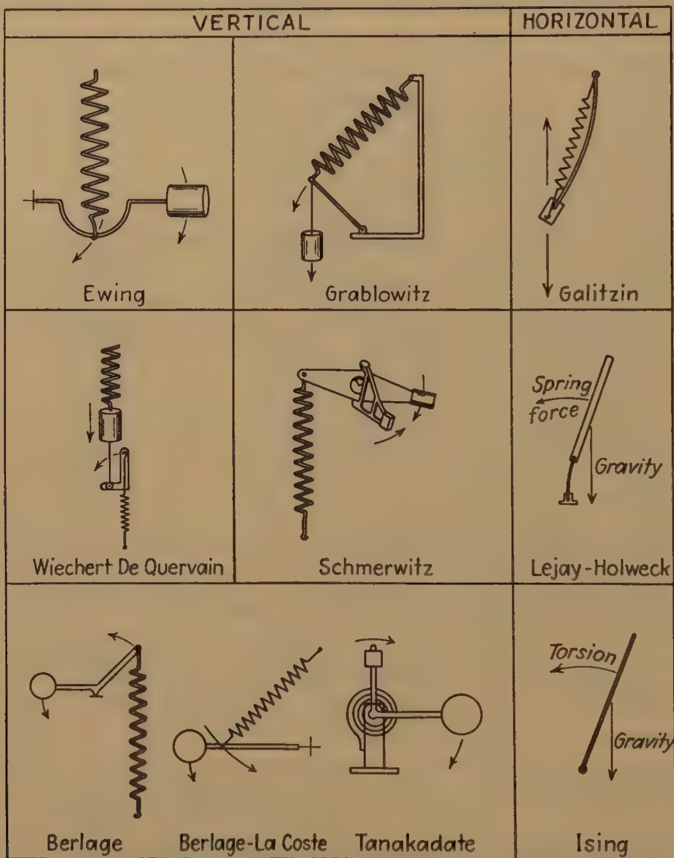


FIG. 6.—ASTATIZATION OF VERTICAL AND HORIZONTAL SEISMOGRAVIMETERS.

proportional to the ratio of the mass lever and the spring lever, the period is increased as this ratio is increased. An identical effect is accomplished in the Berlage and the Berlage-LaCoste vertical seismographs,[10,11] as shown in Fig. 6. Instead of shortening the spring lever arm and keeping the mass lever arm constant, the period may be reduced by the opposite procedure; i.e., by increasing the mass lever arm as the mass moves down. This is substantially the principle of the Grablowitz astatization. A very effective astatization can be accomplished by combining a horizontal

pendulum with a vertical balance, as proposed by Schmerwitz.[12] When the balance beam is tilted to the right, the horizontal pendulum also moves to the right, increasing the moment of the mass. In the Wiechert-De Quervain vertical seismograph the mass is connected by a rigid link to an L-lever in inverted position to which, at the upper end, another spring is fastened. As the mass moves downward, the L-lever is moved to the left, thereby increasing the deflection by the action of the auxiliary spring. A similar effect is brought about by fastening an inverted pendulum permanently to the lever arm of a vertical pendulum, as in the Tanakadate pendulum. As the main mass moves down, the mass on the vertical arm moves outward from the axis, increasing the moment of the mass.

In horizontal seismometers, the best known example of astatization is the Wiechert inverted pendulum. Similar in principle are the Lejay-Holweck gravity pendulum and the Ising astatic gravimeter. In these, the restituting force of a spring or a torsion wire are nearly compensated by the labilizing action of gravity. Different from this is the Galitzin arrangement in which the restituting force of a vertical (downward) spring is nearly compensated by the opposite action of an auxiliary coil spring mounted parallel in direction with the leaf spring.

In both vertical and horizontal seismogravimeters, the restituting and labilizing forces act in the vertical plane; they produce rotations about a horizontal axis. However, it is also possible to produce a labilizing gravity moment with action in a horizontal plane and rotation about a vertical axis when suspending a mass from two fibers. The tendency of such a system to come to rest in the lowest position of the mass produces a horizontal torque. This may be combined with an opposing torsional moment by suspending the mass in addition from a twisted wire or coil spring. Such instruments, called "bifilar" or "trifilar" gravimeters, are capable of high sensitivity and will be discussed later with the vertical seismogravimeters.

The explanation given here of the effect of astatization as a negative restituting force is not the only way in which the increase in period may be visualized. It is known that the period of an oscillating magnet that is proportional to the square root of the ratio of its moment of inertia and the product of magnetic moment and horizontal intensity may be increased by reducing the horizontal intensity with auxiliary magnets. Compensation of gravity with fixed masses under a pendulum, for instance, would be impracticable because of the tremendous mass required; but, in effect, the same thing is accomplished by inverting the pendulum. Finally, astatization is possible in magnetic systems by providing for two members affected in an opposite manner by the same restituting force. However, as mass is not polarized, like magnetic matter, this principle of astatization is without parallel in gravity apparatus.

It may be readily shown that in an astatized system the sensitivity to gravity variations is greatly increased. If h is the (positive) restituting force and k the (negative) labilizing force, the resulting restituting force

$$q \equiv h - k \qquad [11]$$

If the system is to be stable, h must be greater than k. The degree of astatization A may be defined[8] as the ratio of the labilizing force k and the resultant force q:

$$A \equiv \frac{k}{q} \qquad [12]$$

The degree of astatization A increases, therefore, with the labilizing force and the difference between the restituting and the labilizing forces, and small differences in either may be observed with great accuracy. In some astatic systems, differences in stabilizing force h are the objects of observation; in others, differences in the labilizing force; the latter is observed in inverted gravity pendulums. If the labilizing forces are held constant and changes in the stabilizing forces are observed, the resultant restituting force and the degree of astatization change in accordance with

$$\begin{aligned} \frac{dq}{q} &= (A+1)\frac{dh}{h} \\ \frac{dA}{A} &= -(A+1)\frac{dh}{h} \end{aligned} \qquad [13]$$

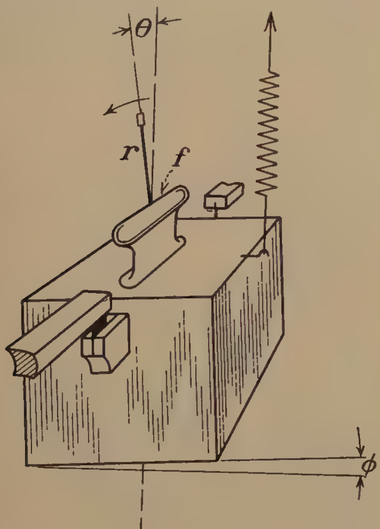
On the other hand, if changes in the labilizing forces are observed, the corresponding relations are:

$$\begin{aligned} \frac{dq}{q} &= -A\frac{dk}{k} \\ \frac{dA}{A} &= (A+1)\frac{dk}{k} \end{aligned} \qquad [14]$$

Astatized Horizontal Seismogravimeters

Astatized horizontal seismogravimeters are gravimeters that when used as seismographs record the horizontal component of ground vibrations. The well-known Wiechert astatic pendulum would be usable as a gravimeter and the horizontal seismogravimeters now applied may be considered as modifications of the Wiechert astatic pendulum. The Holweck-Lejay pendulum, for instance, could be made into an astatic gravimeter easily, although at present it is used primarily with the dynamic method of observation; i.e., observation of period. The only horizontal seismogravimeter that is more extensively used at this time is the Ising gravimeter.

The Ising instrument is illustrated in Fig. 7. It consists essentially of a quartz fiber f stretched between the prongs of a fork-shaped support and forms the horizontal axis of rotation of the thin quartz rod r, which is fused rigidly to the fiber. The astatic quartz pendulum is fastened to a heavy metal block, which is hung from two leaf springs in such manner that it can be turned slightly about an axis parallel to the fiber. Such tilts are produced by tightening



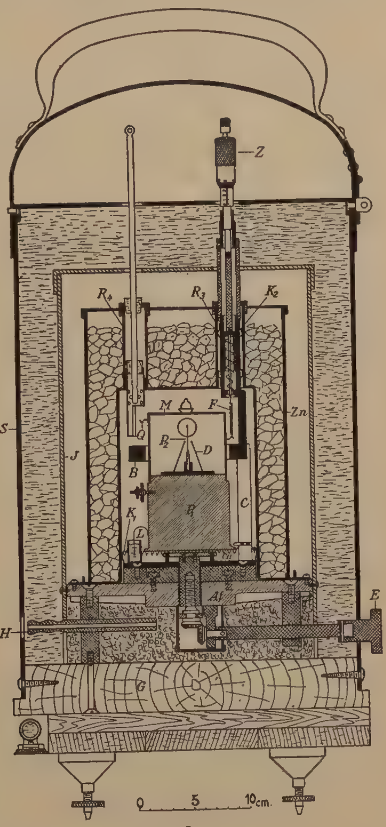


FIG. 7.—ISING GRAVIMETER.[8]

a spring attached to one side of the block. When the block is tilted by an angle φ, the pendulum is deflected from its vertical position by the angle θ. In the position of equilibrium

$$h\theta = k(\theta + \varphi)$$

or

$$\theta = \frac{k}{h - k} = A\varphi \qquad [15]$$

where the significance of h and k follows from formula 11.

Assuming that the gravity at a (base) station is g_0 and that by a tilt φ the deflection θ_0 has been observed, the labilizing force at that station $k_0 = \dfrac{h\theta_0}{\varphi + \theta_0}$. At another station with the gravity g_1, if the same tilt

angle has been used, the labilizing force is $k_1 = \frac{h\theta_1}{\varphi + \theta_1}$. Then the difference in gravity $\Delta g = g\frac{k_1 - k_0}{k_0}$, or in terms of (small) deflection angles for constant tilts:

$$\Delta g = g\left[\frac{\theta_1}{\theta_0} \times \frac{\varphi + \theta_0}{\varphi + \theta_1} - 1\right] \tag{16}$$

The deflection of the inverted pendulum is read with a microscope; the tilt is measured by the rotation of a micrometer screw. The instrument is kept at uniform temperature by a number of envelopes; the innermost cylinder is of zinc and filled with melting ice; it is surrounded by an air space and the space between the latter and the outside case is filled with felt. The mean error was between 0.6 and 0.5 mgal. for the instruments described in the published reports.

Astatized Vertical Seismogravimeters

Astatized vertical gravimeters fall into two major groups: (1) those in which changes in gravity moment produce rotations about a vertical axis, for which purpose a bifilar or trifilar suspension is used; (2) those in which this rotation takes place in a vertical plane and about a horizontal axis. All instruments discussed in this chapter are astatized.

Bifilar Gravimeters

In many physical instruments a moving system is or may be suspended on a vertical wire as axis of rotation (compass, galvanometer, torsion balance, etc.). If the suspension consists of two wires, it is spoken of as "bifilar" suspension. In a single-wire suspension the restituting moment is equal to $\tau\varphi$ whereas in a bifilar suspension (nonastatic) another moment is added as the suspended mass is raised upon rotation. The restituting force due to gravity is, with $2r$ as the distance of the suspension points above, $2R$ as distance below, and D their vertical distance:

$$h = \frac{rR}{D} \cdot mg$$

Therefore, the resulting restituting force

$$q = \frac{rRmg}{D'} + 2\tau \tag{17}$$

where D' is the vertical distance corrected for a reduction in length, which occurs due to the bending of the wires.

The period of oscillation of a mass on bifilar suspension, if the moment of inertia is K:

$$T = 2\pi\sqrt{\frac{K}{2\tau + \frac{rR}{D'} \cdot mg}} \tag{18}$$

The period of oscillation of a bifilar system suspended in this manner is much too short to give sufficient sensitivity for gravity observations.

A bifilar system can be astatized by changing the sign of one of the terms in the denominator of equation 18. In an astatic pendulum this is done by inverting it, thus making gravity the labilizing force. The analogous procedure may be applied to a bifilar system by reversing the upper part of the suspension 180° as shown in Fig. 8; then the period

$$T = 2\pi \sqrt{\frac{K}{2\tau - \frac{rR}{D'} \cdot mg}} \qquad [19]$$

This procedure can give, with high astatizing factors, about 1000 times the sensitivity in the determination of gravity, which is an improve-

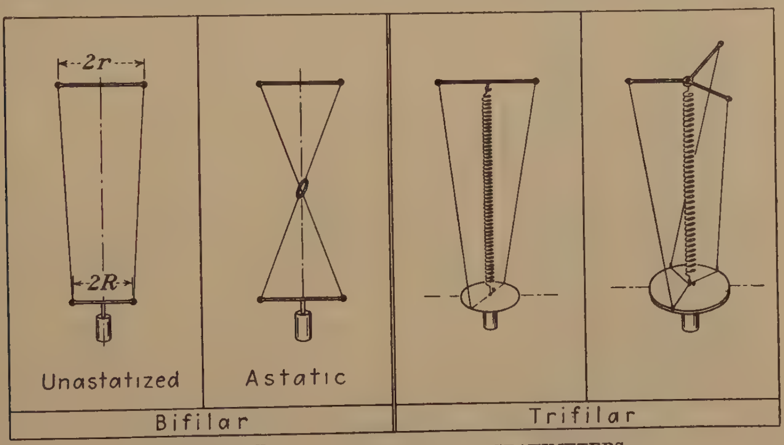


FIG. 8.—BIFILAR AND TRIFILAR GRAVIMETERS.

ment of similar order as realized in the Lejay-Holweck pendulum. Observations may be made statically or dynamically. In an instrument of this kind constructed by Berroth[14] the bifilar suspension and mass were enclosed in an evacuated tube; the distance of the suspension points above and below was the same and equal to only 2 cm., the vertical distance was 6 cm., the mass 10 grams and the period about 2½ sec. A change in gravity of 1 mgal. produced a change in period of about one-half thousandth of a second. Although this instrument showed considerable promise, it was not perfected to the point of field worthiness. Similar difficulties have been experienced with the Ising bifilar instrument illustrated in Fig. 9; the chief source of trouble appears to have been the suspension (quartz).

Bifilar gravimeters with crossed wires appear to be inferior to the trifilar gravimeters discussed in the following section, capable of the greatest sensitivity yet attained in such instruments.

Trifilar Gravimeters

Instead of twisting the suspension 180°, it is more advantageous to balance the restituting moment of gravity by an additional wire or coil spring fastened to the center of a bifilar suspension (Fig. 8). Three wires equally distributed about the circumference of the suspended disk give better stability and better protection against side sway of the instrument. This last instrument has, therefore, four suspension wires but nevertheless will be called a trifilar gravimeter in this discussion. The trifilar gravimeter was first described by A. Schmidt.[15]

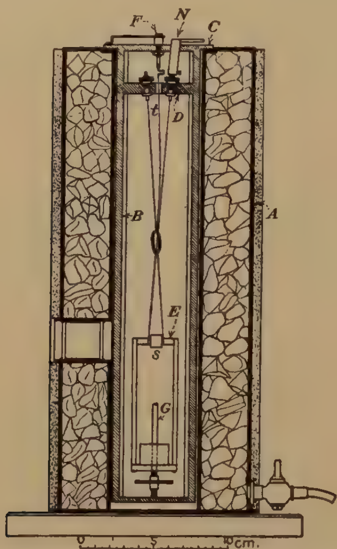


Fig. 9.—Ising bifilar gravimeter.[8]

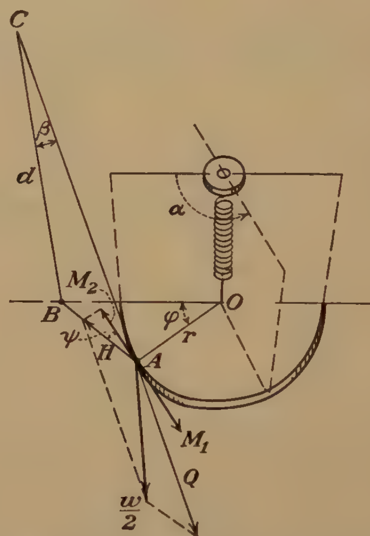


Fig. 10.—Action of trifilar gravimeter.

The working principle of this gravimeter is shown in Fig. 10, which is a view of the gravimeter disk from above, showing only one of the suspension wires. The latter is fastened at a distance $\overline{CB} = d$ above the disk and at a horizontal distance $\overline{OB} = a$ from its center. The radius of the disk is $\overline{OA} = r$ and the horizontal distance of the point where the wire is fastened, from the point B, is $\overline{AB} = e$. Assume that the torsion head has been turned by an angle α, which may result in a deflection angle φ. In this position of the disk the suspension wire is deflected by the angle β from the vertical. If the total weight is W, it is so distributed that the coil spring bears a weight $W - w$ and the suspension wires each $w/2$ or $w/3$, depending on whether two or three wires are used. With two wires, the weight $w/2$ (Fig. 10) may be resolved into a component Q in the extension of the wire and a horizontal component H. Then $H = w/2 \tan \beta = \frac{we}{2d}$. Its tangential component is $H \cos \psi$

producing the couple $2Hr \cos \psi$. In the triangle OAB, $e \sin (90 + \psi) = a \sin \varphi$; thus, with $\cos \psi = \frac{a}{e} \sin \varphi$, the couple

$$M_2 = \frac{wra}{d} \sin \varphi$$

This is in opposition to the moment of torsion of the main coil, which is given by

$$M_1 = \tau(\alpha - \varphi)$$

Thus, in the equilibrium position,

$$\tau(\alpha - \varphi) - \frac{wra}{d} \sin \varphi = 0 \qquad [20]$$

Small changes in the weight of the disk and hence in gravity will produce large deflections in the position of maximum sensitivity. By differentiating equation 20,

$$\frac{d\varphi}{dw} = -\frac{\frac{ar}{d} \sin \varphi}{\frac{wra}{d} \cos \varphi + \tau} \qquad [21]$$

Maximum sensitivity occurs when the denominator is zero, or when

$$\cos \varphi = -\frac{\tau d}{wra}$$

As τ is usually small, the position of maximum sensitivity is very close to 90° from the position of zero deflection.

The trifilar gravimeter is capable of giving extraordinary sensitivity to gravity changes when set up on permanent location and when well protected against temperature changes. It has been used to record the changes in gravity brought about by the changes in attraction of the moon; Tomaschek and Schaffernicht[13] have been able to record such variations with an estimated error of 0.001 mgal. A portable instrument for gravity exploration has not yet been developed. Owing to the limits of accuracy with which elevations and topographic effects can be determined, a field instrument would probably not need a greater accuracy than 1/10 milligal.

Truman Gravimeter

The Truman gravimeter is similar in action to a Ewing astatic vertical seismograph (Fig. 11). The beam consists of a right triangle with the right angle at the axis of rotation; the mass M is attached to the end of this triangle, and near it a vertical coil spring S_1 supports most of the mass and the beam. An astatizing (so-called "period") spring S_2

is attached to the other corner of the triangle, approximately below the point of suspension. Spring S_1 attaches to beam B which is supported in two stirrups by the beam A fastened to the adjusting spindle as shown. Both beams A and B are of bimetallic construction, with the metal of greater expansion coefficient inside. When the temperature rises, the beams curve, whereby their distance from one another is reduced. The accuracy is of the order of 0.2 mgal. This instrument is used chiefly by the Humble Oil Co. and associated companies.[17]

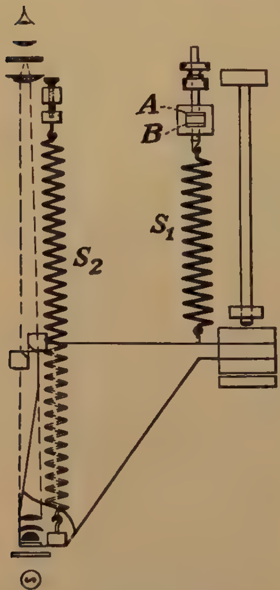


FIG. 11.—TRUMAN GRAVIMETER.[17]

Mott-Smith Gravimeter[13]

The Mott-Smith gravimeter is illustrated in Fig. 12. Between the vertical arms of a T-shaped support, clamped to the case and made of fused quartz, is stretched a torsion fiber B, about 1½ in. long and 0.002 in. in diameter, also of quartz. The fiber carries a weight arm

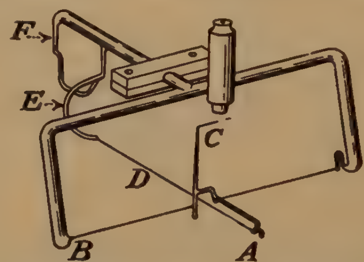


FIG. 12.—MOTT-SMITH GRAVIMETER.[13]

A to which is attached the pointer C, both about 1 in. long. The center of gravity of this system is adjusted so that it is as close as possible to the axis of the fiber B by providing a counterweight to the pointer; its horizontal distance from the fiber may be changed by adding fused quartz to the weight arm as shown in the figure. The negative restituting force is supplied by a fiber D about 2 in. long and 0.0005 in. thick, attached to a spring E which is part of a stirrup F. The tension of the fiber D may be changed by heating this stirrup; the fiber passes through the axis of rotation of the weighing arm, for which purpose an offset arm connecting A and C is provided. The deflections of the system may be read by a microscope.

The action of this gravimeter is similar in principle to that of the Galitzin astatized horizontal seismometer (Fig. 6). If rotated 90° to convert it into a vertical seismometer, the restituting force of the leaf spring corresponds to the torsion of the fiber B; the labilizing spring attached to the weight has its equivalent in the fiber D.

Thyssen Gravimeter

The Thyssen gravimeter is likewise a beam-type vertical seismometer; the beam is horizontal, suspended in the center on a knife-edge, the balancing spring mounted downward instead of upward (Fig. 13). An astatizing arm is mounted upward on the center of the beam as in the Tanakadate vertical seismograph (Fig. 6). Temperature compensation is effected by fastening the coil spring to the end of a tube of nearly equal opposite expansion; in addition, the tube is surrounded by a jacket filled with water. The beam is made of fused quartz, to keep horizontal displacements of the mass at a minimum. The total length of the beam is 15 to 20 cm. Two beams are arranged side by side and reversed 180° with respect to their weights, as in the torsion balance. The beams are in separate chambers; in each of them the temperature may be read on accurate thermometers from the outside. Doors are provided at the bottom of the outside case for adjustments at the point of suspension of the coil spring. The beams are arrested and released by a lifting mechanism operated from the top of the instrument. Both beams are read independently by an optical arrangement that directs the light from the top of the instrument to a mirror opposite the weight at the end of the beam. The scale has 80 divisions, each corresponding to 4 or 5 mgal. The zero position is not observed after the beam has come to rest (as in a magnetometer), but is calculated from several readings of reversals. The mean error of one station is 0.25 mgal., that of repeat stations about 0.5 mgal.,[18–26] the drift in one day 1 to 1.5 milligals.

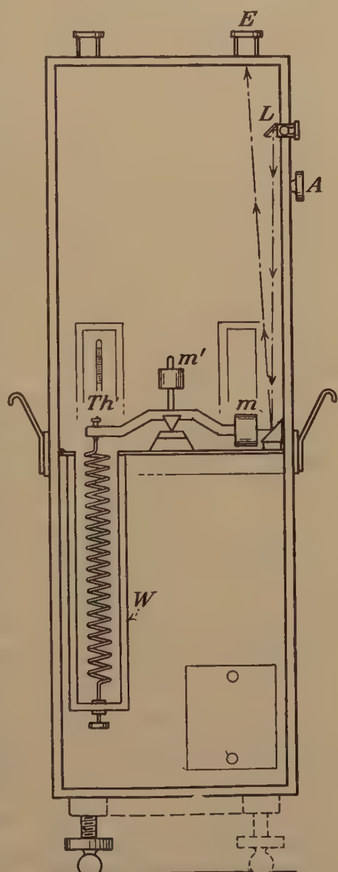


FIG. 13.—THYSSEN GRAVIMETER (SCHEMATIC).

CALIBRATION OF GRAVIMETERS

A number of factors affect the gravimeter readings and must be known in magnitude so that corrections may be applied. They are: effects of temperature, air pressure, and humidity. Furthermore, the sensitivity and the mean error of single and repeat observations must

be determined; i.e., determinations of scale values, mean error, and base correction are necessary.

The effect of temperature on a gravimeter is complex and cannot always be calculated. It is generally determined by experiment. In virtually all gravimeters, some sort of temperature compensation is provided. If this were not done the effect of temperature would be tremendous. In a gravimeter consisting of a mass suspended from the end of a coil spring or leaf spring a change of one degree Centigrade would produce an apparent change in gravity of 200 to 300 mgal.; in a volumetric gravimeter the same temperature change would produce an apparent change in gravity of as much as 3000 to 4000 mgal. The effectiveness of temperature compensation and the temperature coefficient, which generally remains even after elaborate compensation, is determined by experiment. The instrument is slowly heated to various temperatures; the change in reading is observed and plotted against the change in temperature. This gives the temperature coefficient and information on temperature hysteresis.

Changes in air pressure are of some influence on gravimeter readings; the magnitude of the effect depends altogether on construction of the container and the weight of the pendulum. With an increase in barometric pressure the buoyancy of the gravimeter is increased; in such gravimeters, where period instead of static deflection is observed, the former is affected (1) by the effect of the small amount of the surrounding air carried along with the moving pendulum; (2) by buoyancy; and (3) by internal friction (affecting the damping rate). The effect of changes in air pressure on gravimeter readings can be determined by changing the air pressure inside of the gravimeter case and reading it on a manometer, or by observing changes in reading brought about by normal variations of atmospheric pressure.

Changes in humidity may produce unexpectedly large effects on gravimeters whose mass is small as water vapor condensing on it will produce comparatively large changes in equivalent mass. The relative humidity of the air can be determined and a correction can be applied for its variations. However, this procedure is not advantageous and it is better to keep the air inside the gravimeter case as dry as possible by using calcium chloride in solid form or as supersaturated solution. At high temperature, which may be maintained by an electrical thermostat, the effect of changes in humidity can generally be held down sufficiently.

For the determination of the sensitivity (or of the scale value) of gravimeters a number of methods are available: (1) determinations of period; (2) determinations of change in reading due to tilt; (3) addition of weights; (4) electrostatic deflection; (5) determination of change in reading with changes in elevation; (6) comparison with accurate torsion balance data; (7) comparison with pendulum data.

The sensitivity of gravimeters using balanced beams is proportional to the square of the period. However, determinations of scale values by period observations are not in use except for qualitative observation while adjusting an instrument. There are numerous other methods with which scale values can be determined more easily and more accurately.

Some gravimeters can be calibrated by tilting. When the instrument is tilted by the angle φ the effective gravity changes from g to $g \cos \varphi$; therefore the apparent difference in gravity brought about by tilting is

$$\Delta g = g(1 - \cos \varphi) = 2g \sin^2 \frac{\varphi}{2} \approx g\frac{\varphi^2}{2} \qquad [22]$$

Certain other types of gravimeters lend themselves more readily to a determination of scale value by the addition of small weights. If the total mass whose change in weight is determined in the gravimeter is M and a small mass dm is added, the apparent change in gravity produced thereby is

$$\Delta g = \frac{dm}{M} \cdot g \qquad [23]$$

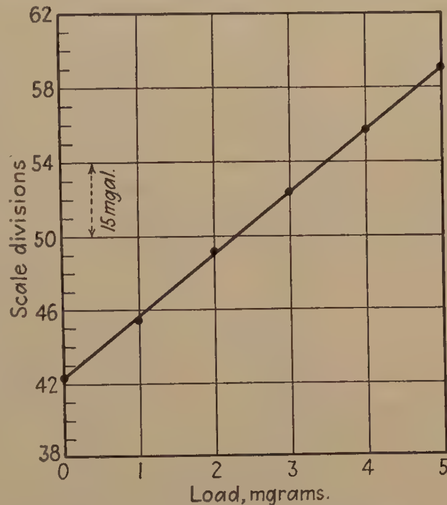


FIG. 14.—CALIBRATION OF GRAVIMETER BY ADDITION OF MASS. (*After Schleusener.*[23])

Fig. 14 shows the results of such calibration of a Thyssen gravimeter.

In gravimeters where displacements are measured electrically as changes in distance of two condensor plates, the scale value may be determined by electrostatic attraction. If the two condensor plates have equal surfaces of S square centimeters and a distance of d centimeters, the apparent change in gravity brought about by a voltage difference E is

$$\Delta g = \frac{SE^2}{8\pi d^2 M} \qquad [24]$$

As it is difficult to measure the distance d directly, the capacity C between the plates may be measured instead.[5] If the capacity is $C = \frac{S}{4\pi d}$, the apparent change in gravity when applying a voltage difference E to two plates of the surfaces S between which the capacity is C:

$$\Delta g = 2\pi\frac{E^2C^2}{SM} \qquad [25]$$

Almost any kind of gravimeter may be calibrated by measuring the change in reading due to a change in elevation. Gravity decreases with

an increase in elevation and the "free-air reduction" may be used directly to determine this change in gravity. Tall buildings may be used to advantage for this test. The gravimeter is read on the ground floor and the highest floor available. The change in gravity due to the mass of the building can generally be neglected. Similar tests may be made on bluffs, hills, embankments, etc.; however, topographic (and Bouguer) corrections are then required, which makes the results less accurate than calibration on buildings. The change in gravity due to a change in elevation is given by

$$\Delta g \text{ (in milligals)} = -\Delta h \text{ (in meters)} \times 0.3086 \qquad [26]$$

The correction due to the Bouguer attraction is

$$\Delta g_{mg} = 0.0419\delta\Delta h_m \qquad [27]$$

The latter is of opposite sign compared with the free-air reduction; for an average density d of 2, it amounts to 27 per cent of it.

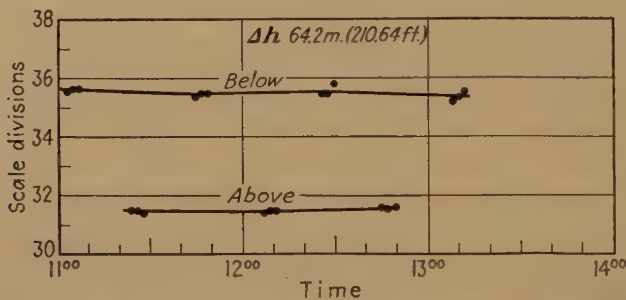


FIG. 15.—CALIBRATION OF GRAVIMETER ON BUILDING TOWER. (*After Schleusener.*[23])

Fig. 15 illustrates the calibration of a Thyssen gravimeter on a building tower.

The remaining methods such as comparisons with torsion balance and pendulum, are used only for occasional checks on calibration. Torsion balance gradients may be used for calculations of differences in gravity provided the station interval is small, the gradient fairly constant over great distances and provided finally that terrain corrections are small. Pendulum anomalies lend themselves less readily to comparisons with gravimeters as their accuracy is less than that of gravimeter observations; conversely, with recent increases in accuracy, gravimeters can be used to check pendulum observations and to determine errors in pendulum surveys.

Because of the elastic hysteresis of the suspension material, most gravimeters show a more or less appreciable drift of the zero position with time. Furthermore, abrupt changes in base position may occur owing to mechanical changes in the moving systems. Both kinds of changes must be corrected for. This is done by repeated checks into a base

station or a number of base stations established at regular intervals in a survey. If the readings (corrected for temperature, etc.) differ when a station is repeated, the difference may be distributed uniformly among stations occupied between repeats, if the assumption is justified that the change is due primarily to the regular drift of the instrument. If abrupt changes are suspected, the base correction is applied in a manner conformable to information available on the time of their occurrence. The procedure of determining and applying base correction is thus identical with the methods used in magnetometer surveys.

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Use of Astatized Pendulums for Gravity Measurements

By Gustaf Ising*

(New York Meeting, February, 1937)

For relative gravity measurements, the author in 1918 described an instrument[1] of which the essential part consists of a highly astatized, standing pendulum turning about a horizontal elastic axis. This type of instrument has been used recently in practical field work, therefore a description of its principle and technical development is given herein. Part I, describing the principle, is a direct translation of the original Swedish paper.[1] Part II gives an account of the practical elaboration of the instrument, together with some description of measurements performed.

I. Principle of Using Astatized Pendulums[1]

In order to reduce the trouble and waste of time attending the use of ordinary pendulums in measuring the force of gravity, various instruments of another type have been suggested for relative determinations, as a rule arranged in such a way that the attraction of the earth on a certain body or mass of fluid is balanced by elastic force from a spring or a shut-in mass of gas. A change in the force of gravity is then made manifest as a change in the position of equilibrium of the body, in that the deformation of the spring (or compression of the gas) is changed. With such instruments there are, however, great difficulties in keeping the elastic force constant or its variation known with sufficient exactitude, because of elastic after-working, influence of temperature, and so on.

The author suggests for relative determinations of gravity another type of instrument, in which two *directional forces* (control coefficients) rather than two forces are compared with one another. By "directional force" or "control coefficient" is meant that quantity in a rotatable system that, multiplied by the angle of deflection, indicates the moment counteracting the deflection.

In this way it will be possible greatly to reduce the influence of the elastic after-working, which in instruments with spring devices acts so disturbingly. The instrument is made with a rotatable system arranged in such a way that the total control coefficient acting upon it, is composed

Manuscript received at the office of the Institute Feb. 20, 1937. Issued as T.P. 828, August, 1937.

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[1] References are at the end of the paper.

of two terms with different origin, one proportionate to the weight of the system and the other generated in some way independent of the attraction of the earth, preferably with the aid of elastic springs. Furthermore, one of these terms shall be *negative;* i.e., the rotatable system shall be *astatized.* The instrument may briefly be defined as an invariable, astatized pendulum.

Such an astatized, movable system under the influence of two control coefficients, one positive and the other negative, possesses some interesting characteristics. Let h denote the positive term in the control, $-k$ the negative ($k < h$) and A the resultant control; then

$$A = h - k \tag{1}$$

As a measure for astatization, may suitably be chosen a figure N, equal to the quotient between the absolute value of the negative control coefficient and the resultant control coefficient; i.e.,

$$N = \frac{k}{A} \tag{2}$$

Astatized needle systems are often used in galvanometers and electrometers, and the author obtained the idea for the gravity meter while working with the construction of electrometers. In these instruments, as a rule, it is not desirable to use higher values for N than 100 or 200. When the astatization is high, small changes that may occur in one or the other of the terms h and k, while insignificant in relation to these terms themselves, constitute a material part of their difference, and therefore cause a considerable percentage variation in the resultant control A, and consequently in the sensitivity of the instrument. Suppose that one of the terms of the control, let us say k, undergoes a slight variation δk, while the other term, h, remains constant. Then the following relations hold:

$$\frac{\delta A}{A} = -N\frac{\delta k}{k} \tag{3}$$

$$\frac{\delta N}{N} = (N + 1)\frac{\delta k}{k} \tag{4}$$

Thus, when the value of N is high, a variation in one of the terms of the control causes a much greater percentage variation of the resultant control (and of N).

This characteristic of astatized systems, which in electrometers and galvanometers is a disadvantage, should be an advantage if it is desired to observe and measure such small changes in h or k. And this is exactly what is desired with the proposed gravity meter, which we shall now examine more closely.

The pendulum should preferably be arranged inverted—with the center of gravity located above the axis of rotation. The gravity will

then give to the control the contribution $-k$, with $k = mgr$, where m is the mass of the pendulum and r the distance from its center of gravity to the (horizontal) axis of rotation. The other term in the directional force, h, is made only very little greater than k, so that the astatization becomes great (N at least some hundreds). Such an arrangement may, for instance, simply consist of a vertical elastic fiber fixed at its lower end, and with its upper end carrying a mass of such weight that the system is on the point of tipping over to one side.

Fig. 1 shows schematically such an instrument. The foot plate is shown shaded, the pendulum itself unshaded, the dotted line shows the position of the pendulum when a deflection takes place, brought about, for instance, by a slight change of inclination of the foot plate.

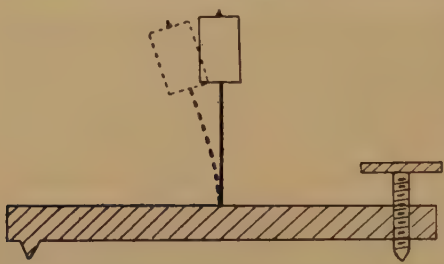

FIG. 1.—ELASTIC FIBER FIXED AT LOWER END, CARRYING A WEIGHT.

There are already in use physical instruments in which the gravity has an astatizing action, though they are made for other purposes and with other methods of use. Among these Wiechert's astatized pendulum for earthquake recording is probably the one that in principle resembles most nearly the suggested gravity meter, though by reason of the different purpose the details and dimensions differ greatly from what is suitable in the latter.

THEORY OF INSTRUMENT

If the value of the gravity in a certain initial case is g_0 and in a later experimental case has changed into g_1, while the term independent of the gravitational force (i.e., in a reversed pendulum the term h) is unchanged, the quotient $\frac{g_1 - g_0}{g_0}$ (the relative change of the gravity) will be known as soon as the astatization in both cases N_0 and N_1 is known, or, which is mathematically equivalent to this, N_0 and the quotient $\frac{A_1}{A_0}$. The initial astatization N_0 is measured once and for all, and the task of the instrument thus becomes to measure the magnitude of the control A in relation to the initial value A_0. A high value for N_0 entails here the very great advantage that the experimental determination of the control or astatization need be carried out with only moderate accuracy; that is, it can be done rapidly and with relatively simple means and nevertheless give an accurate result. This is one of the main advantages of the use of an astatized pendulum as a gravity meter. Another is the possibility of avoiding or reducing the effect of elastic after-working in the springs, the elasticity of which is compared with the gravitation. This question

is further discussed in the description of Fig. 2. If, for example, the astatization N equals 1000, and the measuring of the control has an error of one per thousand, the error according to equation 3 in the (relative) value derived for k and thus for the gravity will be only one-millionth part. A necessary condition for this is only that the term in A that is independent of the gravity (i.e., with a reversed pendulum, h) remains constant (or that its percentage variation is known) with the same high degree of accuracy as is desired in regard to g. For this is required a fairly precise knowledge of the temperature, which affects both the pendulum dimensions and the modulus of elasticity of the springs. Another factor that has some influence is the elastic after-working of the springs. In order to avoid this as far as possible, it is necessary not only to select a suitable material (steel or, still better, fused quartz) but also to arrange the pendulum in such a manner that the term in the control derived from the elasticity is independent of the *permanent* tension of the springs. It is easy to fulfill the latter condition, when the springs supply the positive directional force h; i.e., with a reversed pendulum. The diagrammatic Fig. 2, which shows one possible way of designing the instrument, explains this.

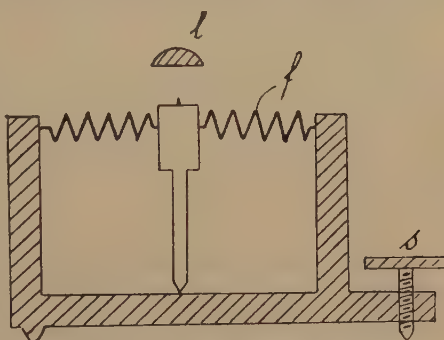

FIG. 2.—INSTRUMENT WITH REVERSED PENDULUM.

The parts of the instrument that are rigidly attached to the framework or instrument case are shown shaded in the figure, the inverted pendulum itself unshaded. The pendulum can turn in the plane of the paper around its lower end and rests there, either on a sharp edge, as indicated in the figure, or, preferably, supported, in order to avoid friction, by two flexible vertical springs attached to a horizontal beam (at right angles to the plane of the paper) on the lower end of the pendulum. At the top end are attached two stabilizing springs f; it is obvious that since the direction of deviation is parallel to the springs, the term h depends only upon their elasticity (= prolongation or compression per unit load) but not on the permanent tension to which they are subjected. This tension, therefore, may be selected approximately equal to zero. The shaded segment l in Fig. 2 denotes the frontal lens in a microscope, which serves for measuring the deflection of the pendulum, while s denotes a micrometer screw, by means of which the observer can change the inclination of the framework a certain known angle for the purpose of measuring the directional force, to which we shall revert later on.

It is also possible to suspend an astatized pendulum in the ordinary way, with the center of gravity below the axis of rotation. However,

such a design entails the disadvantage of requiring the elastic control coefficient to be negative ($= -k$) and this can scarcely be obtained in any way except by springs, *which are at a considerable permanent tension.* Therefore, instruments of the kinds sketched in Figs. 1 and 2, with reversed pendulum, are preferable.

The control acting upon an astatized pendulum can be determined either dynamically by measuring the oscillation period of the pendulum, or (more rapidly and conveniently) statically by measuring the deflection caused by the application on the pendulum of a known deviating moment. Such a moment the observer can bring about quite simply by making a small change in the inclination of the instrument framework; e.g., by turning one of the foot screws of the instrument. If φ denotes the rotation of the pendulum stand around an axis parallel to the axis of rotation of the pendulum, and the corresponding angular deflection of the pendulum with reference to the framework is called ϑ,* the condition of equilibrium with a reversed pendulum will be:

$$\vartheta = \frac{k}{h - k}\varphi = N\varphi \qquad [5]$$

Thus by measuring ϑ and φ the value for N is obtained, and from this, according to equation 4, by comparison with the initial astatization N_0,† the percentage change of the gravitation. The measuring of ϑ and φ need be made only with moderate accuracy, as stated previously.

Finally, regarding the dimensioning of the instruments, it may be mentioned that the pendulum should preferably be made very small, only a few centimeters long. This has the advantage of a fairly short period of oscillation, facilitates the maintenance of a constant temperature (by a double-walled casing filled with ice) and makes the entire instrument easier to handle than if the pendulum were longer. For the static measurement of the directional force, it is, finally, desirable that the swings shall be strongly damped (through the air resistance), and this also is most easily attained with very short pendulums.

II. Development of the Instrument

1. *Experimental*

So far in 1918. Practical experiments along the lines described, were made by the author and Urelius[2] in 1922 and, with some interruptions, continued until 1926. The only material they found satisfactory in regard to small elastic after-working was fused quartz. An astatized pendulum, in its simplest form, may be a standing rod with the lower part thinned out so as to be elastically flexible (Fig. 3 left).

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* ϑ and φ are counted positive in the same direction and are supposed to be small quantities.

† This too can evidently be determined in the same way.

However, the disturbances were considerably smaller with a system corresponding to Fig. 3 right, in which the pendulum stands on a horizontal torsion wire: the elastic strain, corresponding to a given deflection, will be smaller, as it is distributed along the whole length of the wire.*

In the use of the astatized vertical pendulum there are mainly two difficulties to overcome; namely, the temperature variation of the modulus of elasticity of quartz (about $1.1 \cdot 10^{-4}$ per degree centigrade) and the sensitivity of the pendulum to small accidental tilting of its foot. (Compare equation 5.) The first difficulty mentioned was overcome by surrounding the pendulum with a double-walled casing containing melting ice, and the second one was eliminated by mounting the quartz pendulum in a heavy, hollow weight hanging in two metal bands. This arrangement brings about an automatic leveling of the pendulum foot, and affords an easy way of obtaining the very small tiltings (φ) of this foot, used for a *static* determination of the resulting control. By the aid of a fine spiral spring, stretched by a micrometer screw, the tilting (φ) of the weight may be varied from outside by very small, known amounts. For the same purpose there is also on the weight a hook, to permit the addition and removal of a small load (μ). This was especially used in connection with the automatic recording device (see below). Fig. 4 shows a vertical section through the essential parts of the instrument of Ising and Urelius.† P_1 is the weight hanging in the bands A, P_2 the small quartz pendulum; the deviation ϑ of the pendulum end was measured by a microscope, the objective O of which was mounted in the weight. T is a small drying vessel, keeping the humidity of the air around P_2 constant.‡

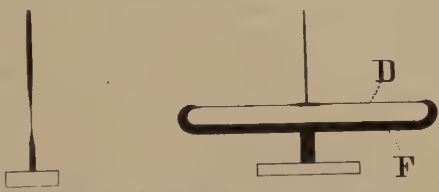

FIG. 3.—SIMPLE ASTATIZED PENDULUM AND PENDULUM STANDING ON HORIZONTAL TORSION WIRE.

* A similar system, though having the pendulum almost horizontal, has been used by Threlfall and Pollock[4] for static gravity measurements. They kept the torsion wire *permanently* twisted in order to compensate the *permanent* moment arising from the weight of the pendulum, and met therefore the difficulty of a continual change of the zero. On the other hand, when using an astatized vertical pendulum, two *control coefficients* are compared with each other, and even with a static measuring method the wire is twisted only during the short time necessary for a measurement of the deviations. No secular wandering of the zero can thus arise. The quartz system of Ising and Urelius was also made in one piece, without any cemented or soldered joints, and therefore less exposed to after-working effects.

† The outer parts (heat-insulating casing and operating devices) have been omitted.

‡ If the humidity varies, the thickness of the water film, adsorbed on the surface of the pendulum, varies and changes its weight (ref. 2, 38). In later instruments the pendulum is hermetically enclosed in the weight, whereby also the influence of barometric changes is eliminated.

In order to determine in a static way the degree of astatization N, the quotient $\vartheta:\varphi$ must be measured. This may conveniently be done by difference measurements in two ways: (1) the screw method (A) or (2) the load method (B). In the first method the operator, with the aid of the micrometer screw (the z-screw), brings the image of the pendulum point alternately to coincide with two fixed lines on the ocular scale, which are situated to each side of and at an equal distance from the vertical zero position of the pendulum, and notes the corresponding positions of the screw. In the second method, the sinking down or lifting up of the small additional load (mass μ) causes the pendulum to make

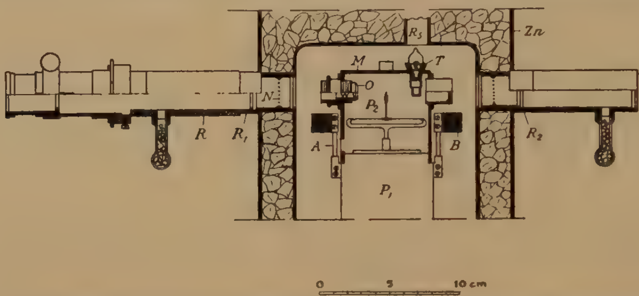

FIG. 4.—VERTICAL SECTION THROUGH ISING AND URELIUS INSTRUMENT.

symmetrical deviations to each side of its zero position; these deviations are either observed visually or recorded photographically.

In each method, for a determination of g, about ten successive differences* are observed and their mean is taken. The method of automatic registering is as follows: a mechanism, controlled by clockwork, lifts and sinks the load with constant time intervals (of ½ to 1 min.) and at the same time moves a photographic plate or film one step forward; a few seconds before every change of load a photomicrograph is taken of the pendulum position. The whole series of 10 or 12 photographs is taken automatically, and the observer has only to set up the instrument and develop and measure the plate or film. Working visually with the screw method, more time is required for the observations (about 20 min. for a series of 10 differences). In the field measurements last summer mentioned later in the paper, the total time for a point, including unloading and setting up of the instrument, observing and packing, was 35 to 40 min. for the visual method and 25 to 30 min. for the photographic method.

The *dynamic* method of determining N, by observing the time of swing of the pendulum, has only been used accidentally, when adjusting the instruments. The static methods seem to be preferable, because more convenient (no time measurement) and more accurate, though it

* The number of differences ought to be *even*, corresponding to a whole number of complete "periods" of the pendulum, in order to eliminate any effect of the slight accidental wandering of zero during a series.

must be admitted that the dynamic method allows a somewhat simpler construction of the instrument. P. Lejay and F. Holweck, who later (1929) also used astatized pendulums for gravity measurements,[6,7] use only the dynamic method.

2. *Some Theoretical Supplements*

In order to get the formulas necessary for computing g from the observations described some theoretical supplements are needed. In part I only infinitesimal deviations were considered. In the following, N always means the astatization degree, valid for the zero position of the pendulum (i.e., for infinitesimal deflections). If the acceleration of gravity on a certain base station be g_0 and on another station g_1, while h as well as mr remain constant ($k = mgr$), the following relations hold:

$$N_0 = \frac{k_0}{h - k_0}, \qquad N_1 = \frac{k_1}{h - k_1}$$

Hence

$$\frac{g_1 - g_0}{g_0} = \frac{k_1 - k_0}{k_0} = \frac{N_1 - N_0}{N_0(N_1 + 1)} \tag{6}$$

To calculate N from finite deflections, we must see how the moment arising from the weight of the pendulum varies with the angle of deflection.

The moments, tending to increase the deflection ϑ (relative to the tilted foot of the pendulum) are:

$$Q_1 = k \sin(\vartheta + \varphi)$$
$$Q_2 = -h\vartheta$$

from which is obtained the condition of equilibrium

$$Q_1 + Q_2 = k \sin(\vartheta + \varphi) - h\vartheta = 0 \tag{7}$$

For infinitesimal deflections (φ and ϑ) equation 5 follows from equation 7. When the deflections are finite, by developing $\sin(\vartheta + \varphi)$ in series and observing that N is a large figure, we obtain the approximate formula:*

$$\varphi = \frac{\vartheta}{N}\left[1 + \frac{N+3}{6}\vartheta^2 + \cdots\right] \tag{8}$$

In the difference methods, two approximately symmetrical deviations are observed: ϑ_1 and $\vartheta_2 (\vartheta_2 \sim -\vartheta_1)$ and the corresponding φ_1 and φ_2. Putting $\vartheta_2 - \vartheta_1 = \Delta\vartheta$, $\varphi_2 - \varphi_1 = \Delta\varphi$, we obtain from equation 8:

$$\Delta\varphi = \frac{\Delta\vartheta}{N}\left[1 + \frac{N+3}{24}(\Delta\vartheta)^2 + \cdots\right] \tag{8a}$$

* Compare ref. 2, p. 17, eq. 18.

The angle ϑ is determined by the deflection u (in scale divisions) of the pendulum image on an ocular scale. If l is the pendulum length and n the enlargement of the microscope, there is obtained with sufficient approximation

$$\vartheta = \frac{u}{nl} \tag{9}$$

$\Delta\varphi$ is known either from the screw reading z (method A), or once for all fixed by the magnitude (μ) of the small load (method B).

The Screw Method (A)

If a is a constant, characterizing the spiral spring,

$$\Delta\varphi = \frac{a}{g}\Delta z$$

and from equations 8a and 9,

$$\Delta z = \frac{g}{anl}\Delta u\left[\frac{1}{N} + \frac{N+3}{N}\cdot\frac{(\Delta u)^2}{24n^2l^2}\right] \tag{10}$$

Δu having a fixed magnitude, one may put

$$\frac{\Delta u}{anl} = \frac{1}{C}, \qquad \text{and} \qquad \frac{g}{a}\frac{N+3}{N}\frac{(\Delta u)^3}{24n^3l^3} = b$$

Hence

$$\Delta z = \frac{g}{CN} + b \tag{11}$$

where C is the instrument constant proper and b a small correctional term, which for N great may be treated as a constant. For a direct experimental determination of C and b by experiments on the base station only, a and nl must be measured, which means determination of $\Delta\varphi$ and $\Delta\vartheta$ in absolute angular measure. The constant C may also be obtained indirectly from the values of Δz on *two* stations with known g:s, having not too small a difference; in this case the calculation of b is unnecessary. From equations 11 and 6,

$$\Delta g = g_1 - g_0 = C[\Delta_0 z - \Delta_1 z] \tag{12}$$

As an example, it may be mentioned that one instrument used in 1935 by the author and Eeg-Olofsson[8] possessed the constants $C = 1.266$, $b = 0.07$, when z was measured in millimeters and g in gals; the astatization of the pendulum in Stockholm was $N = 570$.

The Load Method (B)

Here $\Delta\varphi = a'\mu =$ constant. Thus from equation 8a,

$$\frac{R}{\Delta u} = \frac{1}{N} + \alpha(\Delta u)^2 \tag{13}$$

where

$$R = a'\mu nl, \qquad \alpha = \frac{N+3}{N} \cdot \frac{1}{24n^2l^2} \text{ (α is small)}$$

Hence

$$\frac{\Delta g}{g_0} = \frac{g_1 - g_0}{g_0} = \left(\frac{1}{N_0} - \frac{1}{N_1}\right) : \left(1 + \frac{1}{N_1}\right) =$$

$$= \frac{(\Delta_1 u - \Delta_0 u)\left[\dfrac{R}{\Delta_0 u \cdot \Delta_1 u} + \alpha(\Delta_0 u + \Delta_1 u)\right]}{1 + \dfrac{R}{\Delta_1 u} - \alpha(\Delta_1 u)^2} \qquad [14]$$

R and α are the constants used for the reduction of the observations.*

For a district where the variations of g and Δu are small, equation 14 may be simplified to

$$\Delta g = C'(\Delta_1 u - \Delta_0 u)$$

with

$$C' = g_0 \frac{\dfrac{R}{(\Delta_0 u)^2} + 2\alpha(\Delta_0 u)}{1 + \dfrac{R}{\Delta_0 u} - \alpha(\Delta_0 u)^2} \qquad [14a]$$

As an example, it may be mentioned that a self-recording instrument used last summer (see below) had $R = 0.1028$, $\alpha = 2.17 \cdot 10^{-8}$, when g was measured in gals.

3. *Some Results from Field Work*

As a preliminary testing, Urelius in 1926 took an instrument from Stockholm to Copenhagen and back again, whereby he obtained determinations of g that deviated about 10 milligals from previous determinations with ordinary pendulums. In 1929, when a better ice vessel was introduced, Urelius took the instrument on a journey from Stockholm to Copenhagen, Potsdam, Munich, Berne, and back again over the same stations in reverse order.[3,5] The mean (accidental) difference between the indications of the instrument and the results of previous observations on these stations with ordinary pendulums was about 3 milligal. During the following years no experiments were made in Stockholm on the gravity meter. Meanwhile the astatized pendulum type of instrument was taken up independently in France, where Lejay and Holweck[6,7] in a very creditable way worked out instruments using the dynamic method. In 1934, the author, with the assistance of T. Eeg-Olofsson, took up the experiments again, and since 1935 a considerable number of

* If, instead of equation 9, the slightly better approximation $\sin\vartheta = \dfrac{u}{nl}$ is used, the factor $\dfrac{N+3}{N}$ in the expressions 10 and 13 for b and α, respectively, will be changed to $\dfrac{N+4}{N}$.

observations have been made in different parts of Sweden (see, for instance, ref. 8). With the screw method, the mean error of a g determination per setup, calculated from two or more visits at the same stations, as a rule with an interval of some days, amounted to 1.0 milligal. With small distances between the stations and favorable conditions for stable

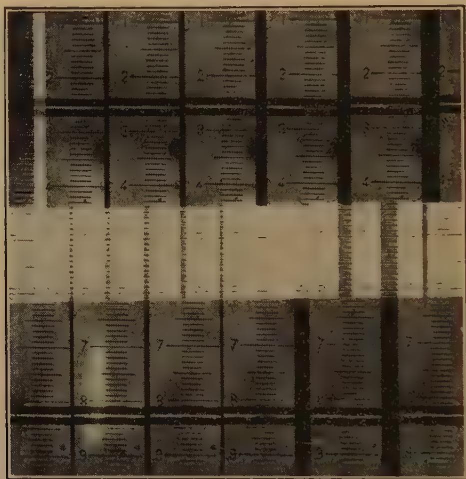

FIG. 5.—REGISTRATION OF ASTATIZED PENDULUM.

FIG. 6. FIG. 7.

FIGS. 6 AND 7.—NEW INSTRUMENT DEVELOPED IN COLLABORATION WITH ELECTRIC PROSPECTING COMPANY.

mounting of the instrument, the mean error went down to a few tenths of a milligal.

In 1936 the load method with automatic recording (on plates) came into use, and with both methods field measurements have been carried out in England and Holland by Eeg-Olofsson, giving a mean error for a single g determination of 0.64 milligal for the visual screw method and 0.46

milligal for the automatic load method with photographic recording. In Fig. 5 a registration is reproduced.

Among the merits of the instrument is especially notable the fact that no secular wandering of the zero and no continuous change of the instrument constant occur when it is properly handled. This makes the instrument type suited for comparing gravity on stations at very great distances apart.

Recently, a new construction of the instrument has been developed, in collaboration with The Electric Prospecting Co., of Stockholm (Figs. 6 and 7), which is being used in field work. It is equipped for photographic recording (on film) as well as for visual observation, and fitted with optical arrangements for obtaining higher accuracy than the previous model.

Acknowledgment

The author is much indebted to Mr. H. Hedstrom, Chief Engineer of the Electrical Prospecting Co., for the opportunity to discuss with him various points concerning these instruments.

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DISCUSSION

(*J. C. Karcher presiding*)

M. Muskat,* Pittsburgh, Pa.—What is the period of this pendulum?

H. Hedstrom,† Stockholm, Sweden.—It is not used dynamically.

* Gulf Research and Development Co.

† Chief Engineer, A. B. Elektrisk Malmletning.

M. Muskat.—What would be its period if it were set into vibration?

H. Hedstrom.—I cannot say. It is very slow; one quarter period per second, or something like that.

C. A. Heiland,* Golden, Colo.—Is the constant determined by using the vertical gravity gradient; that is, measuring on the ground and going up some tower or other structure and measuring the gravity difference?

H. Hedstrom.—Yes, that is one method.

S. R. Phelan, New York, N. Y.—How does the astatic pendulum compare with the torsion balance? I gather that they are both equilibrium instruments; that is, if you were reading them in the static condition, they would be. What data have been obtained on the differential curvature values from the astatic pendulum?

H. Hedstrom.—This instrument gives only total component, the total gravity, and you would have to construct your gradients out of the gravity curve. If you want to have the gradients, of course, the torsion balance has a very much higher sensitivity. The trouble is that the torsion balance is not very suitable for work, for instance, in areas covered by glacial drift where there are buried ridges of crystalline bedrock and buried boulders. In such places torsion balance results are very erratic, while the gravimeter gives steadier results. Of course, it will not at all give the same sensitivity as the torsion balance on mapping of local gravity anomalies like those above ore bodies.

S. R. Phelan.—There is also a horizontal component in the pendulum, is there not?

H. Hedstrom.—No, the horizontal component is not registered.

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* Professor of Geophysics, Colorado School of Mines.

A New Gravimeter for Ore Prospecting

By Helmer Hedstrom,* Member A.I.M.E.

(New York Meeting, February, 1938)

Gravity surveying with the torsion balance or the pendulum for ore prospecting purposes has generally not been considered practical or even possible. It is the intention of this paper to show that a fast and sensitive gravimeter can be very useful in ore prospecting, as an auxiliary to the electrical and magnetic instruments.

A number of gravimeters have been constructed and put in practical use in the last seven or eight years; for instance, by Ising, Lejay-Holweck, Haalck, Hartley, Thyssen, Noergaard, the Humble Oil and Refining Co. and the Gulf Production Co. Their main application, apart from their use for geodetic surveys, has been for structure mapping in oil prospecting. There is no record that any of them have been used for ore prospecting.

About the middle of 1934 the Boliden Mining Co., in Sweden, began experiments in constructing a static gravity meter sensitive enough for ore prospecting. Calculations and considerations that will be discussed later in this paper showed that this would necessitate an accuracy in the measurements of 0.1 milligal—roughly, one ten-millionth part of the total force of gravity. The first successful practical field tests of the instrument were made in January 1936. About the middle of 1937 the Boliden gravimeter was perfected. Ore prospecting with this instrument is now regular practice in Sweden, especially for the testing of indications given by geoelectric and magnetic methods.

Gravity Anomalies that May Be Expected from Ore Bodies

According to Newton's law, a mass M attracts the unit mass in a distance r with the force

$$k = \frac{GM}{r^2}$$

where G = the gravitational constant $= 6.68 \cdot 10^{-8}$ c.g.s units. In the c.g.s., or centimeter-gram-second, system the force of attraction is expressed in the unit "dyne," which has the physical dimension c.g.s.$^{-2}$ and denotes the force needed to accelerate the unit mass in one second to

Manuscript received at the office of the Institute March 8, 1938. Issued as T.P. 953, August, 1938.

* Chief Engineer, The Electrical Prospecting Co., Stockholm, Sweden.

the speed of one centimeter per second. The force of gravity acting on a unit mass at the earth's surface is 1 gram, or roughly 980 dynes. The unit $\frac{\text{millidyne}}{\text{gram}} = \frac{0.001 \text{ dyne}}{\text{gram}}$ = milligal used in gravity surveying is thus roughly one-millionth part of the total force of gravity.

For calculation of the gravity anomaly caused by a rod-shaped body, as shown in Fig. 1, the following procedure may be used:

The attractive force in the origin O due to a small element of volume bounded by the lines dr, $rd\alpha$, dy, at a distance from the origin equal to r in the xz plane and equal to y in the y direction is, according to the above:

$$\frac{G \cdot dr \cdot r \cdot d\alpha \cdot dy}{r^2 + y^2} \cdot \sigma$$

where σ is the mass per unit volume. The vertical component of this force, along the z axis, is

$$\frac{G \cdot \sigma \cdot r \cdot d\alpha \cdot dr \cdot dy}{r^2 + y^2} \times \frac{r \cdot \cos \alpha}{\sqrt{r^2 + y^2}}$$

The component of the vertical force of gravity caused by a prismatic body, extending at right angles from the xz plane to a distance y, with its base in the xz plane bounded by two radii vectores with the angles α_1 and α_2 from the x axis, and by two circles around O with the radii r_0 and r_1, will then be

$$\Delta g_z = G\sigma \int_{\alpha_1}^{\alpha_2} \cos \alpha \, d\alpha \int_{r_0}^{r_1} \int_{y=0}^{y} \frac{dy \cdot r^2 \cdot dr}{(r^2 + y^2)^{3/2}}$$

The solution of this integral gives

$$\Delta g_z = G\sigma \cdot (\sin \alpha_2 - \sin \alpha_1) \cdot y \cdot \ln \frac{r_1 + \sqrt{r_1^2 + y^2}}{r_0 + \sqrt{r_0^2 + y^2}}$$

Putting

$$G\sigma(\sin \alpha_2 - \sin \alpha_1) = \text{constant} = c$$

gives

$$e^{\frac{\Delta g_z}{cy}} = \frac{r_1 + \sqrt{r_1^2 + y^2}}{r_0 + \sqrt{r_0^2 + y^2}}$$

If we choose $r_0 = 0$, we get

$$e^{\frac{\Delta g_z}{cy}} = \frac{r_1 + \sqrt{r_1^2 + y^2}}{y}$$

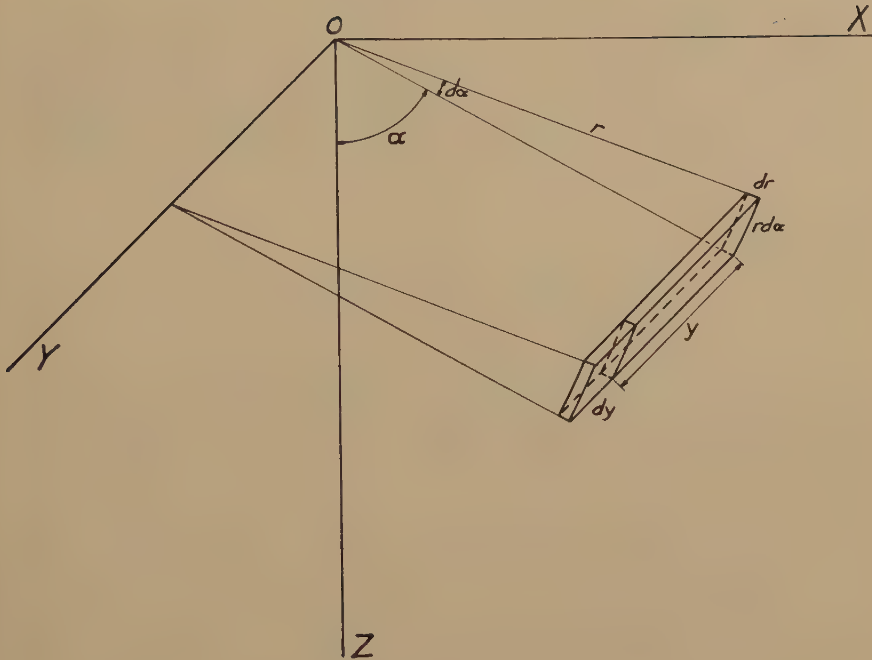

FIG. 1.—GRAVITY ANOMALY CAUSED BY ROD-SHAPED BODY.

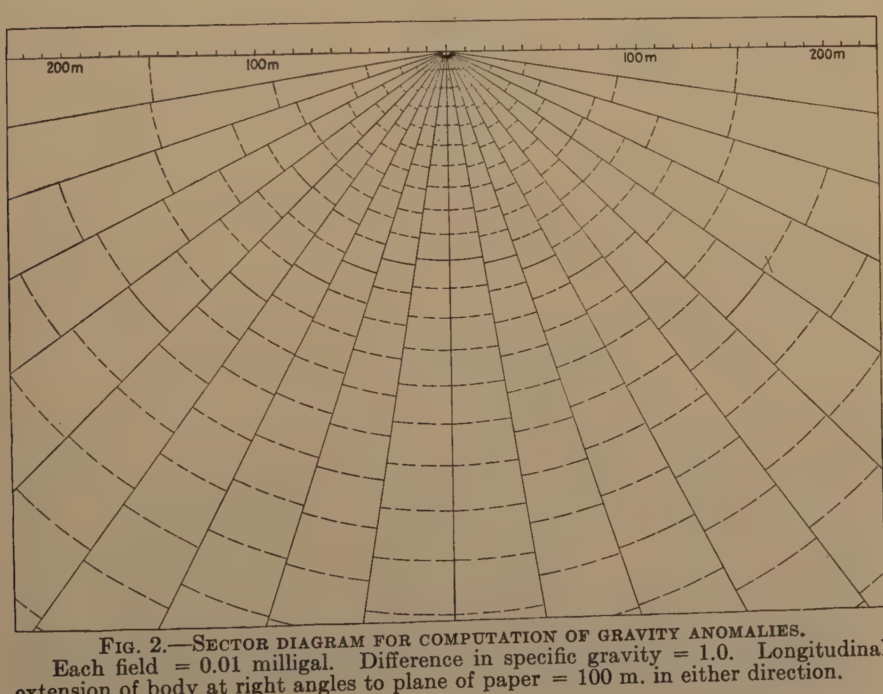

FIG. 2.—SECTOR DIAGRAM FOR COMPUTATION OF GRAVITY ANOMALIES.
Each field = 0.01 milligal. Difference in specific gravity = 1.0. Longitudinal extension of body at right angles to plane of paper = 100 m. in either direction.

If another strip is added on the "outside" of the first one, of the same length y and bounded by the same angles α_1 and α_2, but with the radius of the "outside" cylindrical surface increased to r_2, the anomaly Δg_z will be increased to $2\Delta g_z$, if we make r_2 so large that it satisfies the expression:

$$e^{\frac{2\Delta g_z}{\epsilon v}} = \frac{r_2 + \sqrt{r_2^2 + y^2}}{y}$$

This gives:

$$\left(\frac{r_1 + \sqrt{r_1^2 + y^2}}{y}\right)^2 = \frac{r_2 + \sqrt{r_2^2 + y^2}}{y},$$

from which r_2 can be determined in r_1 and y as units.

In the same way we can determine a radius r_3 corresponding to a total anomaly of $3\Delta g_z$ and r_4, corresponding to $4\Delta g_z$ etc. If we choose $\Delta g_z =$

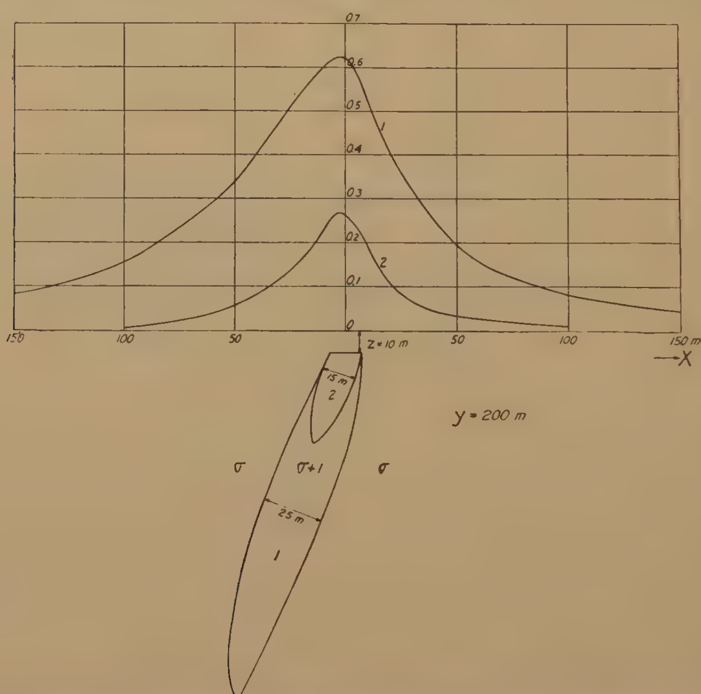

FIG. 3.—GRAVITY ANOMALIES OVER TWO ORE BODIES WITH DIFFERENT DEPTHS.

0.01 milligal, we can in this way compute the radii corresponding to 0.01, 0.02, 0.03 . . . milligal, for a certain division of angle $\alpha_1 - \alpha_2$ and a certain length y of the prismatic body. In this way we can construct a sector diagram like the one shown in Fig. 2, which we may imagine as the xz plane of Fig. 1. The diagram is constructed for divisions of angle equal to 10° and for a length $y = 100$ meters. Every

field in the diagram represents the base surface of a body that increases the vertical component of gravity in the origin with 0.01 milligal, assuming that the specific gravity of the body is one unit larger than that of the surrounding medium.

With the aid of diagrams of this type, gravity anomalies caused by bodies of a certain horizontal length and a certain cross section can be

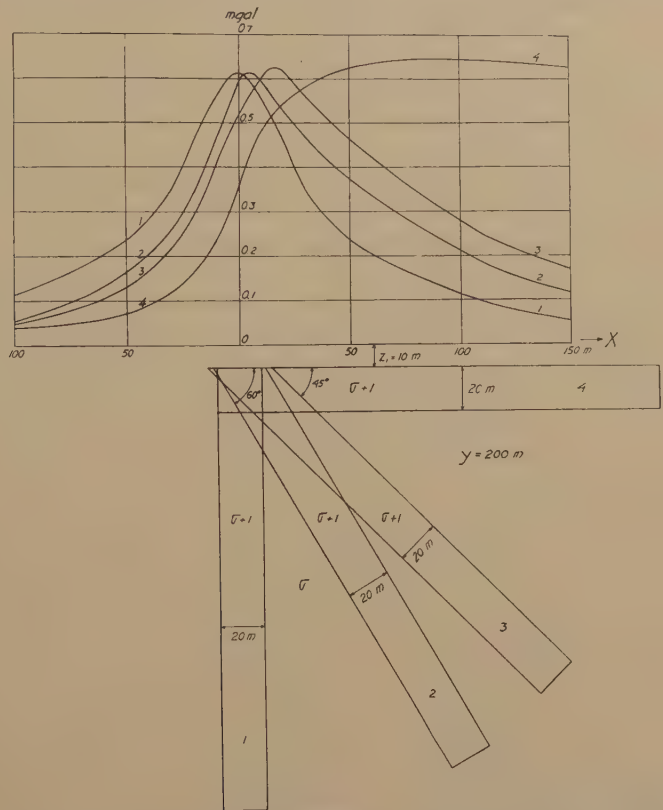

FIG. 4.—GRAVITY ANOMALIES OVER CENTER OF ORE BODIES WITH DIFFERENT DIPS. LONGITUDINAL EXTENSION 200 METERS.

computed, by plotting the cross section of the body in the scale given by the diagram and then counting the number of fields that the cross section will cover. Fig. 3 illustrates the anomaly, computed in this way, along a traverse across the middle of an ore body 200 m. long, with two different assumptions regarding its extension downward. The anomaly is of the order of only a few tenths of one milligal.

The equations above show that a change of scale in a diagram like the one in Fig. 2 changes the gravity anomaly in the same ratio as the scale. Thus, if the length y of the body is increased from 100 m., as in Fig. 2, to 500 m., and the distances given at the top of the figure also are

multiplied by 5, every field in the figure will correspond to 0.05 instead of 0.01 milligal, still assuming the same specific gravity as before. The use of the diagram, therefore, is not limited to the length of 100 m. Similar diagrams may be constructed for any length y; also for infinite length. However, for such "two-dimensional" bodies, especially if they have, for instance, rectangular, triangular or circular cross sections,

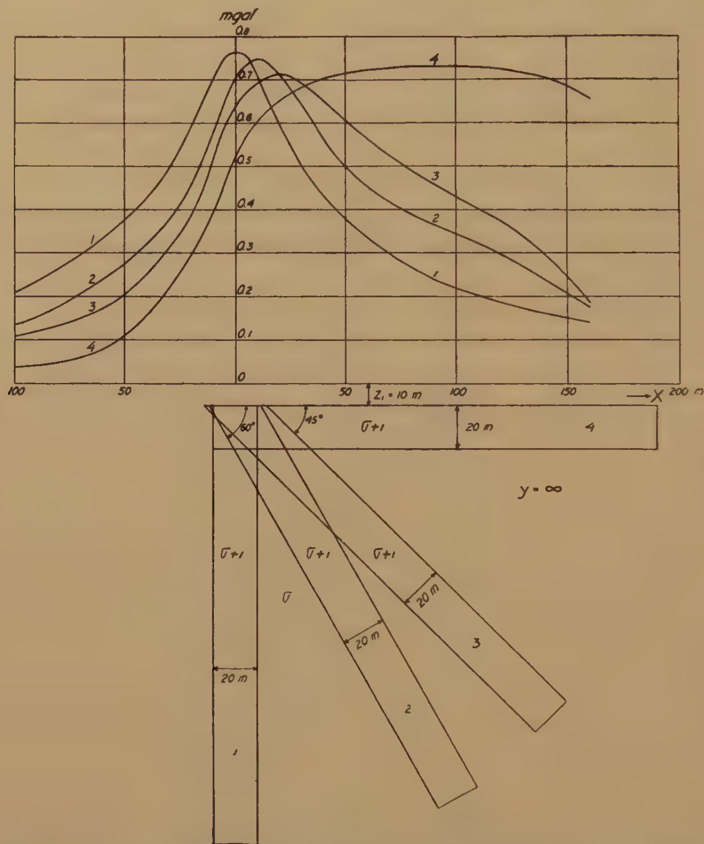

FIG. 5.—GRAVITY ANOMALIES OVER CENTER OF ORE BODIES WITH DIFFERENT DIPS. VERY GREAT LONGITUDINAL EXTENSION.

very simple formulas can be deduced, which serve as well as diagrams, or better.

Figs. 4 and 5 show the computed gravity anomalies over "ore bodies" of rectangular cross section, with the upper edge only 10 m. below the surface, and for different assumptions regarding the dip, Fig. 4 pertaining to a length of 200 m., Fig. 5 to "infinite" length. Fig. 6 shows how the anomaly over a body of a certain cross section increases when the horizontal length is increased from 100 to 200 m. and to "infinite" length. Fig. 7 shows how the anomaly over a heavy body of 200 m. depth and

"infinite" length is affected when the upper edge of the body is thought to be lowered from 10 m. depth to 20, 30 a.s.f. down to 100 m. In all these cases the difference in specific gravity between the heavy body and the surrounding medium has been taken as = 1.

Fig. 7 shows that with the upper edge of the body at a depth of 100 m. the anomaly, directly over the body, amounts to only 0.1 milligal,

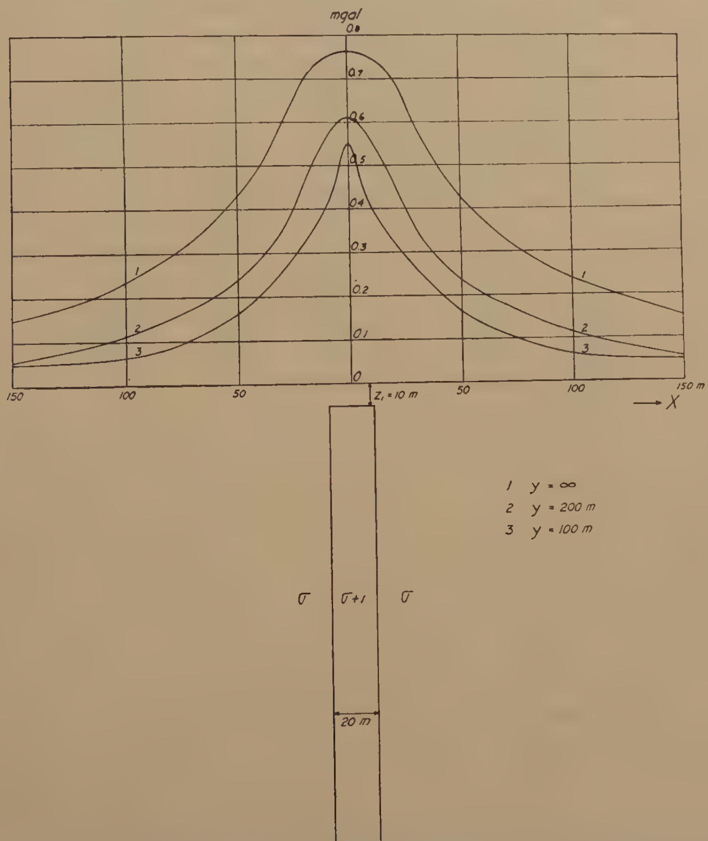

FIG. 6.—GRAVITY ANOMALY OVER VERTICAL ORE BODY. LENGTH ∞, 200 AND 100 METERS.

counted from a point 150 m. (500 ft.) away from the body. Taking this as the limit of instrumental accuracy, it might be interesting to calculate how large an ore body will have to be in order to be "detectable" at different depths. Table 1 shows the result of such computations.

CONDITIONS AFFECTING DETECTABILITY OF ORE BODIES BY GRAVIMETER MEASUREMENTS; CORRECTIONS

The figures in Table 1 for the depths to which a certain ore body can be detected are based on exceptionally favorable assumptions; i.e.,

that the ground is a horizontal plane and that the country rock is homogeneous. Certain limitations to the applicability of gravity surveying will be imposed by difficult topographic conditions and by irregular distribution of rock formations with different density in the vicinity of the ore body. In hilly country corrections must be applied to the

TABLE 1.—*Maximum Anomaly over the Body* 0.1 *Milligal Referred to a Point* 150 *Meters Away*

Length of Body, 200 m. Specific Gravity of Ore, 3.7; of Surrounding Country Rock, 2.7

Width of Body, M.	Depth of Body, 100 M.			Depth of Body, 300 M.		
	Depth to Upper Edge, M.	Cross Section, Area, $M.^2$	Million Tons	Depth to Upper Edge, M.	Cross Section Area, $M.^2$	Million Tons
10	37	630	0.47	60	2,400	1.78
20	58	840	0.62	93	4,140	3.06
40	75	1,000	0.74	133	6,640	4.91
60	79	1,260	0.93	180	7,200	5.33

gravimeter readings for the altitude and for the terrain effect. (In addition, of course, the corrections for latitude are always applied; the corrections for the effect of the sun and the moon are negligible, except at certain zenith distances.) The correction for altitude amounts to 0.1 milligal per foot and can therefore easily be computed with sufficient accuracy by simple leveling. The terrain correction, which generally amounts to only one-third of the altitude correction, can be divided into two parts: one due to the "negative" effect of the masses of surrounding hills above the horizontal plane of the observation point and of valleys below this point, and another for the pull exerted by masses below the point of observation down to a certain horizontal reference plane, common to all the observation points. For the first correction, diagrams like the one in Fig. 3 may be used in connection with ridges; otherwise, it will be preferable to adopt methods like those proposed by Jung and Haalck. This correction seldom amounts to more than one-tenth or a few tenths of a milligal, even when the ground is rather steep, therefore it is sufficient if the specific gravity of the masses involved is estimated within ± 10 per cent.

The second correction, as a rule, is considerably larger than the first one. In this some uncertainty may be caused by the difficulty of determining the density of the layer between the reference plane and the horizontal plane through the observation point. However, an error made in estimating this specific gravity seldom affects the results by more than 0.005 to 0.01 milligal per meter altitude difference. As the profiles of survey for this special purpose need only have compara-

tively short lengths, the *differences* in the terrain corrections between the different points of observation will generally be small, which makes the assumptions involved still less critical.

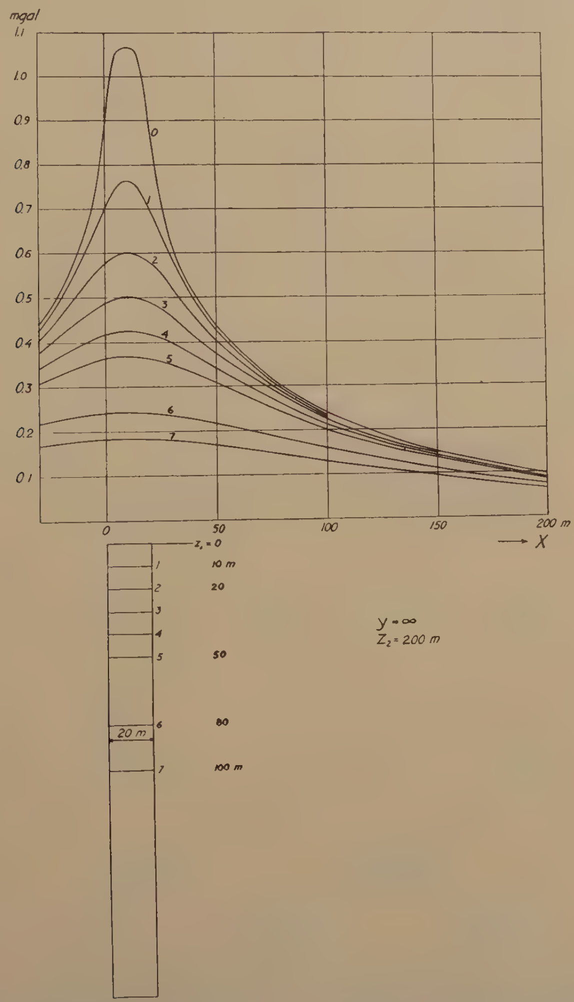

FIG. 7.—GRAVITY ANOMALY OVER VERTICAL ORE BODY OF GREAT HORIZONTAL LENGTH AND WITH UPPER EDGE OF BODY AT DIFFERENT DEPTHS BELOW THE SURFACE.

It is thus evident that, unless the ground is very hilly, the altitude and terrain effects will not appreciably lessen the chances for locating the ore bodies by gravimeter surveying. This is not true to the same degree in regard to the effect of irregular distribution of density in the rock formations close to the ore body. For this corrections are more difficult to calculate and can be made only more or less approximately.

Irregularity of the bedrock surface below the overburden may cause a good deal of trouble, as it may cause anomalies similar to those that can be expected from ore bodies. (See Fig. 8, where the difference in specific gravity between the bedrock and the overburden has been assumed to be equal to 1.00.) If comparison with electrical and magnetic

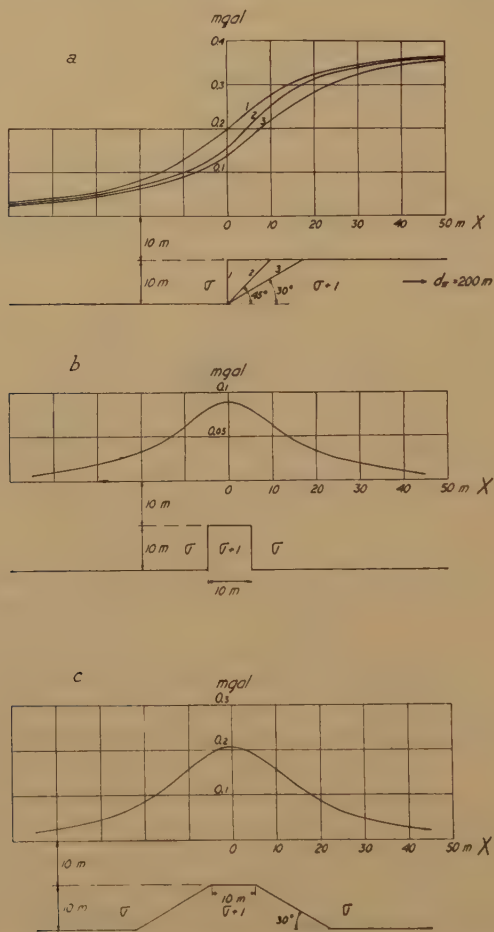

FIG. 8.—GRAVITY ANOMALIES CAUSED BY IRREGULARITY OF BEDROCK SURFACE.

"indications" still leaves some uncertainty as to whether a gravity anomaly is due to an ore body or a ridge, depth-to-bedrock determinations by the resistivity method or by an acoustic-seismic method may be used as a check.

A contact between two rock formations of different density may also cause a considerable gravity anomaly. Fig. 9 shows the computed anomaly over a vertical ore body in the contact surface between two formations with a difference of 0.1 in specific gravity. The indication

of the ore body is quite clear in cases 2 and 3, but less distinct in case 4, where the heavier formation has great width. Fig. 10 shows the same condition as 9, but with the ore body and the contact dipping away at

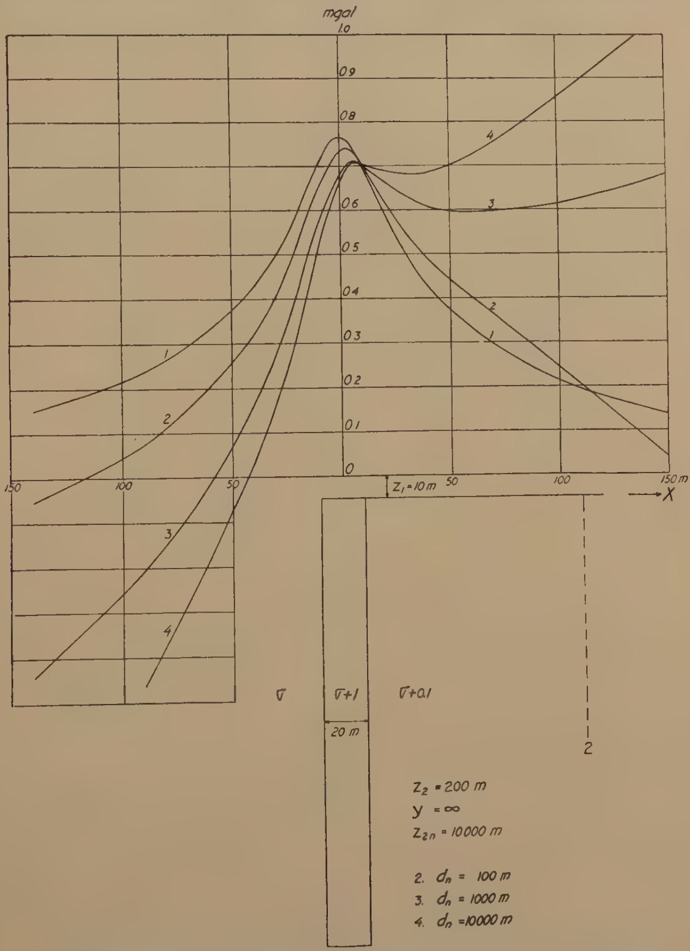

FIG. 9.—GRAVITY ANOMALIES OVER VERTICAL ORE BODY.

At contact between two different rock formations with a difference in specific gravity of 0.1. Curve 1 anomaly due to ore body only; curve 2, total anomaly when the heavier rock formation is 100 m. wide; curve 3 when its width is 1000 m. and curve 4, 10,000 meters.

45°. In this case the effect of the contact is less marked than in the vertical position.

These calculations and considerations show that relative gravity measurements with an accuracy of 0.1 milligal may be of considerable value in ore prospecting. The applicability, of course, may be limited by local conditions, but in this respect the method does not seem to be inferior to other geophysical methods.

The Instrument

The principle of the Boliden gravity meter is similar to that of a spring balance in which deflections are registered with the utmost precision. The principal parts of the actual balance are shown diagram-

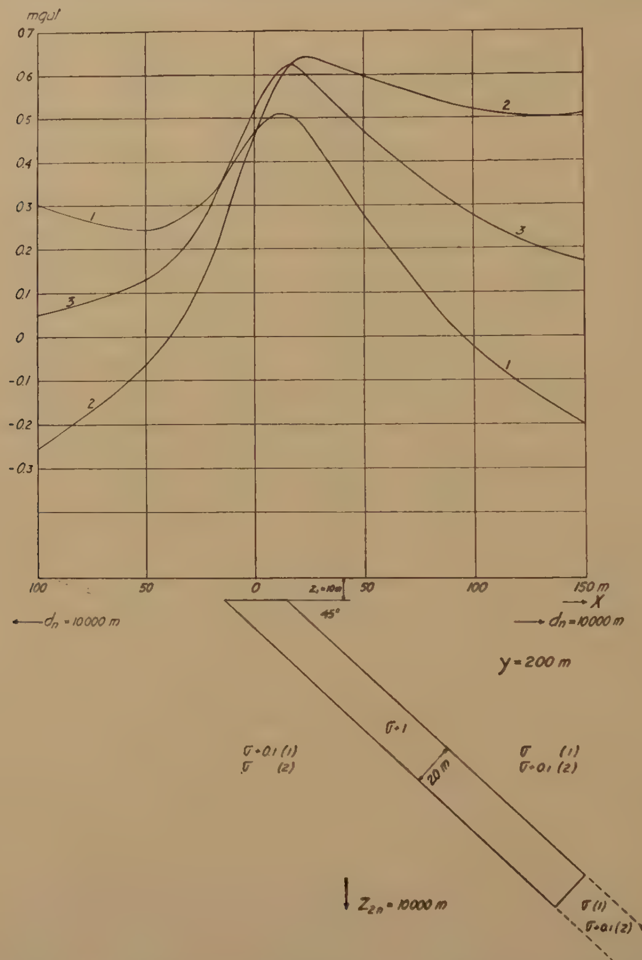

Fig. 10.—Gravity anomalies over an ore body of 45° dip.
At contact between two different rock formations with a difference of specific gravity = 0.1. Curve 3 shows anomaly due to ore body; curve 1, total anomaly when the heavier formation is on the footwall side and curve 2 when it is on the hanging-wall side.

matically in Fig. 11. The movable body is suspended by the plate springs F, which are attached to the cross piece of the support E. The movable body is surmounted by a plate situated closely below and opposite to a corresponding plate fixed to a thick steel disk A by an

electric insulation B. Both plates are connected through the contact C with the oscillatory circuit of an ultramicrometer coupling, which registers minute variations in capacity between the two plates. The ultramicrometer most used is a modification of Dowling's coupling, as improved by Gustafsson. This coupling has been very satisfactory, especially for laboratory use. However, another coupling has also been tried, based on a different principle similar to Whiddington's, in which is determined the audible note caused by interference between two oscillatory circuits, one of which includes the capacity of the two plates of the balance.

In Gustafsson's coupling variations in the anode current caused by changes in the capacity between the two plates are registered on a

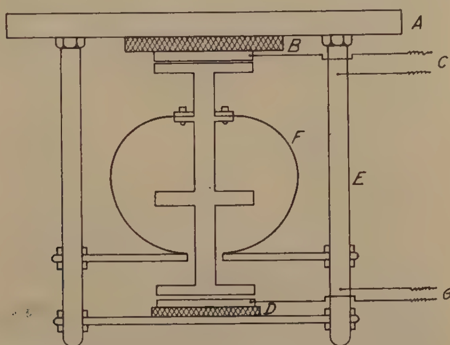

FIG. 11.—PRINCIPAL PARTS OF BALANCE.

milliammeter, the plates being connected to the grid circuit of an audion tube. The coupling can be arranged without much difficulty so as to obtain a sensitivity of at least 10 ma. per cm. Assuming that the ammeter can be read with a precision of 5×10^{-3} ma., this would correspond to an accuracy of 0.5×10^{-3} cm. in the determination of variations in capacity. As we have usually worked with a distance of 0.002 cm. between the two plates of the balance, this sensitivity signifies that we can register variations of 3.5×10^{-9} cm. in the deflection of the balance, assuming that the area of the plates is 2.25 sq. cm. On the other hand, a variation in gravity of 1 mgal will correspond to a change of approximately 5.5×10^{-7} cm. in the position of the movable body, so that variations in gravity of less than 0.01 mgal can be read without difficulty. But as it has not so far been possible to eliminate or reduce errors below the stated limit, there has not as a rule been much use for such a degree of sensitivity. Work has been done, therefore, usually with a sensitivity in the milliammeter of about 2 to 3 scale parts per mgal, which, on the assumption that the readings can be taken with an accuracy of up to 0.2 scale parts, signifies an accuracy in reading of about 0.1 mgal. After the latest improvements in the design of the gravity meter, it has, how-

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FIG. 12.—GRAVITY METER IN USE.
FIG. 13.—MAKING A READING.
FIG. 14.—METHOD OF CARRYING GRAVITY METER.

ever, been possible to increase the sensitivity of the instrument to such a degree that 5 to 6 scale parts correspond to 1 mgal. Thus in recent field work readings have been taken with an accuracy below 0.05 mgal.

In the lower part of the movable body there is another plate, at a certain distance from a fixed plate *D*, insulated from the other parts of the balance. The purpose of this device (the returning plates) is, by applying a certain potential to one of the plates, to return the movable body to a desired position, by means of the electrostatic attraction caused by the difference of potential between the plates.

Let us suppose, for example, that at a certain point the milliammeter has registered a certain figure. We move the gravity meter to another point with a different force of gravity. This entails a change in the position of the movable body. We then change the potential of one of the plates until the milliammeter registers the same figure as before, which signifies that the body has been restored by the applied force to its original position.

This method of measurement entails several advantages. In the first place the limits of measurement can be enlarged almost *ad libitum* by increasing the difference of potential. Secondly, the readings on the milliammeter will be registered at the same place, or within a very limited space with the same sensitivity. As a general rule, the distance between the plates has been so adjusted that a potential difference of 10 volts corresponds to a variation in gravity of 1 mgal. With greater variations in gravity, it should be observed that the attraction increases with the square of the potential difference between the plates.

By a device in the construction of the support on which the movable body is suspended—namely, the use of materials with different coefficients of expansion—the balance with the movable body is so compensated for temperature that a rise of 1° C. changes the distance between the upper plates merely by an amount corresponding to 1 or 2 mgal. As, despite the compensation, it is sensitive to any "unsymmetrical" ingress of heat, its temperature, as also that of the electric coupling, is regulated by an automatic electric device, which keeps the temperature constant to about two-hundredths of a degree. Both the balance itself and the electric coupling are enclosed in a Dewar flask.

Fig. 12 shows the appearance of the gravity meter when in use for field work. The thermos flask with the balance and the electric coupling are suspended by an aluminum ring, which is fastened by three adjustable screws to the support, consisting of an aluminum vessel with a diameter of about 40 cm. The aluminum vessel is mounted on a plate, likewise made of aluminum, which, with the aid of three iron pins driven into the ground, is placed in an approximately horizontal position. This arrangement ensures the stability required for such precise measurements. The adjustment of the movable body in the vertical plane is effected with

the aid of the three screws on which the upper aluminum plate rests. These screws are so adjusted that two levels above the vessel are caused to assume a horizontal position. The electric circuit in the Dewar flask is connected by two electric wires to a battery box, containing batteries for filament current, heating current and anode current, as well as an instrument box with a milliammeter and voltmeter, with which the potential on the return of the movable body is registered. The electric cable to the instrument box usually has a length of 1.5 m. Thus, when the movable body has been adjusted in a horizontal position and the clamping has been loosened, the operator can move to a distance of 1½ m. from the point where the instrument has been set up and make the reading (Fig. 13). In case of necessity, however, the cable to the instrument box can be further lengthened, so as to enable the reading to be made at a very considerable distance from the point where the instrument has been set up. The instrument, which weighs about 25 kg., can be carried by two men with the aid of two poles (Fig. 14).

Constant Determination and Adjustment

For the determination of the constant of the instrument, three different methods can be employed:

1. Determinations while the instrument is inclined.
2. Computation of the electrostatic attraction by the potential difference between the returning plates.
3. Moving the instrument to different heights.

1. The force $k = g_z M$ acting on the movable body is greatest in the vertical position. M = the mass of the movable body and g_z = the vertical strength of the field. If the movable body is inclined to a certain angle α, it will be acted on in the direction of the axis of the body by a force $g_z M \cos \alpha$. The variation $\Delta g M$ in the force acting on the body will then be

$$\Delta g M = g_z M(1 - \cos \alpha)$$

and

$$\Delta g = 2g_z \sin^2 \frac{\alpha}{2} \simeq g_z \frac{\alpha^2}{2}$$

In adjusting the level and in determinations of the constant of the instrument, the procedure is as follows: The deflection of the milliammeter is marked on a diagram as a function of the angle of inclination. The level is adjusted to the a position, where the deflection passes through a minimum; that is, to a position where the capacity between the two plates of the balance is smallest and the force acting on the movable body greatest. From this zero position the remaining values are deter-

mined, from which Δg can then be deduced, if g_z = the absolute vertical field component is approximately known.

2. The additional force ΔgM, which is produced by applying to the returning plate a potential V, can be calculated in accordance with the following formula:

$$\Delta gM = \frac{SV^2}{8\pi a^2}$$

or

$$\Delta g = \frac{SV^2}{8\pi a^2 M}$$

Where S = the surface of the returning plates,
a = the distance between the returning plates,
M = the mass of the movable body.

It is thus evident that it is possible to determine the constant of the instrument, if the quantities in the right member of the above equation have been measured. Instead of measuring the distance a between the plates, it is usually safer to determine the capacity C between them. We then obtain:

$$\Delta g = 2\pi \frac{V^2C^2}{SM}$$

3. In moving the instrument, for instance within a tall building, from a point with the intensity g_z to another point h m. higher with the intensity g_{zh}, the difference in the field strength is:

$$\Delta g = g_z - g_{zh} = h \times 0.309 \text{ milligal}$$

from which the scale value is determined.

Accuracy and Speed of Measurement

As has been pointed out above, there is no difficulty in bringing the accuracy in the reading of the instrument down to 0.01 mgal. In field procedure, however, a considerably smaller degree of accuracy has generally been used; namely, 0.1 to 0.05 mgal. But, on account of instability caused by the transport of the instrument, the mean error of the measurements was at first considerably larger than the limit of error by the readings. In the summer of 1936 it was found that in the measurement of some 100 points one had to reckon with a mean error of 0.48 mgal for a single observation. As the surveyed profiles generally were remeasured some 5 to 10 times, the mean error of the gravity determination was reduced to one-third of the above figure. In the latest field measurements the mean errors have been considerably reduced. After measuring about 600 points, a mean error of 0.10 to 0.20 mgal

for a single observation has been found. (An example of calculations of the mean error will be found in the following section.) By repetition of measurements the mean error is reduced to one-half or one-third of this figure.

A single observation takes from 2 to 4 min. When the points are surveyed 5 to 10 times, the entire measurement of a point takes 10 to 40 min. The average speed of measurement by this system has been 10 to 15 points per day, or 150 to 200 points a month.

Some Examples of Practical Tests

Fig. 15 shows a surveyed profile over a deposit of pyrrhotite at Kärrbo, 12 miles north of Riddarhyttan in central Sweden. The ore,

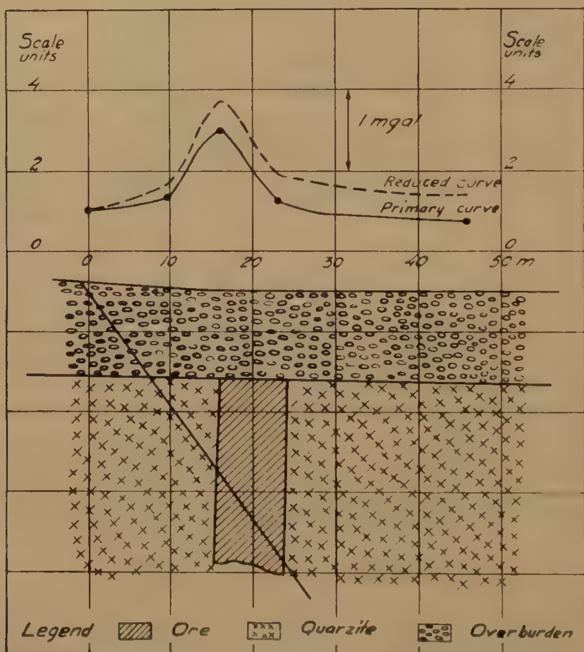

Fig. 15.—Gravity anomaly over a pyrrhotite ore at Kärrbo, Västmanland.

which was discovered by geoelectrical prospecting in 1933, has a thickness of 7 to 8 m., dips steeply to the north, and has an overburden of about 10 m. The gravity measurements, made in Jaunary 1936, gave an indication of barely 1 milligal. The width of the indication, moreover, is very small, which points to a comparatively small extension (Fig. 6).

A surveyed profile over a large ore deposit under the Lake Mensträsk, in the Skellefte district, discovered by the geoelectric method, is shown in Fig. 16. This deposit of sulphide ore consists of three parallel lenticular

veins, dipping steeply to the north, the southernmost of which has a thickness of about 10 m., while that of the other two veins is only about 3 to 4 m. The measurements were made on the ice in March 1936. The indication shows an evident effect of the contact with the schist (Figs. 9 and 10). An endeavor was made to make a reduction for the effect of the contact, on the assumption that the difference in specific gravity between the schist and the leptite is 0.2. The curve, thus reduced, shows in a rather striking manner the indications of the two ores. The more northerly indication, however, is somewhat uncertain, seeing that

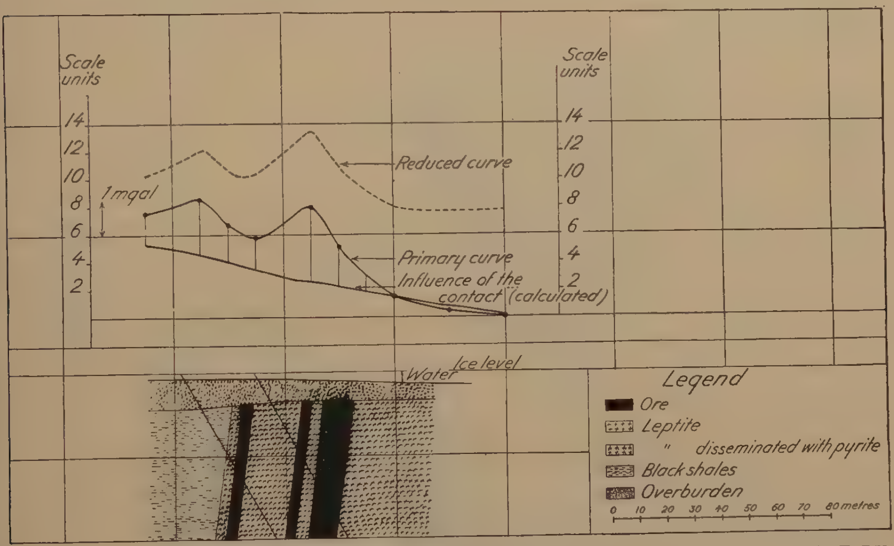

FIG. 16.—GRAVITY ANOMALY OVER PYRITE DEPOSIT DISCOVERED UNDER A LAKE BY GEOELECTRICAL INVESTIGATIONS.

its magnitude does not much exceed the margin for accuracy of measurement. The maximum indication is considerably north of the outcrop, which is caused by the northerly dip of both the ores and the contact between the schist and the leptite.

Among the tests made in 1937 in the Skellefte district, one was carried out over the ore at western Åkulla (Fig. 17), which consists mainly of three steeply inclined lenticular veins of pyrite and chalcopyrite. In the tests, five profiles were measured across the lenticular veins and all three were indicated by the gravity meter. Fig. 17 shows one of the surveyed profiles crossing the north part of the largest vein. The smaller westerly parallel was not indicated in this profile, as only the southern offshoot of the ore body was crossed by the profile, but after two profiles had been measured farther to the north the said parallel was brought out clearly. The relatively large positive terrain correction is necessary

because the profile runs partly just below a waste dump from the exploration work.

At Långsele (Fig. 18) six profiles were measured in the summer of 1937 across the large westernmost vein of ore. The ore in this vein consists chiefly of pyrite and zinc blende with a sulphur content of 30 to 40 per cent. The ore body, which strikes east to west, dips at an angle

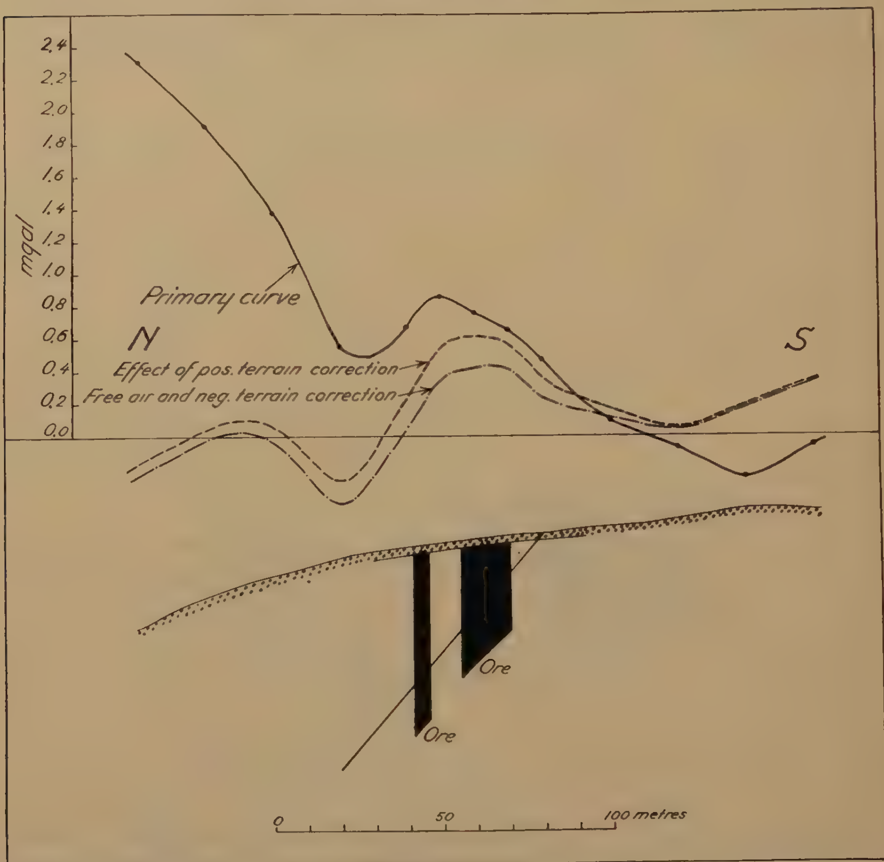

FIG. 17.—GRAVITY ANOMALY OVER WESTERN AKULLA COPPER PYRITE ORE, SKELLEFTE DISTRICT.

of 60° to the south, and pitches to the east. The thickness is from 12 to 15 m., increasing towards deeper levels. There is evidently a rather marked dissemination of pyrite in the foot wall. One of the surveyed profiles is shown in Fig. 18. The indication is over one milligal and the dip to the south is distinctly shown.

Another test was made over the Rakkejaur ore, the largest one in the Skellefte district. This ore body, which has a horizontal area of 19,000 sq. m., was traversed by two profiles, one of which is shown in Fig. 19.

The ore, which occurs at the contact between quartz porphyry and black schists, is for the most part a rather poor pyrite with 25 to 30

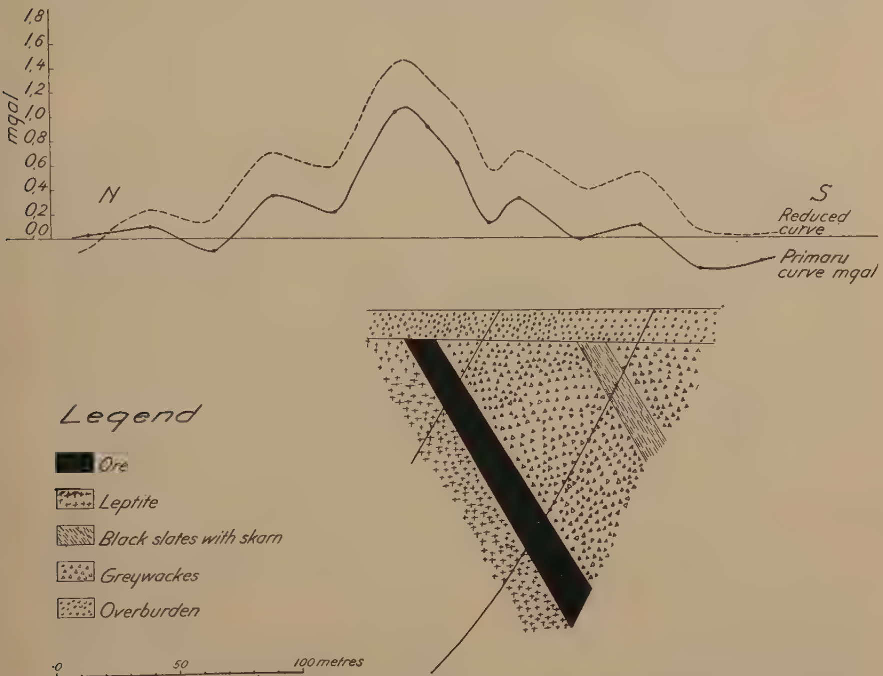

FIG. 18.—GRAVITY ANOMALY OVER LÅNGSELE ZINC PYRITE ORE, SKELLEFTE DISTRICT.

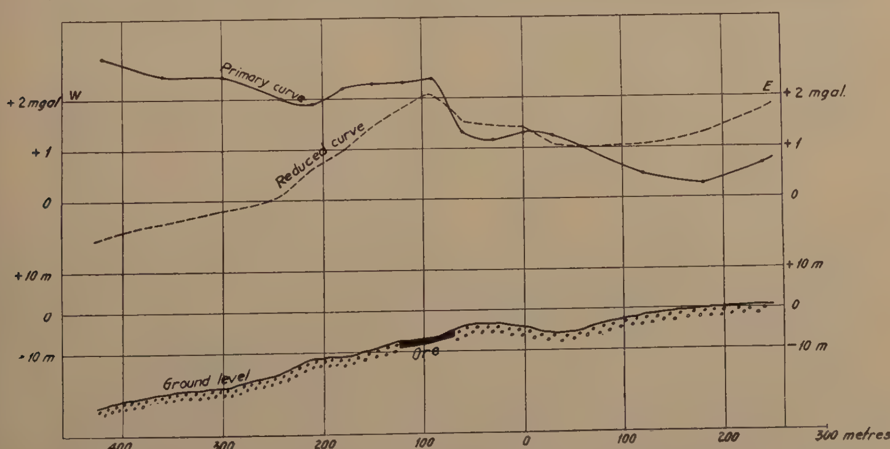

FIG. 19.—GRAVITY ANOMALY OVER RAKKEJAUR PYRITE BODY, SKELLEFTE DISTRICT.

per cent sulphur. Certain minor portions of it contain chalcopyrite and arsenopyrite.

Table 2 shows an example of calculations of the mean errors in measurements at Rakkejaur. The measurements relate to the western

part of the profile shown in Fig. 19. The figures given in the first column of each point are the readings on the voltmeter, while those in the second column are the corresponding gravity values. The time at which some of the readings were taken is also shown.

TABLE 2.—*Example of Calculations of the Mean Errors in Measurements at Rakkejaur*

Point	240		300		360		420		Point
Time	Scale		Scale		Scale		Scale		Time
	Units	Mgal	Units	Mgal	Units	Mgal	Units	Mgal	
11:35	99.1	8.44	97.1	8.10	97.7	8.19			
							93.2	7.47	11:45
			95.7	7.87	96.0	7.92			
11:55	98.5	8.33							
			94.6	7.69	96.9	8.07			
							94.4	7.65	12:04
			97.0	8.08	95.4	7.82			
12:13	99.2	8.45							
			96.8	8.05	95.0	7.76	94.3	7.64	12:23
Mean value..............	8.41		7.96		7.95		7.59		
Mean error for a single determination[a]...........	0.07		0.18		0.18		0.10		
Mean error for total determinations..............	0.04		0.08		0.08		0.06		

[a] For the "mean error for a single determination," the following formula has been used:

$$E = \pm\sqrt{\frac{\Sigma\Delta^2}{n-1}}$$

where $\Sigma\Delta^2$ = the sum of the squared differences between observed values and mean value.

n = number of observed values.

No attempt has been made to improve the results through correction for "drift of reading."

The average specific gravity of the ore seems to be 3.4 to 3.6. The ore body has a maximum width of about 80 m. in the northerly sections, while its width gradually decreases toward the south. Its length is about 550 m. The profile shown in Fig. 19 traverses the ore body approximately over its center. The measurement shows that the indication has a maximum strength of 2.0 to 2.5 milligal. An increase in gravity values east of the ore, where it is adjoined by schists, appears

to be of a regional character. In spite of this, the gravity measurements indicate that the dip of the ore body is probably vertical or eastwards, and not, as had previously been supposed, westwards. On the basis of the measurements, it may be estimated that the ore body has a considerable depth.

Conclusion

It will be apparent from the above that the instrument has so far been tested mainly in studies of gravity anomalies over ore bodies. In such measurements the surveyed profiles are obviously very short, compared with the length of profiles in oil prospecting and geodetic surveys. It should, however, be pointed out that the instrument can be used with advantage also for surveys of the last-mentioned nature.

Recent surveys over large distances, with the instrument carried in a motor car, gave a mean error per single observation of 0.53 milligal, and per station, surveyed three times, 0.31 milligal. With a distance of a few kilometers between stations some 7 to 9 stations were surveyed per hour.

Gravity at Sea by Pendulum Observations

By Albert J. Hoskinson*

(New York Meeting, February, 1938)

Progress on the earth depends to a large extent upon the rapid interchange of ideas and commodities between the various nations of the world. The smooth flow of commerce, by which these ideas and commodities move, is accelerated by accurate and complete maps of the various land and water areas of the earth. The production of accurate maps depends upon a complete knowledge of the figure and dimensions of the object to be charted, therefore a knowledge of the shape of the earth is a prime necessity for accurate charting purposes. Gravity observations were first undertaken to aid in determining the shape of the earth so that the geodetic surveys of its land and water areas might be more accurately connected to form an undistorted map. A large part of the earth's surface is water and it is therefore necessary to get gravity values at sea as well as on land before a complete and accurate figure of the earth can be determined.

Dr. F. A. Vening Meinesz, of Holland, was the first scientist to perfect a pendulum instrument that would accurately measure gravity at sea. He had previously developed a multiple pendulum apparatus for measuring gravity upon swampy and unstable land areas in Holland and it was therefore quite natural that he should adapt the instrument to give accurate results upon a still more unstable support, the surface of the sea. His instrument as perfected gives satisfactory results when mounted in gimbals on any vessel where the roll and pitch are not more than about 5° and where the vibrations from rotating machinery and wave action are not large enough to cause the pendulums to shift on the knife-edges. This degree of stability can be obtained on surface craft only in protected waters or on very calm days. Therefore, Dr. Meinesz mounted his instrument in a submarine so that he could submerge below the disturbed surface waters and get the desired stability in nearly any kind of weather by simply diving to greater depths. Most of the gravity-at-sea work has been done with a Meinesz instrument and a very large number of the observations have been made by Dr. Meinesz himself. He has occupied nearly 1000 sea stations and probably has traveled as many miles by

Manuscript received at the office of the Institute Feb. 17, 1938. Issued as T.P. 955, August, 1938.

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submarine and made as many dives as some of the naval experts of the world.

Gravity work was still in its infancy when it became apparent that the value of gravity at any point on the earth's surface was definitely related to the geological structures underlying that point. This fact, of course, opened up a much larger field for gravity observations and accelerated the development of many new types of instruments for detecting gravitational changes. Many kinds of gravity instruments are in common use today in geophysical prospecting, and a large part of the present-day ocean work is conducted for the purpose of scientific investigation of geological theories concerning the formation of the continents and ocean basins of the world. Several of the leading nations of the world take an active interest in gravity-at-sea observations, and no doubt a more complete and accurate gravity picture of the water areas of the earth will soon be available. These observations will furnish the data needed for a precise determination of the figure of the earth, for a more accurate determination of the various constants used in the isostatic adjustments, and will aid greatly in the studies of the broad geological theories of continents and ocean basins.

Work by the United States

The first gravity-at-sea work undertaken by the United States was in 1928, when Dr. Meinesz was invited to bring his instrument to this country and make a gravity cruise in a United States submarine.* This expedition was known as the "gravity cruise of the U. S. Submarine S-21." The cruise was made in the Gulf of Mexico and in South Atlantic waters in the immediate vicinity of Cuba, Haiti and Puerto Rico. U. S. Navy Submarine S-21 was assigned to this work and the U. S. Eagle Boats 35 and 58 acted as tenders for the submarine. Forty-nine sea stations were occupied on this cruise and some very large and interesting negative anomalies were discovered north of Haiti and Puerto Rico.

Another expedition was organized in 1932, known as the "Princeton-Navy gravity expedition," to investigate the water and land area of the Bahama Islands and also to try to locate the western extent of the large negative strip of anomalies discovered in 1928. Dr. Meinesz was again invited to take charge of the gravity work and U. S. Submarine S-48 was assigned to this project, together with the submarine tender *Chewink*. Fifty-three sea stations were occupied, the extent of the western end of the negative anomaly strip was very nicely determined, and gravimetric conditions in the vicinity of the Bahama Islands were carefully analyzed. At the conclusion of this cruise, in view of the then recent discovery of a large negative strip of gravity anomalies in the East Indies by Dr. Meinesz,

* This expedition was under the joint auspices of the Navy Department and the Carnegie Institute of Washington.

it was thought likely that there might also be a strip of negative anomalies along the entire West Indies island arc. Therefore another gravity-at-sea expedition was organized in 1936, by Dr. Richard M. Field, Chairman of the American Geophysical Union committee on ocean basins. This expedition was known as the "Navy-American Geophysical Union gravity expedition." It started field work at Coco Solo, Canal Zone, on Nov. 29, 1936, and completed the trip at Philadelphia on Jan. 14, 1937. U. S. Navy Submarine *Barracuda* was assigned to this project. This was the first gravity expedition by this country in which Dr. Meinesz was not able to accept the invitation to be a member. Dr. Maurice Ewing, of Lehigh University, was designated as chief scientist of the expedition, Dr. Harry Hess, of Princeton University, who assisted Dr. Meinesz in the 1932 expedition, was selected to make the geological studies, and to the writer was assigned the task of operating the gravity instrument and making preliminary tests of the pendulums before the cruise started. A Meinesz instrument was loaned to this country, by the International Geodetic Association, for this expedition and was received at the Coast and Geodetic Survey laboratory in Washington several months before the sailing, so that ample time was available to redetermine the temperature constants of the pendulums and to make a few minor changes in the recording device, so that a portable crystal chronometer, then being designed and built by the Bell Telephone Laboratories, could be used as the timing device for the pendulum observations.

The instruments were shipped to Coco Solo, Canal Zone, where they were installed on the submarine *Barracuda*. The scientific staff joined the ship there and the cruise started on Nov. 29, 1936. Fifty-five sea and harbor stations were occupied, at each of which the value of gravity was determined by a 30-minute pendulum observation. Several profiles of stations were located at right angles to the trend of the island arc and extending well into the Atlantic Ocean on one side and well into the Caribbean Sea on the other, so that the position of the negative strip, if it did exist, would be well determined. The expedition was a very successful scientific venture and the gravity picture of that part of the world was completed satisfactorily.

The computation of the results is a time-consuming project, so that it is entirely impossible to keep even the preliminary computations up to date on a cruise of this nature. Preliminary reports of the expedition were made at the last annual meeting of the American Geophysical Union and the final results will be published in the near future. In general, it may be said that a strip of large negative anomalies was located just outside of the island arc and extending throughout its entire length. The position of this negative strip with respect to the islands and the active volcanoes of the region corresponds closely with the one located in the East Indies by Dr. Meinesz.

These discoveries have an important bearing on the geological studies of these regions and also on the general geological studies of the continents and the ocean basins. The fact that the West and East Indies are similar from a gravity standpoint indicates that perhaps other island arcs might also show the same characteristics and should be investigated in some detail in future gravity-at-sea work. However, the interpretation or geological significance of these results is well outside the writer's field of work and can best be left to the men that are devoting their lives and energies to this field of scientific endeavor.

When we have secured the results, reduced them to a common datum by isostatic adjustments, computed accurately the figure of the earth, and combined all of the land and water surveys to form an accurate and undistorted map of the world, then our particular interest in gravity observations is satisfied, but the fact that these results are then very useful in other fields of work makes the securing of them all the more important and desirable.

Construction and Use of Meinesz Instrument

There are a number of places along the continental margins of the United States where a few near-by gravity-at-sea values would aid greatly in geological interpretation of conditions within these coastal areas. Therefore it might be well to describe in some detail the construction of the Meinesz instrument and indicate how these water areas may be surveyed.

In the deeper water, where a submarine may safely submerge to 50 ft. or more, no adjustment or modification of instrumental equipment is required to conduct the work. It is simply necessary to secure the submarine, a Meinesz instrument and the experienced personnel required for the work. For very shallow water, tripods could be used for the instrument support, and for the intermediate depths surface craft can be used in calm weather and a damping device can easily be designed so that observations can be made in relatively rough weather; or the observations might be made directly on the ocean floor by the use of a bathosphere or similar protective covering for the instrument.

The Meinesz instrument as designed for sea work consists of three working pendulums, each about 0.25 meter long, constructed of brass or nonmagnetic material and mounted so that they are free to swing on agate knife-edges, which rest on agate planes. The three pendulums are made as nearly identical in every respect as it is possible to make them. Their periods should be adjusted during the construction, so that they will not differ at any temperature by more than 30×10^{-7} sec. For simplicity of discussion, we will designate the pendulums 1, 2 and 3 and will assume that they are mounted so that 1 and 3 are in the end positions and 2 is in the center position. The three agate supports are

in the same horizontal plane, so placed that the three pendulums swing in the same vertical plane. The supports are just far enough apart so that the motion of one pendulum will not interfere with that of the adjacent one. A starting mechanism is arranged so that the outer pendulums 1 and 3 may be started with equal amplitudes and 180° out of phase. Thus the tendency of one pendulum to introduce a small horizontal sway into the support by its periodic movement is eliminated by the opposing movement of the other end pendulum. No. 2, or the center pendulum, is stationary at the beginning of an observation, and for a land station or where a steady support is available remains at rest throughout the entire observation, provided the ideal conditions regarding starting amplitude and phase are realized at the beginning of the observation. Any small error in starting or in lack of isochronism between the pendulum periods will cause the introduction of a small sway into the instrument, thus introducing a small movement to the center pendulum. At a sea station the center pendulum quickly picks up motion from the roll of the boat. This roll also affects the moving pendulums so that a record of any single pendulum will show large variations in amplitude and period. To solve this difficulty the combined movement of each end pendulum with the center pendulum is recorded as the movement of a fictitious pendulum. This is accomplished by placing small mirrors on each pendulum and reflecting a light beam from No. 1 to No. 2 and out to the recording box, also one from No. 3 to the opposite face of No. 2 and out to the recording box. Thus there are available the traces of two fictitious pendulums 1-2 and 3-2. The periods of these two fictitious pendulums determine the value of gravity at any field station and at the same time furnish a check on the accuracy of the work. The roll of the ship has a similar effect on all three pendulums, so that the traces of the two fictitious pendulums are undisturbed in amplitude, phase and period. There are a number of small corrections that must be applied to each set of observations because the ideal conditions regarding the starting amplitude and phase are seldom realized. The pendulums are rarely exactly isochronous and they will, therefore, gradually get slightly out of phase and thus introduce a small sway into the instrument. Temperature, pressure, and humidity of the air are different at each station and the observed period of the pendulum must be reduced to what it would have been under standard conditions. To determine these small corrections it is necessary to know the position of the pendulums with respect to the vertical in the plane of the swinging pendulums and also in the vertical plane at right angles to that one. Therefore two damped pendulums are incorporated in the construction of the instrument, one in each of these planes, and light beams are reflected from these pendulums to secure the necessary information. Thus the final record at any station consists of the traces of two fictitious pendulums, a third trace that

represents the roll of the instrument and a fourth one that represents the pitch. A temperature trace is also introduced to record the variations in temperature during the progress of an observation. Now, if time is recorded on the photographic record, everything that is required to compute the periods of the fictitious pendulums is available on one record. Time is shown on the records by momentarily eclipsing the light source at half-second intervals, thus introducing half-second dots into all of the various traces.

Chronometer

Time is, of course, one of the most important factors in accurate pendulum observations. An error of 0.001 sec. in the total time of a ½-hr. pendulum observation will cause an error of about 1 milligal in the gravity value at that station. Time for the 1936–1937 gravity expedition was obtained from a new portable crystal chronometer designed and built by the Bell Telephone Laboratories of New York City. The chronometer was designed so that the output of the oscillator operated a small synchronous motor, which made one complete revolution every ½ sec. and eclipsed the light source for about 0.001 sec. at every revolution. Thus very accurate ½-sec. intervals were shown on the record. The crystal chronometer proved to be a wonderfully accurate timepiece and thus furnished far better time control than had been available on any previous gravity-at-sea expedition. The chronometer operated continuously during the entire 40 days of the trip and in that time gained a total of 0.36 sec., or an average daily rate of 0.009 sec. The rate showed small variations from temperature changes aboard the submarine but at no time was the variation so large that the total time of a ½-hr. swing was in doubt by as much as 0.0005 sec. This exceptional accuracy of time measurement has never before been available outside of the large time laboratories of the world and is certainly a big step forward in scientific measurements where accurate time or uniform motion is a controlling function. Since the completion of the gravity cruise the chronometer has been further improved, so that its rate and variations are now much smaller than they were during the gravity work. The chronometer is strong enough for ordinary handling, and operates from a small power supply that is easily available to any field party engaged on scientific measurements.

Accuracy of Measurements

A submarine when submerged must travel slowly through the water if it is to maintain a uniform depth of submergence, therefore a gravity observation aboard a submarine is not taken at one spot but is the average value over a line about 2 miles long. However, the change in the gravitational attraction is uniform enough so that the average value will represent very closely the value of the center of the line; anyway, the

position of the vessel at sea located by astronomic sights is seldom known nearer than ½ mile even under good conditions. Thus the movement of the boat during the observations is not serious. The direction of the movement and the exact speed must be carefully recorded, however, for the east-west component of that movement acts directly to increase or decrease the centrifugal force of the earth's rotation and thus has a direct effect on the observed gravity. Similarly, the direction and speed of underwater ocean currents have a direct effect on the observed value of gravity and are at the present time the most uncertain factor in gravity-at-sea work. There are several ways in which these currents could be accurately measured but so far no submarine on a gravity cruise has been equipped to measure them; they have been estimated from the best available current information of the area and from the navigational data of the cruise.

The results of the last gravity-at-sea expedition indicate that the value of gravity at any sea station may be determined by present instruments with an accuracy of 1 to 5 milligals. It is believed that if the gravity instrument were inclosed in a vacuum chamber and thermostated so that a constant temperature was maintained an accuracy of about 1 or 2 milligals could be reached. This is certainly as great accuracy as is required where ocean currents are not exactly known and where positions are only approximate. The large negative values so far observed are as large as 300 milligals, so that very small variations of 1 or 2 milligals will have no appreciable effect on the general interpretations of the results or on any general figure-of-the-earth work. On land work, especially in geophysical prospecting, where smaller structures are being sought, these variations are more serious. Many of the newer gravimeters have an accuracy of a small fraction of a milligal and are ideal for the work in which they are used. These very accurate instruments all have an appreciable drift from day to day and hour to hour, which makes their use on any large scheme where stations are many miles apart a more difficult operation, as a portion of their accuracy may be lost by the unknown drift between stations.

During the last gravity cruise several harbor stations were occupied when the submarine was on the surface and at anchor. In fairly calm water these observations were just as good as those taken at submerged depths. Therefore in protected waters it is not necessary to work on a submerged vessel. It is believed that a damping arrangement can easily be designed that will allow surface observations to be made in fairly rough weather. If the instrument can be perfected so that surface observations are practical in moderate weather, then gravity-at-sea will no longer depend upon the availability of submarines and field work will be greatly accelerated.

Seismograph Prospecting for Oil

Symposium

(Los Angeles Meeting, October, 1938)

Contents

INTRODUCTION

By Walter A. English,* Member A.I.M.E.

The four papers making up this symposum have been prepared especially for those who have no knowledge of seismograph prospecting. To many people mathematics is a formidable subject, and many are discouraged from studying seismograph prospecting under the mistaken assumption that the mathematical requirements are severe. In the papers presented here there is not a single mathematical equation. We believe that considerable competence in dealing with some of the most important problems of this work can be attained without any mathematical training whatever. At the present time, there is a real need for interpretation of results by persons thoroughly competent to bring geologic experience and imagination to bear on data that are subject to multiple interpretation from a strictly physical viewpoint.

Likewise, those charged with the responsibility of planning seismograph exploration campaigns would do well to become familiar with as much as possible of the technical aspects of the work. They would then be in better position to judge of its success in individual cases. It has often been said that the seismograph is a new structure-finding tool for the geologists to use. Using a tool and using its product are two different things. One does not use an automatic screw-threading machine when he buys a box of screws at the hardware store. Neither does the geologist

Manuscript received at the office of the Institute Oct. 24, 1938. Issued as T.P. 1059, April, 1939; reprinted in Petroleum Technology, May, 1939.

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use the seismograph as a tool when his interest is confined to checking up on the cost per spread, and counting the number of closing contours of the structures shown on the final map presented to him.

Much of the territory in the United States suitable to seismograph work has already been shot over in more or less detail. To be successful in the future it will be necessary to find favorable structures that have been overlooked. Improvements in instruments and field methods will be factors, but one must be able to distinguish between real improvements and nonessential talking points or misleading instrumental modification of the vibrations returned by the earth. Possibly the largest factor will be the ability to recognize obscure leads where the best instruments and methods do not produce clearcut results. The geologist has become accustomed to recommending drilling of structures that appear doubtful on the basis of surface mapping. He will have to do the same thing for structures found by the seismograph. Each organization would do well to develop somewhere within itself the ability to judge accurately of the real merit of seismograph results, which may give an obscure clew to a new oil field or which may be only misinterpretation of poor or misleading records.

THEORY OF SEISMIC REFLECTION PROSPECTING

By Willard H. Tracy*

The method of seismic reflection prospecting has many times been compared to sound ranging, a process that became familiar to many during the World War. According to this very simple theory, the dynamite goes off and sends out a compression wave, which meets a reflecting horizon and returns to the surface, where it is recorded and the elapsed time of its travel in the earth accurately determined. Computations are then made on the general theory that the longer the elapsed time, the greater the depth of the reflecting bed. While this serves to give an idea of the fundamental basis of the process, it suggests a simplicity that does not exist in the life history of the various waves set up by a dynamite explosion in the earth.

To begin with, a seismic wave in the earth does not obey the same simple laws of propagation that apply to a sound wave in the air. The property of strength in shear possessed by solids introduces a factor that sets such waves apart from the ordinary air waves. Also, the transmission characteristics of the earth vary widely from place to place, and a fairly accurate knowledge of the space relationships of these variations is necessary if the final results are to be understood. Besides the waves recorded and used in the reflection computations, there are many

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unwanted waves resulting from the explosion, and we must know something of these in order not to confuse them with the wanted reflections.

Though we wish to know something of the theory of seismic waves, as a matter of fact the methods of reflection prospecting have been built up empirically by trial and error, with very little real aid furnished by the theoretical physicists and mathematicians. Not only is the present successful process not the result of following out the obvious conclusions to be drawn from a simple theory, but there are few that will undertake to fully explain the reasons for its success. In areas where the present best practice does not give entirely satisfactory results, the lack of complete knowledge of the various wave paths, and changes in wave character, and the energy relations of the different waves, make it difficult to search for improved methods except by the same old cut and try process. It is only as we do many things according to the methods dictated by hard-won experience that the seismograph record shows any waves that fit into our scheme of reflection calculations. While we have a simple theory to account for the presence of usable reflections on a good record, we are unable to give a clear description of the wave paths and character of waves that produce the bad record that is commonly marked "no reflections" and discarded. We do know that there are many wave paths besides those of the wanted reflections, and that some of these unwanted waves carry considerably more energy than the wanted ones. The chief difficulty of seismic prospecting is, then, the problem of successfully recording the wanted waves and as far as possible excluding the unwanted waves. It is for this reason that a complete knowledge of waves generated by a dynamite explosion would be valuable, and that all those engaged in seismic prospecting should have at least some knowledge of the character of both wanted and unwanted waves.

Waves

The explosion of dynamite is essentially a transient phenomenon. Almost immediately after detonation, a small volume of gas at very high pressure and temperature is substituted for the dynamite. Here is a source of a large amount of energy available for conversion into mechanical motion of the surroundings. How efficiently this highly compressed gas transmits its energy to the surrounding rock is not known even approximately, but it is suspected that it is not very efficient. It is also suspected that almost all of the work done by the dynamite is accomplished within a very short time, measured in terms of a very few thousandths of a second. There is, then, a further loss due to conversion to heat within the zone of crushing of the surrounding rock. But beyond the crushed zone, a few thousandths of a second after the explosion, we should have a hollow sphere of rock, the particles of which are in motion

outward from the center, and are also under compression, having kinetic energy of motion and potential energy of resilience. These are the two types of energy that are present in an elastic wave, and we shall therefore expect to find such a wave or waves proceeding outward from the source.

The term "elastic wave" used here is one of several that are applied to the vibrations that may be transmitted by the earth. Other terms are seismic wave, longitudinal wave, compression wave, rarefaction wave, sound wave, transverse wave. Of these, seismic waves and elastic waves may be of the longitudinal or transverse type; compression, rarefaction, and sound wave being restricted terms for particular longitudinal waves. In books on earthquake seismology the term Principal wave is applied to the longitudinal wave and Secondary wave to the transverse wave, and these terms are often contracted to P and S waves in the discussion. Also, Rayleigh wave and Love wave are used as names for particular types of transverse waves that follow the surface of the ground. The Rayleigh wave is of the same type as the "ground roll" of the prospecting seismologist.

The longitudinal wave is one in which the actual motion of the particles of the earth is back and forth parallel to the direction of progress of the wave. In the transverse wave the particles of the earth move back and forth in a plane at right angles to the direction of progress of the wave. The most important class of transverse waves is that in which the wave is strongest along the interface between two media, as between earth and air, and the amplitude of the motion decreases with distance from the interface. In the Rayleigh wave the motion is at right angles to the interface (up and down on the surface of the earth) and with Love waves the motion is parallel to the interface. Actually the motion includes some longitudinal motion, being of the nature of movement in an ellipse of which the transverse direction is the major axis.

In order to further discuss the character of the explosion-generated wave, we may here distinguish between a transient wave and one of the steady-state type. Much of the literature descriptive of waves and wave motion refers to the steady-state type of wave; partly because of the comparative simplicity of the mathematical formulas that describe steady-state wave propagation, and partly because such waves are important in many engineering problems. A steady-state wave train is one in which there is a recurring series of events spaced at regular time intervals, each exactly like the same event in the preceding cycle. The simplest type of steady-state wave train is one in which during each cycle the variable quantity has successive values proportional to the values of the sine of an angle as the angle increases from zero to 360°. Motion, according to this simple law, is called simple harmonic motion. The oscillation of a pendulum is an example and many mechanical vibrations

tend to take on this type of motion. The short transient wave generated by an explosion of dynamite differs in many ways from steady-state, simple harmonic motion. This warning might be unnecessary if it were not that mathematical treatment of transient waves involves the assumption that a transient wave train may be considered equivalent to a series of steady-state waves so selected that the total effect of all of the assumed steady-state waves is the same as that of the transient wave. The series is an infinite one, however, and when we compare any single transient wave to a single steady-state wave train the difference is great.

In a transient generated by a dynamite explosion we may expect a radiating wave with a sharp compression wave front, followed by a short series of successive rarefactions and compressions, each one of much less amplitude than its predecessor. Such a wave train is called highly damped, the term damping being applied when the decrement is due to decreasing amounts of energy available at the source for successive radiations, as well as to the decrease of energy carried by a wave train due to absorption effect of a transmitting medium only partly elastic. It seems probable that the initial compression phase of the explosion-generated transient is the most important part while the wave is still close to its source, and that the distance between successive compression phases is small as compared to the character of the reflection wave train that returns to the surface after having traveled a mile or two down into the ground and back again. This is borne out by the few fidelity records that have been made of the wave train passing instruments placed a short distance from the dynamite. It is, however, difficult to obtain such a record free from contamination by secondary waves, such as those generated by reflection from the surface of the ground. In order to avoid this interference, the Western Geophysical Co. made experiments in which the dynamite was placed under water in the ocean. These results showed the total wave train passing the recording instrument in about one hundredth of a second.

We start out, then, with an imperfect knowledge of the generated wave. As will be brought out later, we are unable to determine accurately the character of the returning reflection because of the association with it of waves of other character from which it cannot be wholly separated. We believe, however, that there is a considerable modification of the character of the wave during its travels down and back again through the earth.

Reflection Waves

In much of the discussion regarding reflection seismograph records, a reflecting interface is postulated as the cause of the reflection, probably owing to the importance of such interfaces in the problems of earthquake seismology, which was a well developed science at the time that reflection

prospecting began. As a matter of fact, most of the reflections dealt with in prospecting seismograph work are from strata of a thickness less than the wave length of the outgoing wave train, and such a stratum must be considered as a unit, as a plate or diaphragm. The character of the reflection is dependent on the thickness of the bed as well as upon the differences in density and elastic moduli between it and the overlying and underlying media. There may even be several reflections from adjacent strata which when analyzed on the final record are considered as a single "reflection." Likewise reflections may arise when there is no sharp interface present, but merely a gradual change in the character of the beds. Such conditions must be borne in mind when a geologist attempts to visualize the probable reflecting characteristics of a geological section with which he is familiar.

Doubtless actual pure interface reflections also occur, though the thickness of the bed below the interface must be greater than the wave length of the approaching wave if the result is to be considered as an unmodified interface reflection. Such reflections at least have the virtue of comparatively simple equations defining the reflecting power of the interface with assumed differences in character of the media on the two sides. According to theory, whenever a longitudinal wave reaches such an interface at an angle of incidence other than exactly zero degrees that is other than perpendicular to the interface it will be broken up into four different waves, a transmitted and a reflected longitudinal wave, and a transmitted and reflected transverse wave. With a fairly small angle of incidence, as is usual in reflection seismograph work, we believe that the two resulting longitudinal waves will carry the greater part of the energy. At least we have not been able to recognize the arrival of reflected transverse waves at our surface instruments.

When passing from the upper medium to the lower, the transmitted longitudinal wave will be unaltered in phase. But for the reflected wave, we distinguish two cases; namely, that of reflection with the more dense medium below and that of reflection with the more dense medium above the interface. In the first case there is reversal of direction only but in the second there is reversal of both direction and phase. Thus for the more dense medium above the interface, if there is a compression phase leading before the reflection we will have a rarefaction phase leading in the wave train after the reflection takes place. This condition gives a suggestion as to the result of reflection from a thin stratum. The reflection from the lower interface of the stratum, after it passes through the upper interface, will tend to be out of phase with and cancel the reflection from the upper interface. This, of course, is in line with the common sense conclusion that a stratum of even the hardest rock as it approaches infinite thinness will not be able to send back a reflection of material magnitude.

Recording Waves

So far we have considered only the probable origin of reflection waves. Several additional classes of waves arise as a result of the explosion and of modifications in wave type by the transmitting media. The waves commonly found on a reflection record may be classified as follows: (1) first arrival, (2) direct-path compression wave, (3) reflections, (4) ground roll, (5) high-speed ground roll, (6) diffraction waves. Each of these types of wave trains may be recognized by various criteria, but owing to the instrumental characteristics of the receiving and recording apparatus their true character is not accurately recorded. A study of these waves with a high fidelity recording apparatus would be desirable, but we have not the result of any such study available to us, nor have any such results been published.

1. The first arrival or shortest time path wave is the wave used on the old refraction shooting, and is the wave train that arrives first at the recording apparatus when the latter is at a sufficient distance from the source for this wave to outdistance the direct-path wave. A sudden increase in velocity characteristics downward in the earth is necessary for the development of this wave. In reflection shooting this is the interface between weathered zone and the underlying unweathered zone. When the dynamite is placed slightly within the unweathered zone, as is usually done, a direct compression wave will follow along the lower side of the interface and approximately parallel to it, and above the interface a diffraction wave will be developed, which will travel nearly vertically upward to reach the surface of the ground. The time of travel will be that of the least time path, as calculated by the simple differential solution for the given geometry and velocities. Where the explosion takes place above the interface, the path will include an additional refraction at a grazing angle within the unweathered zone. The fastest wave is one of low energy content, and for this reason a maximum of sensitivity of the recording apparatus should be maintained for the proper recording of this wave. This wave is important in calculating the correction to be applied to observed travel time of reflections to eliminate the effect of variations in thickness and character of the low-velocity "weathered" surface layer.

2. The direct-path compression wave should arrive shortly after the fastest wave, and should have the maximum energy of all of the arriving waves. Actually, the shortest time path wave merges into a series of waves of increasing amplitude for several cycles, sometimes with a phase break that may indicate the first arrival of the direct wave; other places the phase break cannot be discerned. Where the initial phase of the direct wave can be discerned it ushers in a wave train of several (say six or more) cycles with gradual build up to a maximum and gradual

decay in amplitude. This is entirely unlike the single compression shock wave that we have postulated leaving the vicinity of the explosion. Possibly this wave train is made up of the direct wave merged with reflections of the same wave, which have taken place within the weathered zone between its boundary surfaces at the top of the unweathered and at the surface of the ground.

3. Reflections are the waves of principal interest, inasmuch as they are the ones that are used to calculate the subsurface structure. Sometimes a reflection may have a characteristic appearance dependent on the stratigraphic succession at the reflecting zone; thus in Oklahoma a characteristic "viola" reflection may be recognized. Sometimes there is a sudden onset with phase interruption from the preceding energy arrival, and short duration with high damping. Such a reflection is most easily recognized, and will have its best appearance on records taken with instruments of minimum amount of filtering. Other reflections may have gradual onset and the wave train may carry through four or five cycles of sinusoidal waves of similar appearance. The one criterion we know for recognizing a "reflection" is that it shall arrive in recognizable form on several traces of the record at regular time intervals, so that when plotted out by the conventional methods used for reflections it will give a reasonable picture of the geological structure. The term "usable reflection" would be more appropriate, as undoubtedly there are many reflections on bad records that are marked, "No reflections," by the computer. The tendency toward build up rather than a single shock wave suggests strongly that there is something resembling resonance in the ground and that the various strata through which the wave is reflected and refracted cause it to take on the form of a wave train of several cycles more than the original compression wave. In general, with increase of the distance that the wave travels in the ground its apparent wave length is increased. Thus deep reflections tend to be of lower frequency than those from shallower horizons. A material amount of the final character is taken on by the returning wave close to the surface. Geophones placed on the surface will give results quite different from those obtained when the geophones are buried only a few feet below the same surface positions.

4. The ground roll is a Rayleigh wave generated by the direct longitudinal wave on reaching the earth air interface. With the average filtered record there is no recognizable ground roll. Yet this type of wave train often has more energy than any of the reflections and it is largely to avoid the effects of ground roll that filtering is used in the instrument construction. A typical ground roll is manifest as large waves of from 8 to 15 cycles per second average frequency, as compared to 25 to 50 cycles per second, the usual frequency limits of reflection waves. The ground roll is a transverse wave, which travels along the

surface of the ground, and is the vibration that is felt at a distance when a shot goes off. It is responsible for such damage as may be caused to surface structures. Its velocity of propagation varies from 800 to 1200 ft. per sec., being greater in firmer ground.

5. The name "high-speed ground roll" has been applied to waves that arrive at successive geophones at times indicating a horizontal component of travel of around 2000 to 2500 ft. per sec. Possibly it is a transverse wave along the interface between the weathered and unweathered zones, which sends off secondary compression waves to reach the surface.

6. Mr. Rieber has called attention to the possible importance of diffraction waves in faulted structures and has illustrated their formation with model experiments. While such waves may be hard to recognize because of attenuation of their energy content due to their mode of origin, they should prove of value wherever their character can be determined and their place of origin computed. As previously stated, the fastest wave path involves a diffraction of the wave.

INSTRUMENTS FOR REFLECTION SEISMOGRAPH PROSPECTING

By Arthur Nomann*

Recording instruments may be conveniently described under the headings of: (1) geophones, (2) amplifiers, (3) recording cameras, (4) miscellaneous parts.

Geophones

Geophones or seismometers are actuated by slight earth movements, and generate electrical currents, which are amplified and recorded. In all geophone designs there is provision for differential movement between parts of the mechanism and the signal voltage is generated as a function of this differential movement. The part that conforms to the movement of the ground is called the support, and the part that remains more nearly stationary with respect to more distant unactuated parts of the earth is called the steady mass. The mechanical period of the geophone is the time required for an oscillation of the steady mass after an initial impulse is given it, and is determined by the effective stiffness of the spring joining the two parts together in relation to the weight of the suspended part. Geophones have been used with frequencies varying all the way from less than 10 cycles per sec. to several hundred cycles per second. Those with a natural frequency of less than, say, 100 cycles

* The Superior Oil Co.

per sec. must be well damped in order to give satisfactory records. The damping usually takes the form of an oil bath, though air damping and electromagnetic damping are used. The latter has the advantage over oil that it is not subject to variations in character as a result of temperature changes, which may be sufficient to greatly change viscosity of a damping oil.

The manner in which the steady mass of a geophone is displaced in response to earth movement depends on the natural frequency and amount of damping. A low-frequency geophone with slight damping will tend to follow earth displacements directly, having its maximum steady-mass displacement at the same time that the earth has its maximum displacement from its position of rest. The steady mass will tend to remain stationary in space, receiving only slight accelerations through the suspension spring or through frictional drag. This is the type of seismometer used in much earthquake study, and the name "steady mass" is appropriate when applied to the suspended weight of this type. Many people may visualize prospecting seismometers as following the earth movement in this manner, but actually such a seismometer would not be at all suitable, because of the difficulty of reducing damping to the requisite small amount, and for other reasons. As the stiffness of the suspending spring is increased in relation to the mass of the moving part the next classification is approached, in which the natural period of oscillation is approximately equal to the period of the waves that it is desired to record. The differential movement that takes place in such a geophone in response to earth movement will tend to have its maximum displacement at the time of maximum velocity of the moving earth, though the relationship will not be an exact one in the transient movement of the earth that we are considering in seismograph prospecting. This is the most popular type of geophone at the present time, various models having natural frequencies varying between 25 and 50 cycles per sec. As the stiffness of the spring is still further increased we reach the class of so-called acceleration sensitive geophones, of which the piezoelectric construction is a common type. Such a geophone will tend to have its maximum steady-mass displacement correspond to the maximum earth acceleration. If the natural period is high it should give a very close approximation to a fidelity record of the earth accelerations. It might at first examination be thought that either the high-frequency or low-frequency type, giving more accurate records of the function of earth movement portrayed, would be preferable to the less exact intermediate type, but that is not true. The latter has the advantage of giving high sensitivity to the range of frequencies that it is desired to record, and the lack of fidelity of the record is more than made up for in the selective action in emphasizing the wanted reflection-wave trains in the record.

The most commonly used methods of transducing the differential movement between geophone parts into electrical impulses are Nos. 1 and 2 below. Nos. 3 and 4 are used less frequently.

1. By means of a moving coil of insulated conductor wire in a magnetic field. As the coil is moved relative to the field, the number of magnetic linkages with the coil varies and a voltage is generated in the coil. The magnitude of the generated voltage will be proportional to the rate of change of linkages, so that the voltage will tend to be proportional to the velocity of differential movement between the standing and moving parts of the geophone.

2. A coil or coils linked with a magnetic field that comprises variable air gaps in the magnetic circuit, the air gaps changing with movement of the steady mass. Changes in the size of the air gap change the reluctance of the magnetic circuit, and with a constant or nearly constant magnetomotive force there will be a change in the number of linkages with the coil. The voltage will vary with the velocity of differential movement between the standing and moving parts of the geophone.

3. Piezoelectric crystal method. Certain crystals, of which quartz is often used, have the property of generating a voltage on their surfaces when they are distorted, the generated voltage being proportional to the amount of compression to which the crystal is subject.

4. Condenser type. The standing and moving parts of the geophone support closely spaced plates which act as a condenser. As the space between the plates varies their capacity as a condenser changes. If a fixed voltage difference is maintained between the plates, there will be a current flow to or from the condenser as its capacity changes, and this current flow can be used to produce a signal for amplification and recording.

For protection in handling, the geophone mechanism is enclosed in a metal case, which serves the further function of shielding the electrical generating mechanism from disturbing electrostatic and magnetic fields that may be present, particularly in the vicinity of electrical machinery and power lines. Adequate means must also be taken to prevent earth currents from getting into the signal circuit. The exterior case commonly takes the form of a cylinder from 4 to 6 in. in diameter, and the weight of a geophone may vary between 5 and 25 lb. The cylindrical geophones are particularly adapted to the practice of burying the geophones in holes slightly deeper than the height of the geophone. With the recent trend toward multiple geophones, there has also arisen the practice of placing them directly on the surface of the ground, and for this purpose the beehive shape is suitable. This is a hemisphere, of which the flat side is placed down and the rounded top will be less disturbed by wind than other shapes.

Amplifiers

The amplifiers come next in the sequence between the earth movement and the final paper record of the movement. The primary purpose is to amplify the signal current as it comes from the geophone until it is capable of actuating the recording galvanometers. The character of the signal is more or less modified in the amplifiers, partly as a matter of necessity and partly from choice. The modification present by choice is given the name "filtering," and is provided to accentuate the reflections and to cut out, as much as is possible without too seriously affecting the reflections, other wave trains that tend to obscure the reflection record. The filters tend to accentuate oscillations of a frequency between 30 and 50 cycles per sec., as these are the commonest reflection frequencies. Such filtering can be had only at the expense of loss in character of the reflections themselves and therefore must be considered a necessary evil rather than an unmixed blessing of which the more the better. The amplification itself is of the familiar vacuum-tube type used in radio sets. From two to four stages of amplification are necessary, according to the character of the other parts of the system, and the total gain in decibels may vary from 40 to 80.

Recent innovations in amplifier design include the provision of several alternative filtering circuits, so that the observer may fit the filtering to the particular requirements of the area being explored. Much of the excellence of the various secret seismograph designs is said to lie in the character of the filter circuit, and extravagant claims are made for the almost human intelligence displayed by such filters in passing the wanted reflections for recording and excluding the undesirable vibrations. While it is possible to design a very bad filter circuit, it would seem that there are simple principles of design which if followed should lead to filters as good as can be made for the particular character of filtering and the amount produced. The most difficult question is to determine exactly what type and amount of filtering is desirable. Since neither the character of the wanted reflection wave trains or of the unwanted wave trains is precisely known, and both probably vary considerably from place to place, the determination of a satisfactory filter is pretty much of a problem of cut and try, and to a considerable extent a matter of individual preference. When every effort is made to carry on the survey by correlation methods, a slightly filtered record, even though it may appear ragged and uneven, will be preferred because the successive cycles of a reflection wave train will tend to have individual character that will make the jumping from one cycle to an adjacent one less likely. Those who rely on the dip method in the more difficult areas will prefer a more symmetrical record, produced with more severe filtering. The pulses will be almost identical from string to string and the arrival times will

be evenly spaced from the top to the bottom string of the record, so that the dip computation becomes a comparatively simple matter.

In still more difficult shooting territory, a device variously known as the compound or running average hookup is used to make it possible to follow a reflection pulse across the record. Each of the interior strings is actuated by energy that comes partly from the particular geophone upon which the string would normally depend for its energy and partly from each of the adjacent geophones. In this way a uniformity is imparted to the record that facilitates the tracing of a pulse through successive strings from the top to the bottom of the record.

Most recording systems now include a sensitivity control against time, which is called an expander. The energy returning in reflections tends to become more attenuated as the travel path increases in length, and therefore the earlier arriving reflections have more energy than the later. Without some sensitivity control as a function of time, a record tends to have excessive amplitude in the early part and die away to undesirably small amplitude later. The expander causes the sensitivity of the recording system to increase with time at such a rate as to compensate for the decreasing signal intensity. One type of expander involves predetermined change in over-all gain of the amplifier unit with flow of time after the first reflection arrives. This may be achieved with variable bias tubes. The same effect may be achieved by varying the sensitivity in the other parts of the apparatus.

Another type of expander controls the amplitude by the same method as the automatic volume control of radio sets. By this system the energy arriving over a particular time interval is used to control the sensitivity of the apparatus for an immediately following time interval, a low rate of energy arrival permitting a greater sensitivity of the apparatus in the following time interval. The control average must be taken over a sufficient length of time so that it does not tend to unduly discourage short bursts of energy arrival from being shown on the record, otherwise a good high amplitude reflection would be smothered by the regulating device. Such an expander will produce a record that is very even in appearance as each channel (recording unit) is made self-regulating to the same level of amplitude as the others, and differences in energy arrival between the different channels are eliminated. The expander should be so constructed that it varies the amplitude of the record but as nearly as possible does not otherwise affect its character.

Recording Camera

The recording camera contains the mechanism that takes the amplified signal currents from the amplifier and produces a corresponding written record. All cameras now use photographic paper as a recording medium. The paper rolls may be from 100 to 250 ft. long and from

3 to 8 in. wide. The paper is fed through the recorder at rates of 12 to 18 in. per sec. and parallel lines across the paper spaced at intervals to represent hundredths of a second are imaged on the paper at the same time that the reflection record is being taken.

The accuracy of the time interval represented by the timing lines should be not less than one part in 5000. This corresponds in accuracy to a watch that gains or loses 20 sec. per day. However, the mechanism of a watch does not measure with the required accuracy the very short time intervals involved, and a "timing fork" is used to control the time lines. A fork is a slotted slab of metal of which the two tines vibrate at a natural frequency determined by its shape and the character of the metal used, and is very similar to a "tuning fork" used for musical pitch control. The forks are self-driven by electrical means, and are equipped to furnish an alternating current, which is used to control the timing lines. Tests should be made to make sure that the mechanical vibrations reaching the recording truck from the dynamite explosion do not disturb the accuracy of the fork.

The records of the geophone signals may be either black lines on a white background or white lines on a black background, the former being the more popular. The black lines are made by a light spot focused on the paper. In the optical train of this light is a mirror to which is attached a tiny flattened coil of insulated copper wire in a magnetic field. When the signal current passes through the coil the latter is caused to rotate, by the electromagnetic interaction, and the movement of the mirror causes the light spot on the paper to oscillate. The combination of the oscillation of the light spot and the simultaneous travel of the paper records a sinusoidal trace indicating the earth movement. The white on black records are made with an Einthoven string galvanometer. The light falls on the paper everywhere except for a tiny focused shadow of a small section of a very fine stretched wire. The fine wire lies in a magnetic field and when the signal current passes through it there is a slight deflection of the wire due to the same electromagnetic relation that causes a coil to rotate, and a shadowgraph record is written. There is no essential difference between the records written by these two types of galvanometers, preference for one or the other being based on such considerations as sensitivity, construction cost, strength and liability to breakdown, ease of repair, etc.

Natural frequencies considered desirable for galvanometers vary between 50 and 300 cycles per second. Those of lower frequency should be critically or nearly critically damped. With the higher frequencies the amount of damping may be less. Damping may be accomplished either with oil or electromagnetically. For the higher frequency mentioned, and especially if still higher frequencies are desired, the string type of galvanometer is the more sensitive. If periods in the lower fre-

quency range are desired the coil type of galvanometer is likely to be the more satisfactory. As with the geophones, galvanometers that have frequencies within the range of the principal frequencies of the reflections necessarily distort the record. The same argument applies that the selective characteristic is sufficiently desirable to offset the undesirability of distortion.

The time of explosion of the dynamite must also be shown on the record; it is usually called the "time break." At one time it was thought desirable to wrap a wire around the dynamite and record the moment at which a small current ceased to flow through the wire by reason of its rupture by the exploding dynamite. This system has the advantage of positive relation to the explosion. At present electric blasting caps are sufficiently uniform in their characteristics, and the moment of rupture of the firing circuit within the blasting cap is considered to indicate the time at which the explosion started. The time break may be recorded on one of the channels, which also records from a geophone circuit, if proper means are taken to eliminate the effect of the firing circuit by the time that the geophone begins to produce its signal. Hand-operated electrical generators (called "blasters") are usually used to fire the cap, their selection being for reasons of safety. A battery-operated circuit may also be used and has certain advantages of reliability in recording the time break under adverse conditions as compared to the blaster method.

SEISMOGRAPH FIELD OPERATIONS

By Frank Ittner*

In this section it is proposed to discuss operating methods considered good practice under the various field conditions commonly encountered. Efficient field direction and types of equipment needed to meet transportation problems are considered only to the extent that the desire to produce the best possible records must sometimes give way to the factor of cost. As in all engineering projects, alternative possible methods of operation must be considered in the light of relative costs as well as of the quality of results obtained. Cost may be in the background of much that is done in the field operations, but only the effect on record quality will be considered here.

Record quality at a particular shot point may vary with the various factors of spread character (relative positions of geophones and shot point), shot-hole depth, formations opposite position of shot, size of charge, the effect of repeat shots, surface geology and culture in the immediate vicinity of the geophones, recorder instrument characteristics, particularly filter.

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It is the problem of the field-operating crew to determine and use the most favorable of each of these factors at each shot point. Another important factor, character of the subsurface formations, is one that cannot be changed for a particular location, but where the importance warrants it is sometimes possible to move a short distance away and get a good record at a position between two positions from which bad records were previously obtained. Such good spots, due to subsurface conditions, are not predictable and only shooting will disclose them. However, often the spread position may be selected with reference to observable surface conditions instead of laying out a uniform pattern of spreads before careful inspection of the ground. This is particularly true in areas of rolling or rugged topography. Sometimes the geophones should be placed in the valley and the shot points on the higher ground, though the best system for each area should be determined experimentally, without too much reliance on previous experience.

Spread

As might be expected, almost all possible spreads have been tried at one time or another; each has its adherents, and may be particularly suitable to certain conditions. One basic subdivision can be made, between the close and the open spread. In the former the geophones are arranged close to the shot point, and it is expected that the effect of the ground roll will have subsided by the time that the reflections to be used have reached the surface. Ground roll starts at the shot point; it travels at an approximately horizontal velocity of 1000 ft. per sec. and continues long enough to obscure the record from 0.4 to 0.6 sec. after its first appearance on the record. A spread with the most distant geophone 1000 ft. from the shot point would therefore be unsatisfactory for reflections arriving earlier than 1.5 sec. subsequent to the shot time. This assumes maximum ground-roll intensity. Actually many areas have hardly any noticeable ground roll, and its manner of variation from place to place is not fully understood. An advantage of the close spread is that the reflections arriving almost vertically have less effect in producing secondary surface waves than do those with the open spread. A reflection, therefore, tends to vary in character less from end to end of the spread in the close spread. The commonest close spread is the split spread, in which the geophones are arranged along a line that passes through the shot point, and are symmetrically spaced on each side of the shot point. In reasonably good shooting territory, a space of 200 ft. between geophones (or groups that feed a single string if there are multiple geophones) is found to be satisfactory. With a 12-string recorder the spread would then be 200 to 1200 ft. on each side of the shot. In areas of more difficult shooting, it is sometimes desirable to shorten the geophone spacing to 100 ft. With fewer strings on the recorder—for

instance, a six-string outfit—it is common to shoot one side at a time, using two shots from the same shot point. Sometimes one geophone is opposite the shot point but offset to the side of the line, and this geophone is retained at the same position for both shots to "tie" them together.

The practice of using multiple geophones was developed particularly for use with the close spread. Instead of a single geophone at each position, a group of two, three or more geophones are arranged in line toward the shot point. The spacing between geophones of the group is calculated to cancel out the reflection frequency (30 to 50 cycles per sec.) contained in the ground roll. Thus if a reflection frequency of 40 cycles is assumed, and a ground roll velocity of 1000 ft. per sec., the crests of this component of the ground roll will be 0.025 sec. apart, and for cancellation we will require that the ground roll arrive at successive geophones of the group at time intervals of 0.0125 sec. This calls for a geophone spacing of 12.5 ft. To cancel the fundamental frequency of the ground roll would require a geophone spacing of something like 50 ft., but this low fundamental frequency is easily removed by filtering. In all conditions except extremely abrupt variations in thickness of the surface weathered zone, the reflections will arrive at almost the same time at all of the geophones of a group; that is, the pulses of the reflection will be in phase and so will be additive for the group. This theory leads naturally to the conclusion that the more geophones in a group the better, and as a matter of fact as many as 16 geophones have been used in a single group in attempting to cope with unusually difficult shooting areas. For most areas experience has shown that four to a group is the point at which the law of diminishing returns sets the limit of good practice.

The compound hookup may be considered as a variant of the principle of multiple geophones. In such a hookup each geophone does double or triple duty, the record written by each string being a running average of the impulses from the geophones that feed the preceding and succeeding strings as well as from the group to which the string is more directly tied. This hookup, when properly applied, is an attempt to obtain the advantage of multiple geophones, without having to resort to an unwieldy number of separate geophones. As an example, suppose a split-spread arrangement, where 16 geophones are placed on each side of the shot point, the nearest 300 ft. from the shot point and the rest arranged at 15-ft. intervals farther out, and an eight-string recorder. By taking a running average, the two center strings on each side of the shot point (numbers 2, 3, 6, 7, as ordinarily numbered) will have the benefit of multiple geophone effect of 12 geophones, and this may aid in making a correlation between the two sides of the shot point. Then if the correlation between the two sides seems reliable the distance, though short, is sufficient to be used for getting a dip that might not have been obtainable otherwise. However, such a record should be considered chiefly of

advantage for correlation, and the advantage of the several-string record is that the particular reflection being sought may have its best appearance on only one of the strings, because of local surface or other geological conditions. A further extension of the compound hookup principle is where a running average is taken from end to end of a single side spread, or of a split spread with no wide gap in the center. The reliability of dips taken from such records must be less than of those achieved by more direct methods. The argument is that if such heroic methods are necessary in order to get the successive strings to "line up" in a satisfactory manner, the geology may be such that the dips are inherently unreliable.

The open spread is commonly on one side of the shot point only, and may be at such distances as 1600 to 2400 ft., or possibly such a spread may be combined with one of 2400 to 3200 ft. All of the shallower reflections will arrive in advance of the ground roll, therefore will be completely free from any interference from that source. Under such conditions a very slightly filtered record may be used to advantage. Experience has shown that, owing to conditions that are not easily recognized, the open spread will give better records in some areas as compared to the close spread, and vice versa. It is to be noted that the character of the same reflection varies somewhat when open spreads and close spreads are compared for the same shot point.

In dip computations with the open spread, the assumed velocities are more critical than with the close spread, and in areas where it is suspected that the velocities may vary between on and off "structure" it is desirable to lay out the shooting program in such a way that the same reflecting surface is shot from both updip and downdip shot-point positions, and the dip determined as the average of the two separate results. The open spread is convenient for rapid shooting along a reconnaissance line. With a 1600 to 2400-ft. spread the shot points will be spaced 4000 ft. apart, and each geophone layout shot from two directions.

For very detailed work it is sometimes desirable to shoot open and close spreads combined. This may be achieved, for example, by spacing shot points 1200 ft. apart and shooting one side close spreads of 200 to 1000 ft. both ways toward the geophones. Then skip to the shot points farther out and shoot two 1400 to 2200-ft. spreads. In this way the maximum of information is obtained along a profile line, but it requires a minimum of four shots per hole, not counting repeat shots, and it is usually necessary to have cased holes in order that the shots may all be put off before the hole becomes unshootable to the correct depth. In areas where deep cased shot holes are necessary, such a shooting program is desirable to get the greatest amount of information possible from each shot hole.

Another variation is the *T* and *L* spreads, in which the position of the shot point is offset from the direction of the line of geophones. The

advantage is that the travel paths to all of the geophones are all about of the same length, and the angle of reflection does not change much for different parts of the spread. Such a spread is particularly adapted to a continuous spacing of the geophones, as successive spreads may be tied together by a common geophone position without having to come undesirably close to the shot point. The weathering data from such a spread is also said to be more reliable than for the in-line shot point type of spread. A disadvantage is the greater difficulty of obtaining access to two lines of positions, instead of the single line that may be placed conveniently close to a road for the ordinary spread. In special conditions the circle spread is useful. In this the geophones are arranged around the circumference of a circle of which the shot point is the center. This is particularly desirable in areas of complicated geology in which dips in several directions might be expected in a single record. The reflecting surface is a disk, which may be taken of moderate dimensions, and so is more compact and less likely to include a warped or irregular surface than the more extended single line of reflecting bed. If the arrival times of a circle shot are plotted on rectangular coordinate paper, they will fall into a sine curve, of which the amplitude is a function of the amount of dip and the phase angle with reference to a fixed azimuth gives the direction of dip.

Depth of Shot

The depth of a shot hole must be correctly determined to a considerable degree of precision. In some areas a difference in shot depth of 5 to 10 ft. may make the difference between getting a good record or a poor one. The so-called layer of surface weathering is important in relation to hole depth, it being necessary to have the hole at least as deep as the top of the "unweathered" zone and usually depths of 5 to 40 ft. within the unweathered are preferable. The weathered zone is characterized by low velocities of propagation of seismic waves. At the top of the unweathered there is a comparatively sudden increase in wave velocity, below which there is a gradual increase in velocity with depth. In Tertiary beds the uppermost unweathered will have velocities of 5500 to 6500 ft. per sec. The velocity in the weathered zone increases downward from as low as 700 or 800 ft. in the top foot or two of dry material to perhaps 3000 or 4000 ft. close to the base of the weathered. The top of the unweathered can therefore be recognized by drilling a hole to considerable depth and determining the travel times to the surface from shot points spaced at, say, 10-ft. intervals from the bottom up to near the surface. It usually turns out that the top of the unweathered corresponds closely to the top of the ground water. In areas where there is a considerable annual variation in ground-water level it may be that

the unweathered is limited to below the level of maximum annual depth to ground water. This suggests that the top of unweathered is really the top of water-filled strata from which air bubbles are excluded. The presence of small amounts of air bubbles in a water-flooded sand will act to greatly reduce the effective velocity of wave transmission.

The precise depth within the unweathered at which to shoot in order to get optimum results is often a matter of experimentation for each individual hole. In general, shale beds are preferable to sand opposite the charge. Shale usually transmits the shock to the surrounding rock more efficiently than soft sand, so that a considerably smaller charge may be used in shale, and this results in less ground roll. In some places, larger charges placed opposite sand formation give the better results.

Character of Record

There appears to be a relation between type of wave emanating from the vicinity of the explosion and the frequency of the recorded reflections. To some extent the character of the shot may be judged by the sharpness of the sound of explosion. A sharp riflelike crack from the explosion usually accompanies an efficient transmission of energy to the surrounding rock, and will give a good record of high reflection frequency (say 40 to 50 cycles). A dull, booming explosion may indicate less energy transmission and a lower frequency of reflections (30 cycles). Where it is necessary to use large charges (15 to 30 lb.) it is usually desirable to have the charge 30 or 40 ft. beneath the top of the unweathered, while small charges (5 lb. or less) usually give best results at 10 to 15 ft. below the top of the unweathered. For deeper reflections (below 2.5 sec. arrival time) larger charges and slightly greater hole depth usually give better results than smaller charges and less depth, which are satisfactory for the shallower reflections.

Where repeat shots are taken as a matter of routine on each spread it is usual to determine by experiment the best charge and depth. Also, if a particular spread seems to give a particular frequency of record, the filter can be changed (if multiple filters are available) with advantage so that the filtering is adapted to bring out the frequency characteristic of that particular spread. The effect of repeated shots in a hole is usually to increase the size of the zone of crushing so that larger charges become necessary to bring out the same amount of reflected energy. Record quality usually becomes worse with repeated shots but in very soft formation it sometimes happens that the effect of the shot is to compact the adjacent material and make the hole more suitable after several shots. Many companies have a program of repeat shots with varying shot depth and charge, and also amplifier filter changes, which are taken as a matter

of routine at each location. With records of this type the computer must be careful to consider the instrument characteristics in comparing records, as there is usually some variation in arrival time of the same event when the instrument characteristics are varied.

The character of the surface in the immediate vicinity of a geophone may have a marked effect on the record. This is particularly noticeable where part of a spread is on hard ground and the remainder on recently plowed and irrigated ground. Firm, evenly consolidated soil is preferable to loose or sandy soil or swampy areas. However, in regions of surface caliche, sand-covered areas are preferred to bare caliche, as the lesser of two evils. The immediate vicinity of irrigating ditches is to be avoided, and concrete roads may carry disturbing vibrations that will affect geophones lined up along the roadside. Trees, telegraph poles and tall grass produce a high noise level, particularly in windy weather, so that the size of the charge may have to be increased from an otherwise sufficient 2 lb. to 20 or 30 lb. in order to get the reflected energy above the level of the disturbing vibrations. The practice of burying geophones in shallow holes so that the top of the geophones may be slightly below the ground surface is partly to reduce the noise. Often better records are obtained with the geophones thus placed than when they are on the surface, irrespective of noise. With still deeper geophone burial there is a marked decrease in recorded energy and the record changes in character. Experiments with geophones on the surface and others directly below at the top of the unweathered show little similarity in appearance. In some regions the geophones farthest below the surface give the better record, but usually that is received from those nearest the surface. In swampy areas of the Gulf Coast, geophones have been attached to the tops of long stakes driven into the ground to reach a hard clay stratum.

From time to time the instruments should be tested in the field, to maintain proper adjustment. Time parallax between the different recording channels should be particularly guarded against. This may be either optical or electrical. The simplest test is to place a group of geophones side by side on hard, similar ground and test with a reflection record. All the strings should give the same record in both character and time of arrival. If they fail to meet this test the geophones should be interchanged to different record channels, to make sure that there are not slight differences in response due to their slightly different positions on the ground. If the instruments are not in perfect adjustment various further tests will suggest themselves in order to localize the trouble. The reliability of the timing system must also be tested occasionally. A nondriven fork to actuate one of the galvanometers to compare to the timing lines should be satisfactory.

DETERMINING GEOLOGIC STRUCTURE FROM SEISMOGRAPH RECORDS

By P. C. Kelly*

The processes involved in turning a set of seismograph records into a geological structure map may be divided into three classes: (1) picking reflections on the records, to be used for the computations; (2) determining the basis on which the computations should be made in order to arrive at correct results; (3) making the necessary computations and plotting sections and maps.

The first is usually done by the crew chief, or at least checked by him. The computing system usually originates in the head office, occasional consultations with the party chief providing modifications as necessary to meet special conditions. The computations and plotting are done by the party computer, with possibly one or two assistants if the calculations require much time.

Picking the Reflections

Marking of the reflections on the records is perhaps the most important step and the one most likely to determine the reliability of the final results. Making the "picks" is important because most of the problems that come up in practical work are connected with the problem of what can be done with poor records. The lack of any definite criteria by which to judge what are good and usable reflections is at the bottom of the difficulty. Give the same set of records to several people to mark and they will probably come out with a variation in the number of reflections per record that are deemed usable. With good records the correct picks are obvious to all, but with poor records experience seems to be the only reliable guide as to what picks are reliable and what are merely born of imagination and desperation. Experience in marking records and seeing how the structures eventually check out is more important than training in physics or geology. Different types of equipment and shooting methods may produce records quite different in appearance, from the same territory, and the computer can do his best work with records of a type on which he has had considerable experience, and has learned the limitations and shortcomings as well as the virtues of the data with which he is working.

Reflections may be picked for dips only, in which case the particular reflection picked is not recognizable with certainty on records from any other spread. For the purpose of computing dips it is necessary that the reflection be marked on all or nearly all of the strings of the records, as

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the dip computation calls for determination of the depth of the reflecting bed at two points and the dip is the indicated slope between the points. The farther the observation points are apart, the stronger is the result, and the less it is affected by small geological irregularities that may not be recognized and corrected for. It is desirable, therefore, to have the picks appear on the two outside strings of the record in order to compute the dip.

If the same reflection appears on records from several spread positions, it is called a correlation reflection. If the reflection is unmistakable on the different records, the correlation method should lead to a considerably more accurate map than if the dip method only is used. If several correlations at fixed time separations can be recognized, the reliance to be placed on each is greater. Where good correlations can be picked it is unnecessary to compute any dips, but usually it is desirable to compute dips also in order to have a check on the reliability of the correlation. If in plotting a profile it is found that the correlation indicates a slope in one direction and the computed dip is in the opposite direction it is obvious that something is wrong with one or the other. Correlations have the further advantage that they will show up faulting, while dips ordinarily will not do so.

Making Computations

Devising the computing system to be used is the most interesting of the tasks connected with computing, as it opens the way to an almost unlimited amount of delving into theoretical questions of geology and physics. The physicist may research on the elastic behavior of rocks under great pressure, and the mathematician may produce equations that leave the simple engineer far behind. Perhaps the most desirable line of research is that leading to more exact experimental data taken in the field. There are surprisingly few data on field experiments in the literature. In order to arrive at a system of computing simple enough to be practical, it is necessary to make simplifying assumptions as to wave travel in the earth. The assumptions themselves are simple and understandable, and their validity rests on checking whether they seem to lead to useful results. When these assumptions are made the computing system reduces to simple arithmetic and a few problems in solid geometry.

Our basic data are travel times of reflected waves from a common source to different observation points. We wish to turn these into travel paths of known position. The first assumption is that the waves travel in straight lines except where reflected, and that at a position of reflection the angle of incidence is equal to the angle of reflection. Actually the wave paths are slightly curved when passing obliquely through beds of varying velocity constant, but calculations show that for the changes in velocity actually encountered, and the small angles of incidence of the

wave to the surfaces of change, the amount of correction required because of curved path is so small that it may be neglected.

The next problem is the velocity of transmission of the waves through the various strata encountered. Travel times must be converted into travel distances. Our most reliable source of knowledge of subsurface velocities is from deep-well shooting. By lowering a specially constructed geophone into a well, and determining travel times from shot points on the surface to the well geophone, the velocity constant (instantaneous velocity, or incremental velocity) is determined for the various strata encountered in the well. Such data show that where there is no marked change in lithologic character, the velocities increase downward at a rather uniform rate. It appears probable that a considerable part of this increase is due to loading; that is, to the effect of the pressure from the overlying beds in changing the elastic character of the buried strata. According to Hook's law there should be no effect on velocity from the loading of a homogeneous elastic material, but according to the best data available it appears that Hook's law is not even approximately true for sedimentary formations and for the pressures involved. For instance, limestone formations that have every appearance of being identical in character have several thousand feet per second higher velocity constant at a depth of two miles than close to the surface.

Besides the effect of loading, the increase downward in velocity must be in part due to the previous history of the beds, which must tend to make the lower beds more compact and better cemented, and consequently with higher velocity constant. It is an elementary fact of geology that heat, pressure and time are a combination leading to hard rocks; clays change to shale, and shales to slates, under such processes. The lower the position of a stratum in the stratigraphic column, the greater the amount of heat, pressure and time that have acted to consolidate the rock and increase its elastic moduli. Subsequent earth movements coupled with erosion of the surface of the ground may have acted to bring a stratum much nearer the surface than its depth of maximum burial but it should retain the greater part of the hardening effect of deep burial. Present depth therefore is not a direct function of the amount of hardening that a rock has been subjected to in its history subsequent to deposition.

In any particular case it is difficult to determine with the data available what proportion of the increase in velocity downward is due to loading and what to stratigraphic position. It seems probable that each of these factors has a normal rate of increase downward with each different type of sediment, being different for sandstones than with shales or limestones. The problem is by no means incapable of exact solution if sufficient data were available, but in the absence of such data we must select some assumption of relation of velocity to depth and geology usable for the problem at hand. It works out that the computing is much simpler

if we select increase of velocity with depth and ignore stratigraphic position for the computation of at least a first approximation to the structure. To the extent that the stratigraphic component is important, this will produce a map with a slightly incorrect amount of structural relief but will give a correct qualitative picture. It has the advantage that the computations for plotting of both dips and correlation depths in originally disconnected areas will be in agreement and can be checked against each other. This assumption is satisfactory for both the Gulf Coast and the California Tertiary. On this basis it is possible to prepare a table or graph that will give velocity constant for each depth, irrespective of geographical location, and also an average velocity to each depth. It is then possible to convert travel times into travel distances and arrive at a solution for dip of the reflecting surface when travel time to two surface observation positions is given.

The assumption of increased velocity with depth does not mean that eventually the stratigraphic effect will not be considered, but only that the first approximation to the structure will be computed on the simpler assumption. Actually toward the edge of the basin the older strata will be involved, and the velocities will tend to be higher for the same depths. On the other hand, the lithologic character will also change areally and affect velocity, and these two effects can be corrected for at the same time if well-shooting data are available for various parts of the region shot, or the velocities are otherwise determined. This correction can be made by preparing a depth correction sheet to be applied to the preliminary contours in much the same way that an isopach sheet is applied to extrapolate structure contours from a horizon for which the control is good to a deeper one for which only a few determinations of the stratigraphic interval between the two are available.

It is also possible to determine subsurface velocities from the arrival times of reflections themselves. The method amounts to determining the radius of curvature of the returning wave. In order that the result may be at all accurate, it is desirable to have data on as large an arc of the wave as possible. The time elements involved in the calculation are small, and slight errors of observation or slight irregularities in surface conditions will cause considerable errors in the computed result. However, if a large number of solutions are made and properly averaged, velocities should be accurate to possibly 2 per cent error. This method is of necessity used in areas where no well-shooting data are available, and it may be used to extend information to depths greater than those penetrated by any well. It is also used on the Gulf Coast, where the velocities over the crests of salt domes seem to vary considerably from the normal off-structure velocities.

Another assumption necessary for dip computation is that for each spread the source of a reflection is a bed, which acts as a smooth, unwarped

surface in producing the reflection. Actually the energy return considered as a single reflection on the seismogram may be a summation of several reflections from adjacent strata. The several amplitudes and phase relationships of the separate reflections will determine the character of the summation. This summation feature renders the computed dip particularly sensitive to variations of reflecting power of any one of the contributing beds. The change in reflecting power of a stratum may be due to changes in its elastic characteristics with change in lithologic type or amount of cementation, or to change in thickness. To illustrate such false dip effects with an example, suppose that there are two similar reflecting beds, each 10 ft. thick and separated from each other by an interval of 25 ft., which contribute to a combined reflection. Now suppose that the upper bed decreases in reflecting power 50 per cent between the positions where the reflection to the top string and the bottom string of an 800-ft. spread takes place. This will compute out to give approximately one degree of false dip. The amount of variation postulated in this example is moderate when compared with what seems geologically probable for many of the formations of the shallow marine type that are prospected for oil. Under such conditions, any one dip is not to be considered accurate, and an average of many such dips must be taken to arrive at trustworthy results. Cross bedding is another feature that may lead to lack of parallelism between computed dips and the actual true stratification. The geologist is able to recognize cross bedding in surface outcrops, but the seismograph is unable to make any such distinction.

When a reflection is correlated over a considerable area, there may be more gradual changes in lithology, which affect the parallelism between the computed position of the source of the reflection and the actual bedding of the formation. Suppose the source of a reflection is a series of thin, hard beds in a shale formation. As one follows the formation laterally new hard stringers may appear, their top gradually ascending in the formation. Under such conditions one would expect the reflection to change slightly in character from place to place, and such a reflection should be used with caution. The possibility of "skipping a cycle" familiar to all computers is an accompaniment of changes of this type. The same energy band may be correlated from record to record with certainty but the individual cycles at the start of the reflection seem to fade out or increase in energy from place to place. Such errors should be avoided in continuous shooting traverses, but with considerable jumps between shot locations two different records may show prominent reflections of nearly identical appearance and at the same general arrival time, yet one will actually begin a cycle in advance of the other.

Before dips or depths can be accurately computed travel times of reflections must be corrected for variations in velocity constant and thickness of the beds immediately below the surface of the ground. These

corrections are known as weathering corrections. In many areas they are of a magnitude comparable to the time differences that determine the amount of the computed dip. Most of the data for making weathering corrections come from the arrival times of the fastest wave, which is the first wave to reach each of the geophone positions. Every few miles the velocity characteristics of the weathered zone should be determined in detail by shooting at 10-ft. intervals from top to bottom of a shot hole and determining vertical travel times to the surface of the ground. It must be assumed that elsewhere the weathered zone has the same relation between thickness and travel time as at these holes for which it is definitely determined. The details of computing the weathering correction vary somewhat with each party or system, but in no case are they complicated either in theory or in the type of calculation required. Unconsidered irregularities are usually present in the weathered zone in sufficient amount so that the correction does not give perfect results.

In areas where there are thin streaks of high-velocity material in the weathered zone, the data from the fastest wave are unreliable, and there is no solution for the weathering correction except measuring it by uphole time at the shot point and assuming that the same travel time applies to the nearest geophones, which should not be far away. In such areas, of which West Texas is an example, neither dip computations nor open spreads can be used to advantage. In still other areas geological conditions may cause a two-layer correction; as, for example, where there is an irregular thickness of gravels or other similar formation beneath the weathered zone, which may extend to a depth of several hundred feet and have an intermediate velocity constant between that of the weathered zone and that of the more regular underlying formation. More elaborate weathering data are required, and because of differences in velocity each of the two irregular surface layers must be separately corrected for.

In areas where still deeper stratigraphic irregularities occur any correction becomes difficult. It should therefore be borne in mind that structural data from any horizon above which are beds subject to irregular variations in character are likely to be unreliable. The more gradual and uniform lateral variations may be determined from the geological logs of wildcat wells combined with an approximate knowledge of the velocity constants of the various types of deposits indicated on the well logs.

Further assumptions, which appear to be borne out experimentally, are that the velocity of wave travel is independent of amplitude of the wave, and also of frequency. There is some reason to believe that theoretically wave velocity might vary slightly with amplitude, but such variation if it exists appears to be too small to be of practical importance.

Before making a structure contour map it is usual to plot traverses in structure sections, showing the observed dips and correlations. This presumes that all of the reflecting points of the traverse lie in the same

plane. Actually, for any straight-line spread any plane tangent to the side of a cone of revolution of which the line of geophones is the axis and the line on the reflecting bed from which the reflections came is the generating line will satisfy the requirements of the observed travel times to the various recording strings. In other words, a straight-line traverse determines only dip components, and the reflecting bed will lie directly under the line of geophones only when shooting is directly across the strike of the beds. From the single traverse we do not know whether we are dealing with the actual dip or with a dip component from one side or the other of the surface position of the traverse. If the traverse happens to cross a fold obliquely, the reflecting beds will lie first on one side and then on the other side of the position of the line of traverse. Under such conditions difficulties may arise from plotting all of the data of a traverse as though they all came from a single plane. As an example, a profile run at right angles to the axis across a syncline may show dips in two different directions coming from the two flanks of the syncline on the same record. If these dips are plotted on the structure section each will be on its correct side of the syncline and a true picture will be shown. Now suppose that a profile is run down the axis of the syncline. The same two reflections will show up on a single record and will be plotted on the structure section as though they were both in the same plane, though actually they are separated in space by a considerable distance and have no direct structural relationship to each other. If the dips happen to be in agreement, nothing will be suspected, but if they happen to fall over other plotted dips in the opposite direction it may not be realized that all that is wrong is the attempt to plot three-dimensional data on a two-dimensional structure section.

Where the survey includes a network of intersecting profiles, it is geometrically simple to determine the correct position of the surface trace of each bed at the intersection of two profiles. This gives a basis for offsetting the profile lines on the structure map, so that the reflecting beds are shown at their correct position. Such offsets are considerable for deep horizons, and any attempt to show the position of small, deeply buried structural features accurately must consider this factor.

On conclusion of each shooting job it is well to check over the results to determine whether anything that appears to contradict the original assumptions of relation of geology to wave travel has developed. If the final picture does not appear to be geologically probable the geologist will suspect that some mistake has been made even though he cannot put his finger on the spot. There has been considerable controversy between geologists and physicists as to the propriety of thus requiring the final picture to fit into the geologist's preconceived ideas as to what ought to be found. The idea of the physicist seems to be that if the problem is properly analyzed there should be nothing wrong with the premises. If

the observation and computing are properly done, it then follows that the final picture must be right, however much the geologist may object to it. Our only comment is that there seems to be plenty of room for either viewpoint to be wrong in any individual case. The geologist cannot be certain of what ought to be found or it would have been unnecessary to make the survey, and the physicist is unable to put on a diving helmet and follow a vibration down a couple of miles into the earth and back again and report on what happened as a matter of precise knowledge.

Problem of Inclined Layers in Seismic Reflection Methods

By Zdzislaw Specht*

(New York Meeting, February, 1940)

This paper discusses elementary laws pertaining to seismic reflection methods and presents a general and simple criterion for determining the direction of dipping of a reflection horizon from observational data of distance and time.

The important development of practical geophysics and the successful results obtained in America, Germany, and other countries, caused introduction of geophysical methods of prospecting in Poland.

Of all methods, seismic prospecting has proved most successful in the area of the Carpathian foreland, where new oil reserves are to be expected. Because of the great depth of the sedimentary column in that area, ranging sometimes over 2000 meters, it was obvious that only the seismic reflection method could give reliable information regarding the structural features of these strata at such a depth.

During the experimental surveys it appeared that the complicated geological conditions of that area do not always allow prediction of the depth and the angle of dipping of reflecting horizons, and for that reason information about the inclination of a given horizon has great importance in the determination of subsurface structure. Therefore the problem arises of obtaining from seismic reflection data reliable information as to the direction of dipping of reflecting strata, without any previous knowledge of the average velocity and depth of geologic horizons under consideration.

Reflection Seismograms

The earth, being a nonhomogeneous medium, is composed of a series of layers, which differ from each other in their physical properties, particularly in their elastic coefficients. These variations in elasticity are the cause of refraction and reflection of a seismic wave. Under the simple but only approximately correct assumption of the straight-line path of an elementary elastic disturbance, the phenomena mentioned

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Manuscript received at the office of the Institute Nov. 24, 1939. Issued as T.P 1177, April, 1940.

* Physicist, former collaborator of the Pionier Exploration Co., Lwów, Poland.

above may be treated in the same manner as those in geometrical optics. In this paper only the laws governing the reflected part of any seismic disturbance will be studied.

At a point on the surface an elastic wave is generated through a detonation of an explosive material. At other points on the surface instruments—geophones—record it. A geophone is an electromagnetic system consisting of a permanent magnet, an inertia mass, and a movable coil that oscillates in the magnetic field. Alternating currents produced in the instrument by ground vibrations pass through cables to the central station, which is mounted in a special car. Here the currents are amplified and produce movements of oscillographs, which are recorded on sensitive paper. In this way the reflection seismograms are obtained.

Fig. 1 is a general view of such a seismogram. Any reflecting horizon may be determined by observing the line that corresponds to the suddenly increased amplitudes and conformity of the phase of the recorded wave system. Such "reflection" is shown in that seismogram along the line LL.

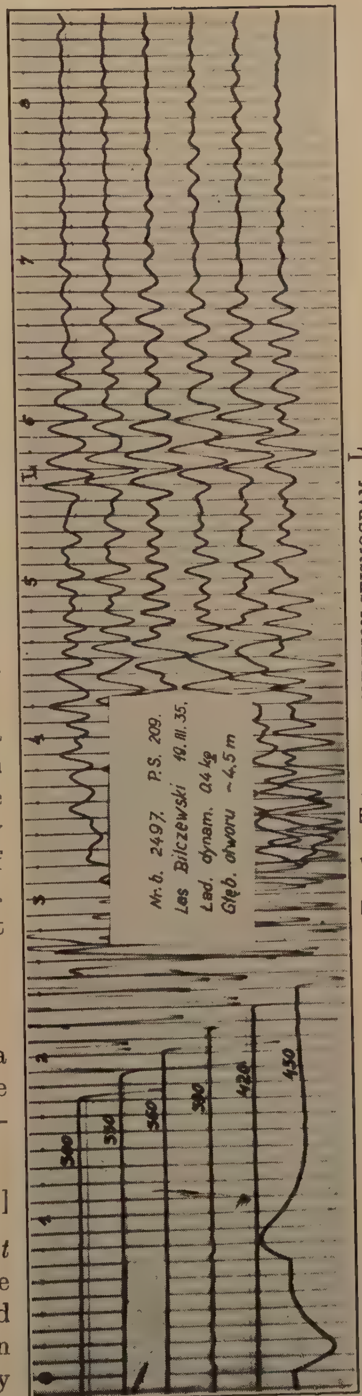

FIG. 1.—TYPICAL REFLECTION SEISMOGRAM. The impulses along line LL are the reflections.

THEORETICAL CONSIDERATIONS

Consider first the simplest case of a horizontal layer (Fig. 2). Using the method of images P and P', the following expression can be obtained:

$$s^2 = v^2t^2 = x^2 + 4z^2 \qquad [1]$$

v is the average velocity in medium M, t the reflection time, x the distance of the seismograph A from shotpoint P, and $s = vt = AP'$. The relation between time t and distance x is graphically expressed by a hyperbola, as shown in

Fig. 3. From the shape of this curve is obtained the following time-distance series

$$\begin{Bmatrix} x_1 < x_2 < x_3 < \cdots \\ t_1 < t_2 < t_3 < \cdots \end{Bmatrix} \qquad [2]$$

or in seismograms following the reflection line in Fig. 4.

For nonhorizontal layers in general the following expression is obtained by means of Carnot's theorem,

$$s^2 = x^2 + 4z^2 + R \qquad [3]$$

R being a function of the angle of inclination of reflecting horizon. Particularly for layers with the inclination shown in Fig. 5,

$$s^2 = x^2 + 4z^2 + 4zx \sin \alpha$$
$$(s = vt = AP')$$

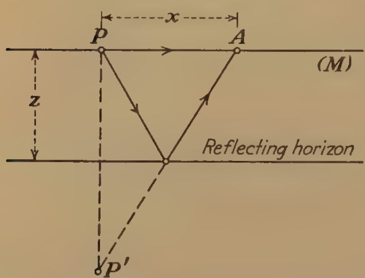

FIG. 2.—PATH OF REFLECTED RAY FOR THE CASE OF A HORIZONTAL LAYER.

and such form of inclination is called a "positive dip direction" (Fig. 5). For layers with the opposite inclination (Fig. 6),

$$s^2 = x^2 + 4z^2 - 4zx \sin \alpha$$
$$(s = vt = AP')$$

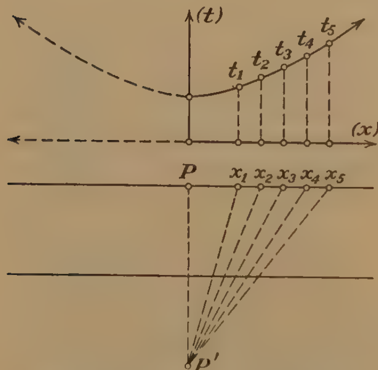

FIG. 3.—PARABOLIC TRAVEL-TIME CURVE FOR REFLECTIONS FROM HORIZONTAL LAYER.

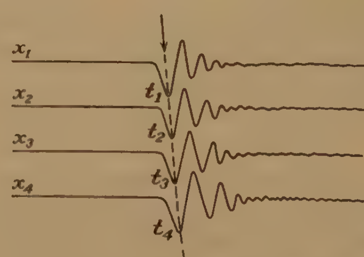

FIG. 4.—SCHEMATIC DRAWING OF REFLECTED WAVES FROM SINGLE HORIZONTAL LAYER.

and this form of dipping is denoted as a "negative dip direction." In both cases of dipping, in general, the following applies:

$$s^2 = x^2 + 4z^2 \pm 4zx \sin \alpha \qquad [4]$$

Assuming v, z and α as constants, such function presents graphically a hyperbola, which has at a definite point x_c the minimum s_m, obtained

by differentiation of the above function. The following are minimum conditions:

$$\frac{ds}{dx} = \frac{1}{s}(x \pm 2z \sin \alpha) = 0$$

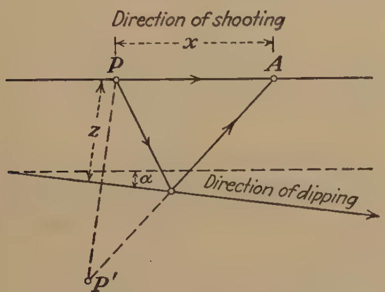

FIG. 5.—PATH OF RAY REFLECTED FROM LAYER DIPPING IN POSITIVE DIRECTION.

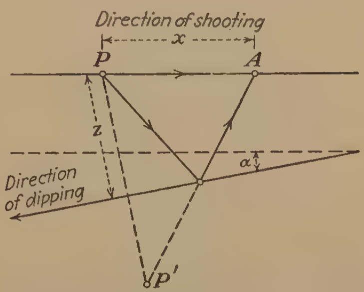

FIG. 6.—PATH OF RAY REFLECTED FROM LAYER DIPPING IN NEGATIVE DIRECTION.

With s always $\neq 0$, is obtained:

$$\left.\begin{aligned} x_c &= \mp 2z \sin \alpha \\ s_m &= \quad 2z \cos \alpha \end{aligned}\right\} \qquad [5]$$

GENERAL DISCUSSION

From eq. 5 it follows that only when there is negative dip direction is the minimum of the function s observable. Otherwise, the real value of the seismograph distance x, which could be negative, does not exist.

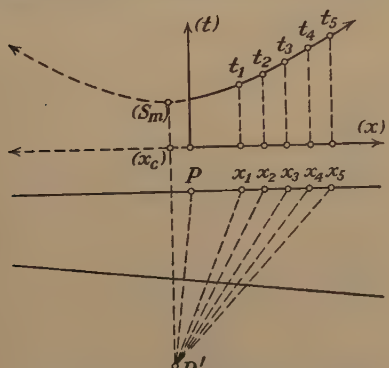

FIG. 7.—REFLECTION TRAVEL-TIME CURVE FOR POSITIVELY DIPPING LAYER.

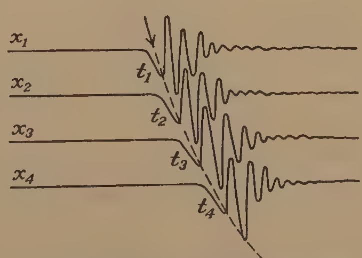

FIG. 8.—WAVES REFLECTED FROM POSITIVELY DIPPING LAYER.

The relation between the time t and distance x in positive dip direction is graphically illustrated in Fig. 7. According to real conditions the shot point P is assumed as the origin of the Cartesian coordinate system (x,t). It is easily seen that in this case of dipping the work is done on one of the hyperbola branches, on which to the increasing values of distance x

correspond simultaneously increasing values of time reflection t. Thus, if the distances x are denoted by x_1, x_2, x_3, . . . the following series is obtained:

$$\left\{\begin{matrix} x_1 < x_2 < x_3 < \cdots \\ t_1 < t_2 < t_3 < \cdots \end{matrix}\right\} \qquad [6]$$

Practically, the reflection lines are obtained in the form shown schematically in Fig. 8.

For negative dip direction the curve of Fig. 9 is obtained. From the shape of this curve, it is obvious that this work is done on two branches (AC and AB) of the hyperbola. In the area enclosed by the first branch (AC) there are the following time-distance series

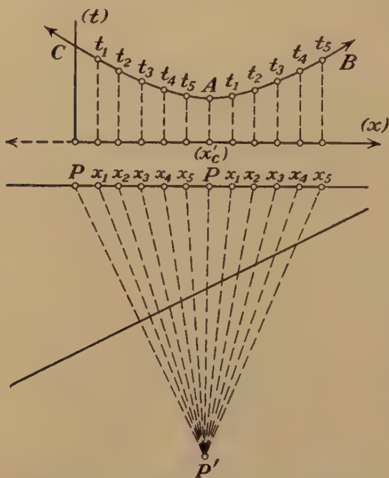

Fig. 9.—Reflection travel-time curve for a negatively dipping layer.

$$\left\{\begin{matrix} x_1 < x_2 < x_3 < \cdots \\ t_1 > t_2 > t_3 > \cdots \end{matrix}\right\} \qquad [7]$$

or in seismogram the reflection line shown in Fig. 10. In the area enclosed by the branch AB the series are

$$\left\{\begin{matrix} x_1 < x_2 < x_3 < \cdots \\ t_1 < t_2 < t_3 < \cdots \end{matrix}\right\} \qquad [8]$$

and the seismogram is that of Fig. 11.

An interesting result follows from these considerations. If the seismographs for negative dip direction are placed so that one part of

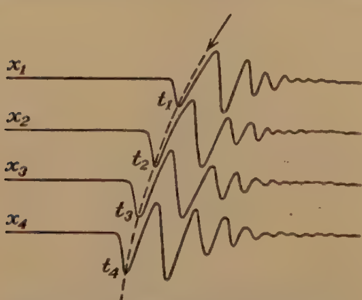

Fig. 10.—Reflected waves in branch AC of curve in Fig. 9.

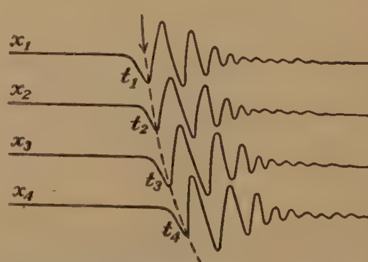

Fig. 11.—Reflected waves in branch BC of curve in Fig. 9.

them is on the left and the other part on the right side of the critical point x_c', the reflection line shown in Fig. 12 is obtained.

If the distances between the geophones are comparatively small, a vertical reflection line of virtually the form shown in Fig. 13 is obtained.

The practical result of these considerations is as follows. If we have a seismogram with the so-called "left reflection line" (Fig. 10) or "ver-

tical reflection line" (Fig. 13), it may be stated that the dipping is negative (Fig. 6). But if we have the so-called "right reflection line" (Figs. 4, 8 and 11), we cannot say which direction of dipping actually exists, because there are three possibilities: a horizontal (Fig. 2), and

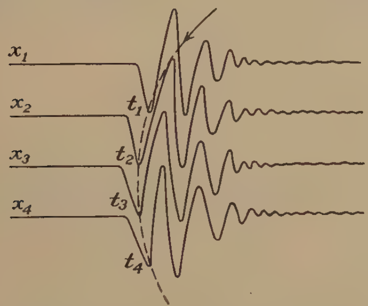

FIG. 12.—REFLECTED WAVES, GEOPHONE SPREAD CENTERED ABOUT POINT A OF FIG. 9.

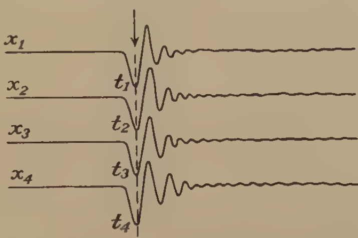

FIG. 13.—"VERTICAL REFLECTION LINE" OBTAINED FROM SHORT GEOPHONE SPREAD AT POINT A OF FIG. 9.

two in opposite directions, inclined layers; namely, those with positive (Fig. 5) or negative (Fig. 6) dip direction.

SIMPLE METHOD FOR DIPPING DETERMINATION

The following method permits the solution of this problem by using one shot point and any geophone displacement. With v, z and α as constants, by differentiation of eq. 4 can be obtained for positive dip direction

$$v^2 t \frac{dt}{dx} - x > 0 \tag{9}$$

and similarly, for negative dip direction,

$$v^2 t \frac{dt}{dx} - x < 0 \tag{10}$$

Consider now the following time-distance series:

$$\begin{Bmatrix} x_1, & x_2, & x_3, & x_4, & x_5 \\ t_1, & t_2, & t_3, & t_4, & t_5 \end{Bmatrix}$$

For positive dip direction are obtained from eq. 9 the following approximate expressions:

$$\left.\begin{aligned} v^2 t_2 \times \frac{t_3 - t_1}{x_3 - x_1} - x_2 > 0 \\ v^2 t_4 \times \frac{t_5 - t_3}{x_5 - x_3} - x_4 > 0 \end{aligned}\right\} \tag{11}$$

By simple modification of eq. 11 can be found

$$\frac{t_2}{x_2} \times \left(\frac{t_3 - t_1}{x_3 - x_1}\right) > \frac{t_4}{x_4} \times \left(\frac{t_5 - t_3}{x_5 - x_3}\right)$$

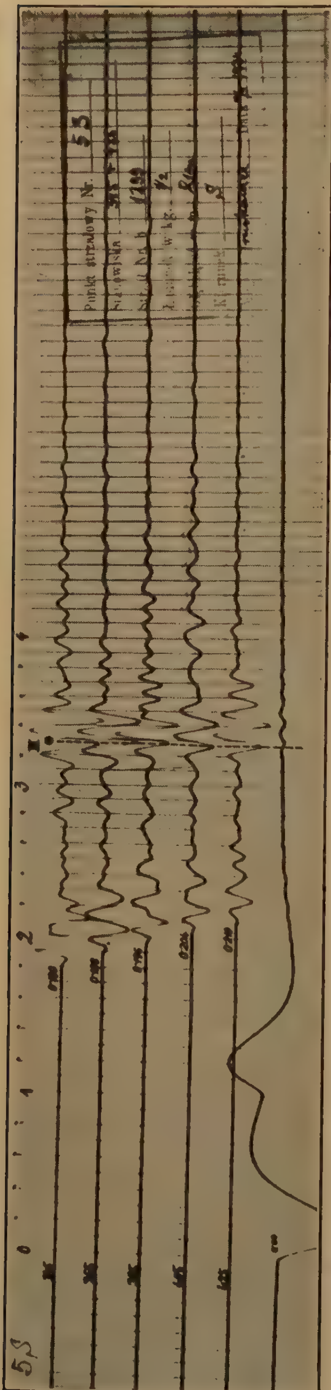

Fig. 14.—Seismogram of reflections from negatively inclined layer, as illustrated in Fig. 10.

which must be valid for each layer with positive dip direction. Similarly, for layers with negative dip direction is obtained:

$$\frac{t_2}{x_2} \times \left(\frac{t_3 - t_1}{x_3 - x_1}\right) < \frac{t_4}{x_4} \times \left(\frac{t_5 - t_3}{x_5 - x_3}\right)$$

and for horizontal layers,

$$\frac{t_2}{x_2} \times \left(\frac{t_3 - t_1}{x_3 - x_1}\right) = \frac{t_4}{x_4} \times \left(\frac{t_5 - t_3}{x_5 - x_3}\right)$$

The general criterion of dipping in the form is

$$\frac{t_2}{x_2} \times \left(\frac{t_3 - t_1}{x_3 - x_1}\right) \gtreqless \frac{t_4}{x_4} \times \left(\frac{t_5 - t_3}{x_5 - x_3}\right) \qquad [12]$$

which is not only necessary, but also sufficient.

Practical Results

The criterion of dipping in eq. 12 was used during our practical seismic investigations in Poland. In all cases the direction of dipping was in perfect accordance with the results obtained by using different methods of shooting and was also in agreement with known geological data.

In the neighborhood of Truskawiec, near Boryslaw oil fields, we obtained a very beautiful left reflection line, which corresponds to a negative inclined layer (Fig. 14).

It is clear that the useful application of this criterion affords relatively great accuracy in time-reading, and correspondingly chosen distance between the single geophones.

Acknowledgments

The writer wishes to express his gratitude to Dr. Z. A. Mitera, scientific collaborator of the Geotechnika Exploration Co., Lwów, Poland, for encouragement and useful suggestions in the work described herein.

Formula for Calculation of Slope of Reflecting Horizon in Seismic Reflection Prospecting

By H. H. Pentz*

(New York Meeting, February, 1936)

This paper gives the derivation of a practical formula for the calculation of slopes in seismic reflection prospecting. The derived formula is an approximation and can be used where the slope of the reflecting horizon is less than 250 ft. per 1000 and the depth to the reflecting horizon is greater than 2500 ft. The slope of sediments in oil regions is generally considerably less than 250 ft. per 1000. This is particularly true of the Gulf Coast, with the exception of the salt domes, which usually have the sediments at a greater slope than 250 ft. per 1000.

Derivation

In Fig. 1, o represents the image of the shot point a. Then the length of the path aeb is equal to ob and $ae'd$ is equal to od.

In the triangle aob, p is equal to ob, x is equal to ab, and c is equal to ao. The angle u is the apparent dip of the reflecting horizon. Angle oab is equal to $(90° - u)$.

Then, by the law of cosines:

$$p^2 = x^2 + c^2 - 2cx \sin u \qquad [1]$$

Differentiating:

$$2pdp = 2xdx - 2c \sin udx$$

$$\frac{dp}{dx} = \frac{x - c \sin u}{p} = m$$

$$c \sin u = x - mp \qquad [2]$$

In the triangle aod, od equals p' and ad equals x'. From this triangle:

$$c \sin u = x' - m'p' \qquad [3]$$

Adding equations 2 and 3:

$$2c \sin u = x - mp + x' - m'p' \qquad [4]$$

The quantity m is the slope of a line tangent to the curve, defined by equation 1, at the point (p, x). The quantity m' is the slope of the line tangent to the curve at the point (p', x').

Manuscript received at the office of the Institute April 1, 1936. Issued as T.P. 766, February, 1937.

* Bethlehem, Pa.

The portion of the curve under discussion can be approximated by a straight line. In that case the slope is constant and m is equal to m'.

Therefore equation 4 becomes:

$$2c \sin u = x + x' - m(p' + p)$$

The sines and tangents of small angles are nearly equal, so that:

$$\text{Slope} = \frac{x + x' - m(p' + p)}{2c} \qquad [5]$$

In equation 5, $m = (p' - p)/(x' - x)$ and c is assumed equal to p.

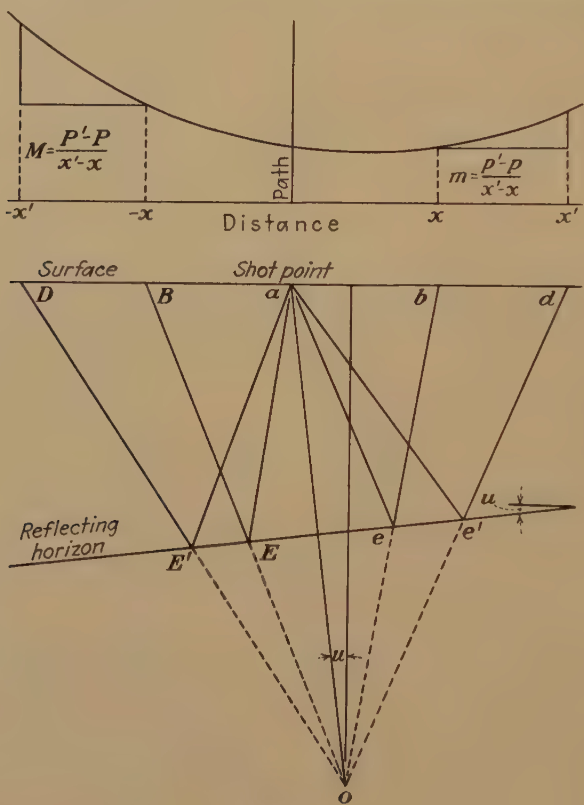

FIG. 1.

Substituting in eq. 5:

$$S = \frac{x + x'}{2p} - \frac{(p' - p)(p' + p)}{2p(x' - x)} \qquad [6]$$

In deriving equation 6, the slope was arbitrarily chosen positive when shooting up the slope. In the determination of an unknown slope, if the

sign of the slope is negative the direction of shooting is downslope; and if the sign is positive the direction of shooting is upslope.

In actual practice, it is necessary to average an upslope with a downslope. The necessary data can be obtained by shooting in opposite directions from the same shot point, or by reversing the relative positions of the shot point and the recording positions. The former procedure is considered in this paper. It is impractical, and not always possible, to follow the correlation from upslope to downslope, therefore an average of two calculations that occur at about the same depth is necessary.

The average velocity to any reflecting horizon can be determined from data obtained by shooting in opposite directions from the same shot point. The reflecting horizon must be correlated from updip to downdip in obtaining the average velocity to that reflecting horizon. Using capital letters to designate data obtained by shooting in the opposite direction to that of equation 6, the slope formula is:

$$-S = \frac{x + x'}{2P} - \frac{(P' - P)(P' + P)}{2P(x' - x)} \qquad [7]$$

The distances from the shot point to the recording positions are the same in these cases. Since c is assumed equal to p or P, in this derivation p is equal to P.* By eliminating S in equations 6 and 7, substituting velocity multiplied by the time for the length of the path, and solving for V, the velocity can be determined by the following equation:

$$V = \sqrt{\frac{2(x' + x)(x' - x)}{(T' + T)(T' - T) + (t' + t)(t' - t)}} \qquad [8]$$

Discussion of Results

The approximations made in obtaining the slope formula are not serious if it is not used beyond certain limits. The calculation of an assumed case follows (distances in feet):

Assume $x = 1500, \quad x' = 2500, \quad u = \tan^{-1} 0.250$

then $c = 5177, \quad p = 5007, \quad p' = 5154, \quad P = 5706, \quad P' = 6250$

Substituting in equation 6:

$$S = \frac{2000}{5007} - \frac{147 \cdot 10161}{10014 \cdot 1000} = 251 \text{ ft. per } 1000$$

Substituting in equation 7:

$$S = -\frac{2000}{5706} + \frac{544 \cdot 11956}{11412 \cdot 1000} = 219 \text{ ft. per } 1000$$

The average of these two calculations is 235 ft. per 1000.

* The use of equation 5 and the similar equation for the opposite direction will give the same formula for V.

The error in equation 6 increases as the distance of the recording positions from the shot point increases, as the slope increases, and as the depth to the reflecting horizon decreases. As shown by the average of the upslope and downslope calculations, the slope formula can be used with a maximum theoretical error of 15 ft. per 1000 under the following conditions:

1. The depth to the reflecting horizon must not be less than 2500 feet.
2. The distance of the last recording instrument must not be greater than 2500 feet.
3. The slope of the reflecting horizon must not be greater than 250 ft. per 1000.

Equation 6 is the equation to be used in calculating the slope. The calculations are made separately in each direction, plotted and then averaged. The notation of the equation may be simplified by redefining terms, and then by substituting velocity multiplied by the time for the length of the path, the equation is given in terms of quantities measured in the field, as follows:

$$S = \frac{X}{Vt} - \frac{\Delta t VT}{\Delta x t} \qquad [9]$$

in which

$X = (x + x')/2 =$ average distance of the recording instruments from the shot point.

$V\Delta t = (p' - p) =$ velocity multiplied by the difference in the time between the arrival of the first and last reflection.

$VT = (p' + p)/2 =$ velocity multiplied by the average time.

$\Delta x = (x' - x) =$ the distance between the first and last recording instrument.

$Vt = p =$ velocity multiplied by the time for the first reflection.

$V =$ average velocity to the reflecting horizon.

This formula can be used to give reliable results with the proper corrections. The factors that need correction are the differences in depth below the detectors of the "weathered zone" and differences in elevation between the detectors. The weathered zone is composed of a layer of soil and rock at the surface that has a much lower velocity than the rock below; the correction for this low-velocity layer is very important. This weathered zone is not to be confused with the similar geological term.

Continuous Profiling Method of Seismographing for Oil Structures

By Sylvain J. Pirson,* Associate Member A.I.M.E.

(New York Meeting, February, 1937)

The number of seismograph field crews employed in the active survey of potential oil territories is still on the increase, owing to the ever pending threat of a shortage in the supply of crude oil. It is to be expected that for years to come the seismograph will be responsible for discovering many new oil fields, both in wildcat territories and in deeper formations, and that the seismographic campaign will go on unabated.

Yet improvements in instrumental design, in the field and interpretative techniques, are brought about year after year, and lead to the reshooting of areas many times surveyed previously.

Notwithstanding the experience gained by the past eight years of active work in reflection seismographic surveys, the results given by this method of mapping deep formations have in many instances been at variance with the information subsequently obtained by the drill. To overcome this shortcoming, geophysicists have used closer spacing of the shot points and closer control of the depth points. The method discussed here is being used actively in the Mid-Continent region and endeavors to reduce the interpretation work to a mere mechanical technique, leaving as little as possible to the opinion of the interpreter.

The New Method

Field Technique.—In sectionized territory where country roads are available every mile, shot points are preferably located every ¼ mile when six detectors are available or every ⅓ mile when eight detectors are at hand. The disposition of individual spreads is represented on Fig. 1; in each case a shot point is located at the section corners.

Fig. 2*a* represents the layout of a spread between shot points, the vibration detectors being preferably equidistantly located. Normally in the ¼-mile control, the distance between shot points is 1320 ft. and the distance between detectors 220 ft. When shooting from shot point S.P. 1, detectors 1 to 6 are connected to the recording galvanometers and detector 0 (which indicates the vertical travel time of the wave corre-

Manuscript received at the office of the Institute Jan. 20, 1937. Issued as T.P. 833, August, 1937.

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sponding to the shot-hole depth) is connected by means of a highly damped circuit to one galvanometer. The shot instant is connected in a like manner to another trace or galvanometer.

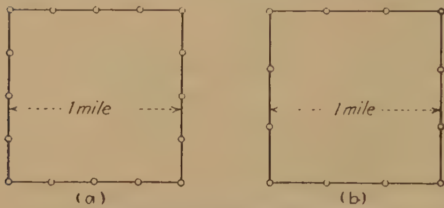


FIG. 1.—INDIVIDUAL SPREADS.
a, quarter-mile control; *b*, third-mile control.

For speedy work two sections of three connecting cables cut to length are made up in such a way that they can be dragged on the ground without necessity of reeling up. The connections made in shooting from S.P. 1 are indicated on Fig. 2*b*. The recording truck is placed close to location 3, therefore only a short connecting cable is needed. When a shot is to be made from S.P. 2, it is only necessary to invert the connections at the recording truck.

Principle of Method.—The principle underlying the method is made clearer by Fig. 3. The seismic rays traveling through a cross section of the subsurface are figured. Two shot points, A and B, are located, and spread *a* has its last geophone or detector located at shot point B whereas *b* has its last geophone located at shot point A. Image points A′ and B′ required in drawing the paths of the reflected rays are also

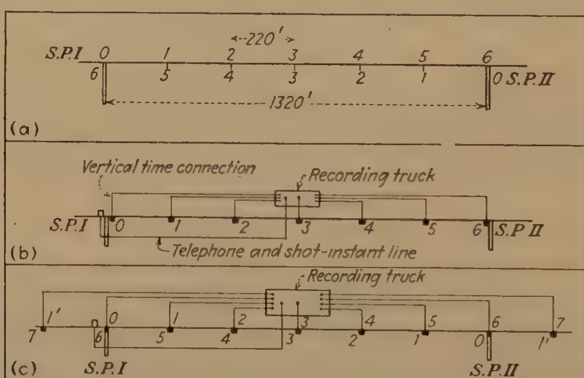


FIG. 2.—SPREAD BETWEEN SHOT POINTS.
a, shot point and geophone layout for continuous profiling.
b, electric connections for continuous profiling.
c, electric connections for overlapping of continuous profiles at both ends.

represented, together with the reflection points $R_{A_{1,2,3,4,5,6}}$ for spread *a* and $R_{B_{1,2,3,4,5}}$ for spread *b*. The reflection points R_{A_6} and R_{B_6} coincide. Consequently, the depth calculations of this point from the time elapsed during the travel of the seismic wave from A or B must give identical answers.

Computation of Seismic Records

Fig. 4 shows two interlocking records obtained in Oklahoma with shot points 1320 ft. apart. The shot instant is noticed on the next to the

last trace of each record and the vertical time in the shot hole is recorded on the fourth trace. The first arrivals at each detector are very noticeable except on the fourth trace of one record, owing to interference with the vertical time recorded on the same trace. After the first arrival disturbances, reflections can be noticed, although at first sight one would not call these records very good samples; it is the characteristic of this particular method that the seismic strips do not show beautiful reflections because of the unusual length of the spread. The computation of seismic records includes the time marking on the photographic strips,

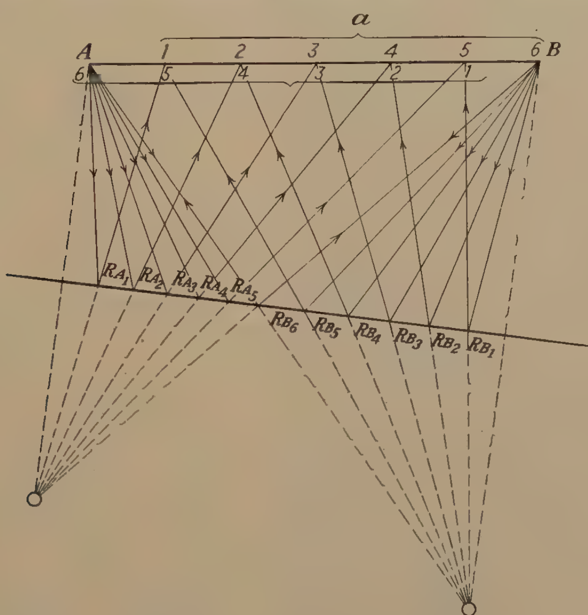


FIG. 3.—SEISMIC PATHS IN CONTINUOUS PROFILING.

the calculation of the weathering corrections, the reduction to a fiducial plane and elevation corrections.

Weathering Corrections.—Fig. 5 shows the computation sheet for weathering corrections where the corresponding first arrival times are recorded above each other on lines 1 and 2. The figures in the first and last columns are subtracted from each other which gives the travel time (T) from the bottom of the shot hole of S.P. 1 to the bottom of the shot hole at S.P. 2. The other columns are added, the results of which are the summations of T and of twice the travel times in the weathering zone. The paths in the weathered zone are somewhat oblique to the vertical, and in order to reduce these paths to the vertical it is necessary to multiply the weathering times by $K = \sqrt{\frac{V_2 - V_1}{V_1 - V_2}}$.

Twice the uncorrected weathering times ($2t'_w$) are obtained in line 5 by subtraction of the respective figures in line 4 from those of line 3.

Line 4 is obtained by subtraction in the first and last columns and by addition in the others. The uncorrected weathering times t'_w are deducted and recorded in line 6. Lines 7 and 8 show respectively the values of the quantities $T_1 - t'_w$ and $T_2 - t'_w$; these are plotted on a travel-time curve at 5. From these lines one derives the average velocity of the refracted wave underneath the spread (8130 ft. per sec.) and the uncorrected weathering times at shot point S.P. 1 (0.025 sec.), at shot point S.P. 2 (0.013 sec.) and at geophone 3 (0.018 sec.). This last value could not be found previously because the first arrivals were illegible on the third trace of the first record of Fig. 4, owing to interference with vertical time measurements. . The average velocity of transmission of the elastic wave in the weathered zone is found to be about 2000 ft. per sec. Thus the value of coefficient K is 0.777, from which the corrected weathering times are calculated and recorded in line 9.

FIG. 4.—TWO INTERLOCKING SEISMOGRAPH RECORDS.

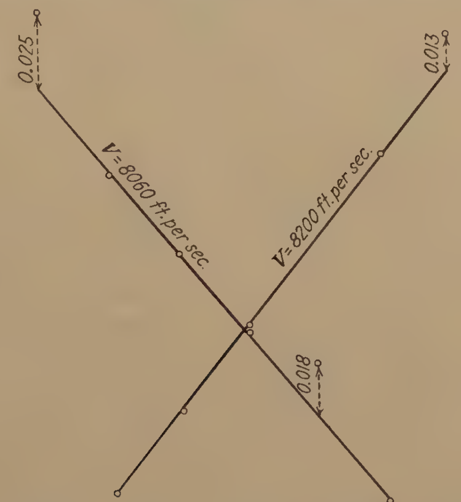
Elevation Corrections.—Choosing an elevation of 1000 ft. above sea level for the fiducial plane, the figures of Table 1 give the summary of the elevation corrections. The velocity of propagation in the zone immediately underlying the weathered layer is assumed to be 8000 ft. per sec.

The figures in the columns $t_w + t_{el}$ represent the delay of arrival of the seismic wave that originated at the shot point and was received at detectors strung on the surface of the ground. Subtracting these figures from the travel times reduces the measurements to the conditions that would be obtained by exploding the dynamite on the fiducial plane and receiving the wave by detectors strung on the

same plane. The delay time figures are helpful in following a reflection across the record as well as in making depth determinations.

Correlation

Correlation of Seismic Records.—It was stated in a preceding paragraph that the key to the correlation of two interlocking records is the consideration of the overlapping depth points corresponding to the last trace of each seismogram. In fact, if the vertical time recorded at S.P. 2



Geoph. No.	0	1	2	3	4	5	6
1 T_1	0.025	0.052	0.072	0.097		0.143	0.174
2 T_2	0.183	0.155	0.123	0.095	0.067	0.030	0.017
3 ΣT	0.158	0.207	0.195	0.192		0.173	0.157
4 $-T$		−0.157	−0.157	−0.158		−0.157	
5 $2t'_w$		0.050	0.038	0.034		0.016	
6 t'_w		0.025	0.019	0.017		0.008	
7 $T_1 - t'_w$		0.027	0.053	0.080		0.135	
8 $T_2 - t'_w$		0.130	0.104	0.078		0.022	
9 $t_w = t'_w \times 0.777$	0.019	0.019	0.015	0.013	0.014	0.006	0.010

Fig. 5.—Weathering calculations.

(0.017 sec.) is subtracted from any time pick on the last trace of tape 1, it should be identical (within reasonable limits of errors) to the figure obtained by subtracting the vertical time at S.P. 1 (0.025 sec.) from the corresponding time event on the last trace of tape 2. Two such corresponding events are 1.031 sec. on tape 1 and 1.032 sec. on tape 2. Undoubtedly they represent the same phase instant of the seismic wave, and since they are picked on strong evidence of the occurrence of reflected energy they correspond to the same geologic formation, in this particular instance the Viola limestone.

The figures in the column $t_w + t_{el}$ of Table 1 indicate on tape 1 that the correlation across the record to the first trace is in a slanting direction to the right from the vertical, whereas similar considerations for tape 2 require a correlation in a slanting direction to the left. The correspond-

ing time picks, corrections, and depths below the fiducial plane are recorded in Table 2. The depths are calculated by using 11,000 ft. per sec. as the over-all velocity to the Viola limestone. The so-called angu-

TABLE 1.—*Elevation Corrections*

Geophone No.	Geophone Elevation, Ft.	$-t_w$	$-w$	$W = \frac{45' + w}{2}$	El. − W	Elevation Correction	t_{el}	$t_w + t_{el}$
SEISMOGRAPH TAPE NO. 1. DEPTH S.P. 1 = 45′								
1	1073	−0.019	−38	−41	1032	−32	−0.008	−0.027
2	1072	−0.015	−30	−37	1035	−35	−0.009	−0.024
3	1072	−0.013	−26	−35	1037	−37	−0.009	−0.022
4	1068	−0.014	−28	−36	1032	−32	−0.008	−0.022
5	1063	−0.006	−12	−28	1035	−35	−0.009	−0.015
6	1067	−0.010	−20	−32	1035	−35	−0.009	−0.019
SEISMOGRAPH TAPE NO. 2. DEPTH S.P. = 48′								
1	1063	−0.006	−12	−30	1033	−33	−0.008	−0.014
2	1068	−0.014	−28	−38	1030	−30	−0.008	−0.022
3	1072	−0.013	−26	−37	1035	−35	−0.009	−0.022
4	1072	−0.015	−30	−39	1043	−43	−0.011	−0.026
5	1073	−0.019	−38	−43	1030	−30	−0.007	−0.026
6	1073	−0.019	−38	−43	1030	−30	−0.007	−0.026

larity or spread correction is derived from the approximation formula $\frac{1}{2}\frac{X^2}{tV^2}$, in which X is the distance from the shot point to the geophone, t is the over-all travel time and V the over-all velocity of propagation of the seismic wave.

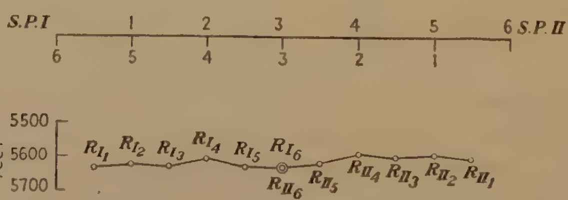


FIG. 6.—DEPTH POINTS PLOTTED BELOW FIDUCIAL PLANE.

The depth figures are plotted on Fig. 6, where the amount of offset of the depth points due to the dip of the formations is neglected. Since formations are dipping by an angle of about 4° to the horizontal, the amount of offset is negligible as far as its effect on the amount of dip is concerned. Thus the depth points are plotted on the median of the shot point-geophone distance.

Correlation across Shot Points.—Each shot point is shot in at least two opposite directions and at section corners in four orthogonal direc-

tions. The depth points corresponding to the first trace of the records obtained from a given shot point are distant from each other by 220 ft. at the most. In such a distance, one can normally expect an easy correlation between records. It is, however, the author's experience that this does not hold, and many a time it has been necessary to resort to short straddle spreads across the shot hole in order to clarify correlations of records obtained from the same shot point.

TABLE 2.—*Time Picks, Corrections and Depths below Fiducial Plane*

Geophone No.		Over-all Time t, Sec.	Distance to Shot Point, X, Ft.	Angularity Correction $\frac{1}{2}\frac{X^2}{tV^2}$, Sec.	$t_w + t_{el}$, Sec.	Corrected Time	Depth below Fiducial Plane, Ft.
S.P. 1	1	1.051	220	Negligible	−0.027	1.024	5632
	2	1.047	440	−0.001	−0.024	1.022	5621
	3	1.047	660	−0.002	−0.022	1.023	5626
	4	1.044	880	−0.003	−0.022	1.019	5604
	5	1.043	1100	−0.005	−0.015	1.023	5626
	6	1.049	1320	−0.007	−0.019	1.023	5626
S.P. 2	6	1.056	1320	−0.007	−0.026	1.023	5626
	5	1.052	1100	−0.005	−0.026	1.021	5615
	4	1.045	880	−0.003	−0.026	1.016	5588
	3	1.042	660	−0.002	−0.022	1.018	5599
	2	1.039	440	−0.001	−0.022	1.016	5588
	1	1.032	220	Negligible	−0.014	1.018	5599

In order to improve the method, overlap of spreads may be taken at both ends of the profile. Fig. 2*c* gives a graphic representation of such a suggestion. Nine geophones are in use, one serving to measure the vertical time at the shot hole. The first two traces correspond to geophones located 220 ft. on opposite sides of the shot hole. A further advantage of this system would be provided by the occurrence of three overlap depth points: $I_6 - II_6$, $I_7 - II_5$ and $I_5 - II_7$.

CONCLUSIONS

The continuous profiling method of seismographic surveying reviewed here gives excellent results when applied judiciously. It has the disadvantage of being somewhat slow, although the author was in charge of a field crew that in one day obtained 26 profiles, covering a distance of 3.25 miles. This was an exception, however, and too much strain was put on the crew. Normally in open country one can expect an average of 2 miles per day.

Application of the Seismic Refraction Method of Subsurface Exploration to Flood-control Projects

By Edgar R. Shepard* and Albert E. Wood†

(New York Meeting, February, 1940)

The interest of the Federal Government in improvement of waterways dates from 1820, when Congress appropriated $5000 for making a survey of the Mississippi and Ohio Rivers and assigned this work to the Corps of Engineers of the Army. Since that time the Government has gradually assumed increasing jurisdiction in the matter of improvement and control of inland waterways until, by the River and Harbor Act of Jan. 21, 1927, Congress assigned to the Secretary of War and to the Chief of Engineers the duty of making surveys, in accordance with House Document No. 308, 69th Congress, to formulate plans for the most effective improvement of the navigable streams of the United States and their tributaries, for navigation, development of water power, control of floods, and the needs of irrigation. Virtually all the preliminary investigations and surveys required by this act have been completed. The reports made to Congress have been published and often are referred to as "308 Reports."

The map shown in Fig. 1 is merely a skeleton distribution of the projects covered by these reports, but it serves to indicate the magnitude of the proposed undertaking. In many instances, a single character on the map stands for a major development involving several dams and reservoirs or many miles of canals or levees.

The devastating floods of 1936, and the need for worthy public works to take up the slack in employment, gave further impetus to this program and in 1936 led to the enactment of a bill authorizing a large number of these projects.

The National Flood Control Act of 1936 specifically authorizes some 270 flood-control projects in 31 states, and affecting nearly every state in the Union, with an estimated construction cost of about $300,000,000. In addition, the 1936 Act directs the Secretary of War to make preliminary examinations and surveys for flood control in some 230 additional localities, and to continue surveys and studies of 18 reservoir sites. The Corps of Engineers is proceeding rapidly with foundation investiga-

Manuscript received at the office of the Institute Feb. 26, 1940. Issued as T.P. 1219, June, 1940.

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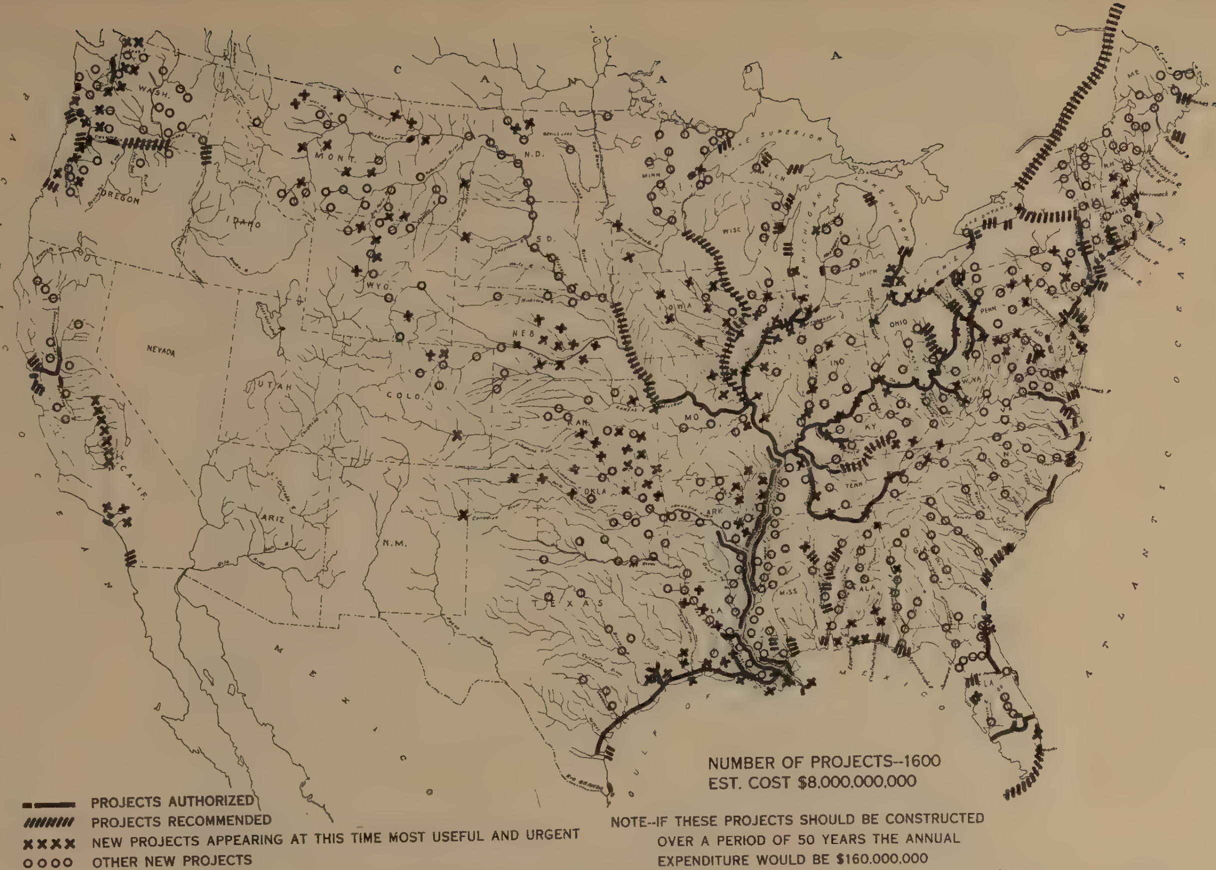

FIG. 1.—FLOOD-CONTROL, NAVIGATION, IRRIGATION, AND POWER PROJECTS PLANNED BY THE ARMY ENGINEERS.

tions and preparation of detailed plans for the projects that appear most urgently needed for the protection of life and property, and detailed investigations of other projects will follow as funds become available.

Need for Rapid Method of Subsurface Exploration

When confronted in 1936 with the necessity of investigating hundreds of dam sites, the Army Engineers were in a position to welcome any adequate means of subsurface exploration that showed promise of facilitating the task. For many reservoir projects there were one or more alternate sites. To explore all such sites by drilling not only would involve unduly large expenditures but also would delay the program at a time when funds and labor are available for construction, and when there is a popular demand that work begin at once, before the possible recurrence of destructive floods.

Fig. 2.—Apparatus used in seismic explorations.

Experiments were made with the electrical resistivity method of exploration, and, although satisfactory results were obtained in some instances, they were, in general, such as to discourage the extensive adoption of this method.

In 1937, at the request of the War Department, the Bureau of Public Roads made seismic tests along the Connecticut River near Hartford and at three dam sites in southern New York, using a portable refraction seismograph developed by the Bureau for shallow exploration (Fig. 2). These tests were so successful and promising that in 1938 the Corps of Engineers adopted the method as a standard procedure and it is now in general use throughout the country. In the period from October 1938 to December 1939, over 225 dam sites and several canal projects have been explored by the seismic method, and many of these have been eliminated from further consideration without drilling.

The seismic exploration is being done, for the most part, by engineers and geologists assigned from the staffs of the several district or division offices. A training period of from two to four weeks is usually sufficient to qualify a man to work independently unless he encounters unusual underground conditions. Increased efficiency and greater accuracy in interpretation come with more experience.

Theory of Seismic Exploration

Only a brief description of the theory and principles of seismic refraction shooting will be given in this paper, as the subject has been ade-

quately treated elsewhere.[1,2] The method is based on the fact that the velocity of wave propagation in the earth's crust differs greatly in different substances. Unconsolidated materials such as sand, gravel and clay transmit wave disturbances at velocities from 600 to 6000 ft. per sec., whereas consolidated materials like shale, sandstone and crystalline rocks

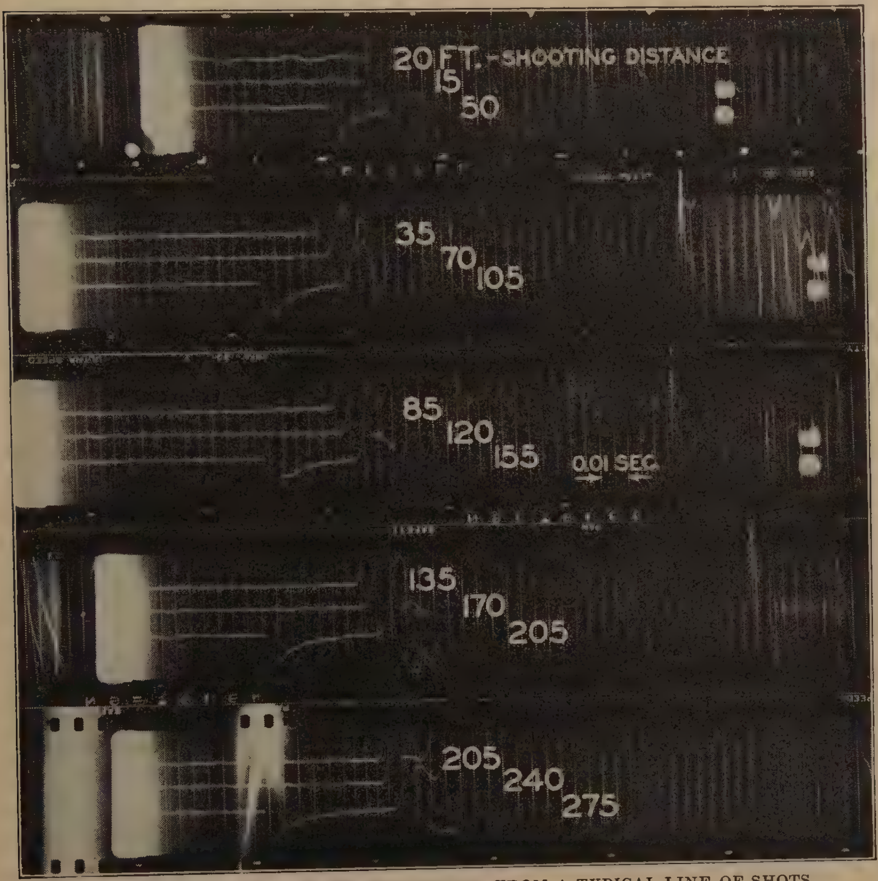

FIG. 3.—EXAMPLES OF SEISMIC RECORDS FROM A TYPICAL LINE OF SHOTS.

transmit such disturbances at 7000 to 20,000 ft. per sec. This wide range in velocities in different kinds of soils and rocks provides a means of determining the nature of the different materials encountered.

In the practical application of the principle, three or more detectors or geophones are placed in line on the surface of the ground, usually at

[1] M. Ewing, A. P. Crary and H. M. Rutherford: Geophysical Studies in the Atlantic Coastal Plain. *Bull.* Geol. Soc. Amer. (1937) **48**, No. 6, 753–802. 1 pl., 35 figs.

[2] E. R. Shepard: The Seismic Method of Exploration Applied to Construction Projects. *The Military Engr.* (Sept.-Oct. 1939).

intervals of 35 ft. Small charges of dynamite, buried about 3 ft. deep, are exploded along the detector line, beginning with a shot 10 ft. from the center detector and extending the shooting distances to such lengths as may be required, by intervals normally either 20 or 50 ft. A shooting distance from three to four times the depth to which information is desired is usually necessary. From an oscillograph record the time of wave travel from each shot point to each of the detectors is read and the results are plotted in the form of a time-distance graph, from which the depth to rock or other subsurface discontinuities may be determined along each line of shots.

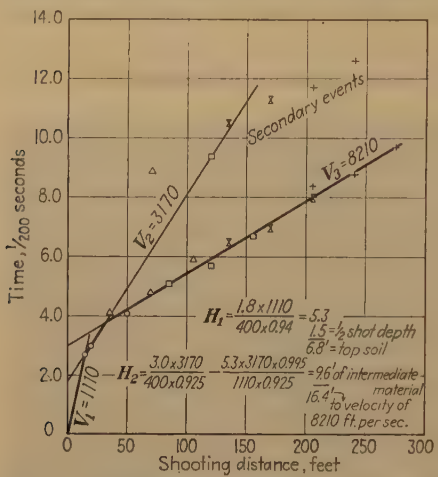

FIG. 4.—TIME-DISTANCE GRAPH PLOTTED FROM FILM RECORDS SHOWN ON FIG. 3.

Fig. 3 shows a series of typical records so obtained, and Fig. 4 is the time-distance graph plotted from these same records. The graph shows three layers consisting of a topsoil, in which the velocity of wave propagation is 1110 ft. per sec.; an intermediate material, in which the velocity is 3170 ft. per sec.; and a rigid material (in this case sandstone), in which the velocity is 8210 ft. per sec. The thicknesses of the two layers of overburden are determined from the intercepts on the time axis and the three indicated velocities, by means of the formulas presented by Ewing, Crary and Rutherford (ref. 1, 763–764).

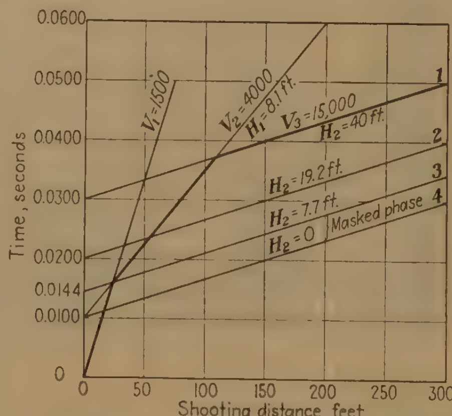

FIG. 5.—TIME-DISTANCE GRAPH SHOWING MASKING EFFECT ON INTERMEDIATE LAYER WITH VARIOUS THICKNESSES.

Sometimes the intermediate phase or phases between the topsoil and the high-velocity rock layer are poorly defined or apparently missing on the time-distance graph. Where such a zone is present but is neglected in the computations, a considerable error in depth determinations may be made. As this condition has been found to exist in numerous instances and frequently is a possible source of error in the interpretation of seismic data, a further discussion of it seems appropriate.

The graphs in Fig. 5 represent a situation where there are three layers in which the velocities are 1500 ft., 4000 ft. and 15,000 ft. per sec., respectively, the top layer being 8.1 ft. thick. If the intermediate layer is 40 ft. thick, as indicated by the rock line intersecting the time axis at 0.03 sec., the second phase is unmistakable and well defined. For an intermediate layer 19.2 ft. thick, the second phase is less well defined, and for thicknesses less than 7.7 ft. it is entirely masked, so far as first arrivals are concerned. For thicknesses only slightly greater than the critical value of 7.7 ft., the recorded first arrivals on the graph might fail to disclose definitely the presence of the intermediate layer. Such a graph is that shown in Fig. 4.

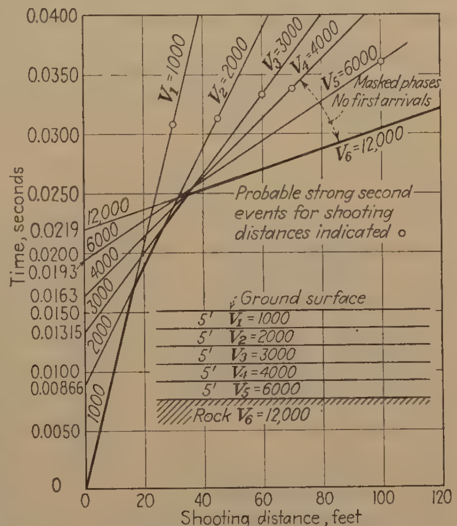

FIG. 6.—TIME-DISTANCE GRAPH SHOWING MASKED PHASES.

Where there is a question as to the presence of an intermediate layer, it is often helpful to examine the film records for strong second events, which are sometimes present. The strong secondary arrivals evident in the film records of Fig. 3 are plotted on the graph of Fig. 4. Although they suggest the presence of intermediate material, the arrangement of the points does not suggest a material of any one velocity but rather one of increasing velocity with depth. This condition has been observed frequently and has led to considerable speculation concerning its significance. A possible explanation may be found in Fig. 6, which is fitted to the conditions of Fig. 4. In this figure the theoretical arrival times through the several strata are shown in their true positions. First arrivals occur only through the upper three layers and the high-velocity rock, and all intermediate layers are masked. First arrivals through the 3000-ft. per sec. phase occur over a very limited shooting distance and in actual practice would be obscure or uncertain. The strength of the secondary arrivals through the various layers depends on the shooting distances, and it is probable that, as the shooting distance increases, the velocity corresponding to the most prominent subsequent event will increase, thus forming a pattern of secondary arrivals as indicated, and one that bears a striking resemblance to the observed pattern shown in Fig. 4.

In calculating depths from graphs like those shown in Figs. 4 and 6, it is generally impracticable to use more than one, or at most two, interme-

diate velocities, as the time so consumed would be out of proportion to the accuracy required where this method of exploration is used. The question arises, therefore, as to how this common type of graph should be analyzed and interpreted. Table 1 gives the results of various methods of calculating the depths of overburden from the graphs in Fig. 6. The "rigid" interpretations are based on the formulas given by Ewing, Crary, and Rutherford. By "Ind. Int." (individual intercepts) is meant the shorter method by which layer thicknesses are computed from individual intercepts on the time axis rather than from the total times.

TABLE 1.—*Results of Different Methods of Calculating Depths from Seismic Data in Figure* 6

Solution	Method of Calculation	Method	Calculated Depths					Ft., Total
			H_1	H_2	H_3	H_4	H_5	
1	Using all phases	Rigid	5	5	5	5	5	25
2	Using all phases	Ind. Int.[a]	5	6.02	7.14	8.05	9.0	35.21
3	Using 1000, 4000, and 12,000 phases	Rigid	8.4	10.8				19.21
4	Using 1000, 4000, and 12,000 phases	Ind. Int.	8.4	11.9				20.3
5	Using 1000, 3000, and 12,000 phases	Rigid	7.0	12.4				19.4
6	Using 1000, 3000, and 12,000 phases	Ind. Int.	7.0	13.55				20.55
7	Using 1000, 6000, and 12,000 phases	Rigid	9.8	8.3				18.3
8	Using 1000, 6000, and 12,000 phases	Ind. Int.	9.8	9.0				18.8
9	Using 1500, 4000, and 12,000 phases	Rigid	13.1	10.4				23.5
10	Using 1500, 4000, and 12,000 phases	Ind. Int.	13.1	11.9				25.0
11	Using 1500, 3000, and 12,000 phases	Rigid	11.4	10.6				22.0
12	Using 1500, 3000, and 12,000 phases	Ind. Int.	11.4	13.55				24.95
13	Using 1500, 3000, 6000, and 12,000 phases	Rigid	11.4	7.8	5.7			24.9
14	Using 1500, 3000, 6000, and 12,000 phases	Ind. Int.	11.4	10.66	9.0			31.06

[a] Individual intercepts.

This table shows that the correct depth of 25 ft. to the high-velocity rock can be closely approached by using the approximate individual intercept method and assuming two strata of overburden to which average velocities are assigned. In practice it has been found that the very low velocities observed in the topsoils apply to relatively shallow

depths and that it is often better to disregard this first tangent of the time-distance graph and to select a somewhat higher value as the average topsoil velocity. Where there are several intermediate materials with gradually increasing velocities, the choice among them is far from critical. This is evident from a comparison of solutions 4 and 6 or 10 and 12 in which a change in the intermediate velocity from 3000 to 4000 makes practically no change in the calculated depth to rock. Where it is desired to use the seismic data to obtain information regarding depths to overburden discontinuities, or where it is desired to use the velocities in overburden as an indication of the nature of the materials present, it is obviously very important to use care in the selection of the intermediate velocities.

Field Practice

The size of the field crew necessary for successful operation of the seismograph depends on the program planned and, to a lesser extent, on the physical conditions at the site. A minimum crew consists of an instrument operator, a dynamite man, and a laborer. A crew of this size can be conveniently used for demonstration work, or for very preliminary work, where a large proportion of the time is consumed in moving between points of operation. With such a small crew the work must be on terrain that will not require extensive hand transportation of the instrument.

At many sites information as to the exact positions and elevations of the shot points is not necessary, since depth to rock in a general area and not at a specific point is desired. Usually, however, such information is desirable if the work is other than very preliminary in nature. For efficient large-scale operation, a small survey party is used to locate the seismic lines, including detector and shot points, and to determine the elevations of all points, and the actual shooting is performed by a party consisting of an instrument operator, a powder man, an assistant powder man, and two to four laborers. This permits the shooting of an average of 80 shots per day. To keep up with the field operations takes the full time of one man in the office to make the computations and interpretations.

The seismic explorations of dam sites performed to date have been of two main types, preliminary and detailed, the former having been much more generally utilized. In preliminary explorations, the aim is to determine as rapidly as possible the depth to rock in the valley bottom and on the abutments, in order to permit the selection of the best type of design and to determine whether or not it is practicable to place spillway and outlet structures on rock at the site without excessive excavation. These data are also sufficient to permit a selection to be made from alternate sites. A preliminary program normally consists of from two

to six or more lines of shots, involving one to three days work for a small party.

In detailed explorations, it is desired to obtain the maximum number of elevations of the rock surface over the entire site area, permitting the preparation of reasonable estimates as to the amount of overburden and rock excavation, and the determination of the best location for structures. Such explorations also disclose the nature, depth and relationships of the various overburden zones and the amount of weathering in the rock. The seismograph is well adapted to this type of work, permitting extensive information to be obtained in a minimum time and at a minimum cost. Detailed shooting has been performed mainly in New England and New York. A detailed seismic program for a single dam site requires 30 or more lines of shots, and may take several weeks or months for completion. The seismic explorations permit the selection of the best possible location for spillway and outlet structures. After this program is finished, a relatively small amount of detail drilling supplies the necessary information to furnish a check on the seismic records and to permit the exact location of rock for structural foundations.

Although the exact nature of the overburden cannot always be told from seismic data, it is possible to correlate the seismic results with those of drilling, and thus to extend the information from the latter. It is, moreover, often possible to make interpretations directly from the seismic velocities. Data are gradually accumulating as to the significance of different velocities and other record characteristics, from which the nature of the overburden is indicated. It is frequently possible to separate sand and gravel from silty deposits, when above the water table. In some places, sand and gravel below the water table are separable from silty materials; in some they are not. It is usually possible to locate the water table. Glacial till is almost as easily distinguishable as is rock. As a rule, however, there is considerable difficulty in determining the nature of the overburden from seismic data. On the other hand, it is very generally possible to determine the depth to an overburden discontinuity, even though the nature of the discontinuity may be uncertain.

Application to Different Types of Geologic Formations

The seismic method can be successfully used in most localities, although the regional geology has a pronounced effect on the accuracy and ease of interpretation. The ideal conditions are present when the surface of the rock is hard and unweathered, with little or no jointing or fracturing, and the overburden is loose and uncompacted. Under these conditions there is no difficulty at all in determining the depth to rock. These conditions prevail when the bedrock is unweathered crystalline rock, sandstone, limestone, or well indurated shale, and are present in most of the New England and Middle Atlantic states, in considerable

areas of the central part of the country, and in much of the Rocky Mountain and Pacific Coast areas. At any site, however, areas are likely to be encountered where the conditions are not so favorable because of weathering of the rock. Where the rock is badly weathered and the overburden is stony, it may be virtually impossible to separate the two. Where a well compacted glacial till or hardpan overlies a jointed and weathered rock surface it is often impossible to be certain of the location of the rock surface. Usually, these conditions are of local extent; an entire site seldom presents such difficulties. In the Appalachian and Illinois coal fields, however, it appears to be almost impossible to determine either the depth to sound rock or to the top of rock, because of the extent to which the coals and intercalated shales have disintegrated. In the Sierra Nevadas, it has been found that granites are usually badly jointed and deeply weathered and that the seismograph does not show the extent of such weathering with any reliability. As a result, the seismograph has not been as useful in granitic areas in this region as in other types of formations. There are areas similar to this in the southern Appalachians where there would probably be uncertainties in determining the elevation of the top of rock and of the top of unweathered rock.

Where the bedrock has never been fully consolidated, it may be difficult to separate rock and overburden. These conditions hold over much of the lower Mississippi Valley, the Great Plains areas, and the coastal plains. In one place in the Great Plains, it was impossible to distinguish between a slightly indurated sandstone and the overlying saturated sands and gravels.

Even in areas where difficulties of interpretation are likely to arise, it is unusual that careful work will not yield valuable results, and it is unusual to have these conditions prevailing over an entire dam site. Therefore, in spite of exceptional circumstances such as those given above, it is generally simple to locate the rock surface.

Accuracy of Seismic Data

Where it has been possible to test them, seismic data are usually close to the depths given by drill holes. In general, when the records are reasonably satisfactory, the depth to rock, as given by the seismograph, agrees within 5 ft. of the depth from drilling. The average arithmetical difference in 30 or 40 records in New York and New England that have been checked is 2 ft. The records that gave these comparisons were not selected as being good seismic records, but were all the cases where it was possible to compare the two. Depths to rock are generally more accurate than those to overburden discontinuities, but even the latter fall within the above limits, though the average deviations are somewhat larger. Some instances have been observed where the upper few feet of the rock are so badly weathered that the driller and drilling inspector cannot identify the exact surface of the rock, nor can its precise elevation be

determined from inspection of cores and samples. Under these conditions, the seismic data sometimes may actually give more accurate data as to the location of the rock surface than do the results of drilling. One difference between the results of the two methods is that a seismograph survey gives more determinations of rock elevation than are available from a comparable drilling program. Even though there may be some doubt as to the exactitude of the individual seismic figures, the large number of records greatly increases the value of this method, permitting the elimination of incorrect or questionable records. In regions where the rock surface is irregular and unrelated to the ground surface, the results of a few drill holes can be very misleading, locating either unusually high or unusually low points on the rock surface. The more numerous rock elevations determined by the seismograph tend to eliminate this source of error.

There is no doubt that in some respects the data obtained by the seismograph are not as satisfactory as those resulting from drilling. The seismograph does not give samples of overburden that can be studied to determine stability of foundations or suitability of borrow, although information now available suggests the possibility of future progress in this line, since there appears to be a marked correlation between the seismic velocities and the physical properties of the overburden. Furthermore, although the seismic records give a general idea as to whether or not the rock is weathered or jointed, they do not present specified information on this point. In addition, the seismograph does not yield samples that may be inspected by contractors and others not familiar with seismic records and their interpretation. The most efficient and economical method of conducting subsurface exploration for the foundations of large structures involves the following sequence, which has been adopted for several flood-control reservoirs. The first step would be preliminary exploration by seismograph to allow selection from alternate sites. If stability of foundations were an important factor in the choice of sites, one drill hole in the valley bottom should be included in a location shown by seismic data to be suitable to obtain overburden samples for laboratory analysis. The next stage would be detailed seismic exploration of the entire site, followed by detail drilling for foundations of concrete structures and by drilling and test-pit exploration of foundation and borrow areas.

Seismic Exploration Costs

It is difficult to find an entirely satisfactory basis for an exact comparison of costs between drilling and seismic explorations, since a drilling program gives additional information not supplied by the seismograph, and the latter gives a larger number of determinations of the rock elevation. For preliminary explorations, the method that has been adopted

is to compare the total cost of a drilling program sufficient for preliminary design with that of a comparable seismic program. Experience has shown that seismic operations cost about $100 per day, and that two or three days are sufficient for preliminary exploration of most dam sites. This gives a cost of $200 to $300 per dam site for preliminary seismic exploration. The cost of a drilling program is much more variable, depending largely on the depth to rock and the ruggedness of the topography, which affect seismic costs very little. Drilling, however, costs on the average from $3.00 to $5.00 per foot. Allowing the extremely low figure of 200 ft. of drilling on a site for preliminary exploration, the drilling program would cost from $600 to $1000 per site, or approximately three times the cost of a seismic program. The above drilling program would take about 14 days for one drill or 7 days for two drills, a period considerably longer than the two to three days required for seismic explorations. It appears, therefore, that a preliminary seismic program costs no more than one-third as much as a preliminary drilling program and takes one-third as long. Seismic data give sufficiently accurate location of the rock surface for preliminary design, or for determining which of two or more sites is the most suitable. For this type of exploration, therefore, the seismograph is preferable to the drill because it gives equally acceptable data for one-third the cost, in one-third the time. In most cases, preliminary design does not depend to any great extent on the nature of the overburden, so that the lack of information on this question is not an important handicap.

It is even more difficult to make a fair comparison between seismic and drilling costs for detailed exploration. At one dam site in New York a detailed study of this sort has been made. Here it was shown that the seismic explorations cost 10 per cent as much as the cost of drilling program that would have given the same number of elevations on the rock surface. This comparison avoids the question of sampling, since the drilling estimates were based on the cost of overburden and rock drilling without taking overburden samples. This is perhaps not a fair comparison of the cost of a program by each method, since no drilling program would consist of as many holes as there are rock elevations resulting from a seismic program. Although in some parts of the country the seismograph is not considered adaptable for final design exploration, work in New England and New York seems to show that it is a valuable aid for such exploration, permitting fewer drill holes more advantageously located than would be necessary without the extensive seismic program.

Summary

In view of the large amount of exploratory work required in the program of flood control and navigation improvement proposed by Con-

gress, the Army Engineers have welcomed the seismograph as a rapid method of making subsurface explorations. It has definitely established its usefulness for preliminary investigations, and apparently is proving its value for final design exploration. There are, of course, still numerous possibilities for improvement in interpretative technique, as in connection with the problem of masked phases discussed above, which will permit an increase in the accuracy of the method.

DISCUSSION

(*M. King Hubbert presiding*)

G. P. Woollard,* Bethlehem, Pa.—The treatment of masked phases presented by Mr. Shepard is interesting because this is a problem that confronts those doing deep subsurface investigations as well as those doing relatively shallow investigations. In my own experience with Dr. Ewing we have been able to use second and on occasion third "breaks" on the record for plotting these phases for which there were no first "breaks." This practice has been limited, however, to the study of deep seismic horizons with a velocity differential as a rule of around 2000 ft. per second. For that portion of the geologic column generally associated with the "weathering" layer, our experience has varied with different localities. In areas of clays with perched water tables the initial velocity line has always approximated the velocity of water—about 5000 ft. per second. In areas of extensive Pleistocene gravels with a low water table the initial velocity line has approximated 2000 to 3000 ft. per second. These, however are admittedly average values for all near-surface material since we have had no interest in subdividing this part of the geologic column. I believe Dr. E. C. Bullard, of Cambridge, England, has studied the near-surface velocities with close geophone spacings, and it is my impression that his results indicate a curve of increasing velocity with depth similar to that reported by Ewing for the Gulf Coast sediments rather than a limited number, say three, velocity layers being present, which might be interpreted as three different kinds of material when actually the change may be due entirely to compaction or secondary ground-water deposition in one material.

Our experience with the use of secondary "breaks" on the record has been that frequently "turn around" points occur, which plot an entirely different pattern from the velocity lines desired in that they are transverse to them. Finally, in determining velocity lines for near-surface horizons from these points the position of the water table relative to the depth computed from these data and the effect of water on observed velocities must be borne in mind.

This is not to be construed as destructive criticism of Mr. Shepard's analysis, but rather as some precautions that should be observed in the shallow depth work in which he is interested when using second "breaks" to deal with masked phases. From the standpoint of depth to bedrock where no significance is to be placed on intermediate thin horizons, his conclusion that results of a sufficient degree of accuracy are possible under several different assumptions as to velocity distribution is well founded, although, as he points out, a judicious choice of velocities is needed—and this, I may add, can generally be obtained from an inspection of conditions in the shot holes as to whether the surface material is primarily sand, clay or gravel, and also its moisture.

* National Research Council Fellow in Geology, Lehigh University.

F. W. Lee,* Washington, D. C.—The Section of Geophysics of the U. S. Geological Survey has also made shallow seismic refraction surveys relating to the location of buried channels between Chelmsford and Lowell, Mass. It was found very advantageous to use a large number of seismic pickups to record each shot. No less than 12 were used between the shot points located at each end of the observation interval. This procedure often greatly clarified the picture concerning intermediary beds between the surface alluvium and the bedrock. The mathematical relations under this method have singular characteristic properties, which permit the computation of the slopes and dips of the beds.

* Chief, Section of Geophysics, U. S. Geological Survey.

Geophysical Study of Soil Dynamics

By Rudolf K. Bernhard*

(New York Meeting, February, 1938)

Static soil investigations with more or less heavy loads and extensive borings do not always have efficient results; also, they are very expensive. The new geophysical method of dynamic tests described herein is simple and cheap and avoids several disadvantages of the routine investigation, thus forming a useful supplement to the older methods and even at times replacing various static tests.

The fundamental principle of dynamic soil tests consists in the application of artificial vibration. The technique employed is essentially geophysical in nature and in some respects is akin to seismic studies. Artificial vibration may be excited by centrifugal forces resulting from eccentrically supported rotating masses. The reactions of soil and structures caused by these vibrations may be measured. Their dynamic constants, such as amplitudes, phase speeds, damping, reflection, and interference effects, can be investigated. This geophysical method has been used for several years with increasing success. The results of these tests, summarized at the end of the paper, indicate that this method opens a new field for the practical engineer in investigating soils (capacity and formation), foundations, dams and structures, and especially buildings that must withstand earthquakes.

Methods

Static Method.—The first and oldest tests to investigate the bearing capacity of the soil were carried out by static weights. The penetration of these weights, however, depends not only on the soil but also on the size and the form of their lower surface. The length of time the load acts is also important. To imitate actual field conditions, heavy weights and lengthy tests are required, both of which are expensive. Furthermore, the influence of static weights affects only the surface and does not penetrate into deeper strata, therefore substitutes and supplements for static tests have to include the time factor and consequently lead to dynamic investigations. Such dynamic investigations also affect

Manuscript received at the office of the Institute June 5, 1937. Issued as T.P. 834, January, 1938.

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the deeper strata. Dynamic testing is being introduced more and more in research work of all kinds of material.

The second standard soil tests consist of borings to investigate the formation; i.e., the succession of various beds. Borings are time-consuming and expensive; furthermore, important inclusions may be missed. With boring devices the natural position of the soil is often disturbed, which further distorts the findings.

Dynamic tests avoid these disadvantages, give closed profiles and leave the soil undisturbed.

Dynamic Method.—Actual loads are seldom entirely static. Street and railroad traffic cause dynamic loads. Even the foundations of buildings may be affected either by direct transmission of propagation waves or indirectly by vibrations of the whole structure. Hence the imitation of actual field conditions requires the introduction of dynamic loads.

On the other hand, the effect of insulating material for damping oscillations, as in the protection of structures against traffic vibrations, for example by trenches, can be tested only dynamically.

Finally, the research work for earthquakeproof structures such as houses, bridges and towers requires exciting forces having dynamic characteristics.

This new method takes into account the geophysical experiences of well-known *macro*seismic investigations. Dealing with relatively small areas, it may be called a *micro*seismic method. In the geophysical field, dynamic testing has been in use for a long time. Macroseismic investigations are often carried out with artificial vibration excited by blasts or shots, thus producing nonperiodic waves with rather complicated forms.[1] This microseismic method uses only vibration of periodic character having the form of a pure sine wave.

The dynamic qualities are measured—i.e., bedding value, natural frequency of the soil, wave length, propagation speed, reflection, interference, and resonance effects caused by these artificial vibrations. All these dynamic qualities give a characteristic picture of the soil. They make it possible to draw important conclusions on the bearing capacity, the composition and the compressibility of the soil, and in certain cases on the formation, succession or depth of various beds.

The new method has been used for several years in the United States[2] and Europe. Being still in the development stage, nothing definite can be told as yet of its possible importance in this particular field, or related engineering fields.

Already it has become a useful help for all kinds of geophysical soil-investigation problems, supplementing the old standard static test without, however, being able to replace it in all cases.

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[1] References are at the end of the paper.

TECHNIQUE

Instruments to Excite Vibrations.—The apparatus for exciting these alternating forces consists of a relatively small machine having two disks eccentrically supported. The disks are revolved in opposite directions by an electric motor of 1 to 2 hp. Changing the eccentricity of the disks will change the amount of the centrifugal forces from about 1 to 5000 lb. Changing the motor input will change the speed of revolution (hence the frequency of the exciting forces) from approximately 1 to 50 cycles per second. All horizontal forces are neutralized in the body of the machine by the inverse rotation of the two disks. Two external vertical forces alternating in a pure sine form remain (Fig. 1). To prevent any "jumping" of the machine, weights must be fixed to the body. The lower surface of the body can be increased by adding cover plates underneath.

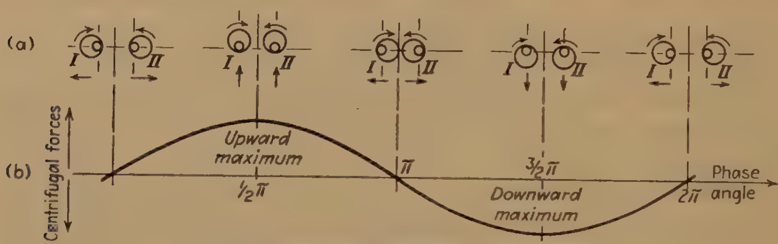


FIG. 1.—EXCITING ARTIFICIAL VIBRATION.
a. Position of the two revolving masses during one revolution.
b. Corresponding centrifugal forces, alternating in a sine form.

The direct current for the motor can be supplied by a small motor generator mounted on a truck. This truck transports the oscillator and all accessories.

The power input of the oscillator motor is measured by a standard wattmeter; the rotation speed—the frequency of the exciting forces—by a standard tachometer.

Instruments to Measure Vibrations.—To measure the amplitude of the excited vibration on various distances and depths of the soil and on the vibrator itself small standard seismographs may be used. These seismographs must have a low natural frequency and a high sensitivity. They may be used in connection with an oscillograph, so that up to 24 seismometer readings at various points are indicated simultaneously on one diagram. An accurate calibration curve of the seismometers indicating the correlation between amplitude and frequency has to be established. This can be easily checked from time to time on any vibration table.[3]

The propagation or phase speed of the forced oscillation can be determined by measuring the phase difference of two corresponding maxima or minima. The time required for the wave to move from the

oscillator to any point on the soil causes a phase difference in both sine curves plotted by the seismographs (Fig. 2). To check the frequency on the record any time mark on the diagrams can be used.

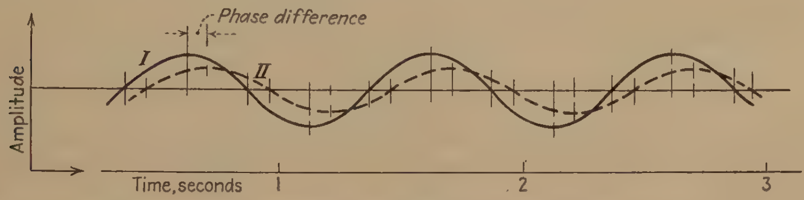


FIG. 2.—AMPLITUDE-TIME CURVES RECORDED BY TWO SEISMOGRAPHS AT VARIOUS DISTANCES FROM OSCILLATOR.

The seismograph recording curve II is farther away from the oscillator, indicating a difference in phase in relation to curve I.

The settling of the oscillator may be measured from time to time with a standard leveling instrument. The level should be set up at a point where no settling will be produced by the artificial vibration.

APPLICATION

General Field Tests

Field investigations are carried out in two ways:

1. First series: The standard oscillator excites vibrations, beginning with low frequencies and gradually increasing them. For each frequency the amplitude and the settling of the oscillator or the power input of the driving motor can be measured, so that so-called resonance curves can be plotted (frequency against amplitude, settling or power).

2. Second series: The oscillator vibrates with constant frequency. The amplitudes at various points on the field under investigation and at the oscillator are measured with seismographs. Oscillator and seismographs must be moved systematically all over the field as indicated in Fig. 3. All these tests can be carried out with varying eccentricities of the revolving disks. By moving the seismographs so that successive positions overlap, it is possible to plot closed profiles or contours with no danger of omitting salient points. Finally, the routine static field tests are carried out to determine the weight, water content, volume of the densest and loosest beds, grain sizes and consistency, and thus finish the actual field work.[4]

Investigation of Soil

Soil research may include the investigation of the quality (bearing capacity) or the formation (succession of beds).

Capacity of the Soil.—As already mentioned, the quality of the soil depends on its dynamic constants. Determining the natural frequency of the oscillator on the soil, i.e., the resonance point, one of these relations

will be indicated. A higher frequency shows a higher bearing capacity. For example, soft marshland is very elastic, has a low natural frequency and hence a poor bearing capacity; old sand is relatively rigid, corresponding to a high natural frequency and a good bearing capacity.

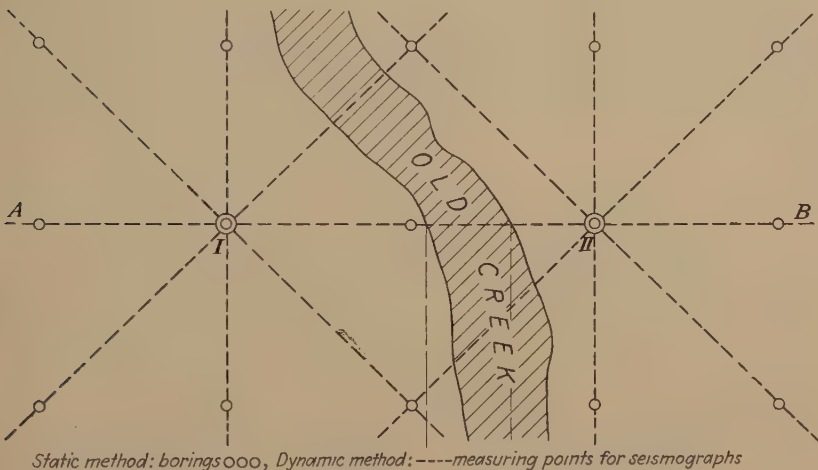
The phase speed shows a similar relation to the bearing capacity of the soil. A loose sand stratum has a low speed, a dense sandstone a high speed—indicating a bad or good bearing capacity, respectively. Table 1, the result of various tests, indicates this relation between propagation speed, natural frequency, and bearing capacity of different kinds of soils and rocks.

TABLE 1.—*Relation between Formation of Soil, Phase Speed, Natural Frequency and Allowable Soil Pressure*[13]

Test No	Formation of Soil	Phase Speed, Miles per Hr. (Frequency 20 to 25 Cycles per Sec.)	Natural Frequency, Cycles per Sec.	Allowable Soil Pressure, Lb. per Sq. In.
1	10 ft. marshland on sand	180	4.0	0
2	Very find sand	245	19.3	14
3	Tertiary clay, moist	290	21.8	
4	Clay sand	310	20.7	
5	Medium sand, moist	310	21.8	28
6	Jura clay, moist	335		
7	Old sand and cinder	355		
8	Medium sand in water	355		28
9	Medium sand, dry	355	22.0	28
10	Argillaceous sand	380	22.6	36
11	Gravel with stones	400	23.5	36
12	Clay, moist	420	23.5	
13	Marl boulder	420	23.8	42
14	Fine sand with 30 per cent medium sand	420	24.2	
15	Clay sand with lime inclusions	445	25.3	
16	Medium sand, undisturbed	490		57
17	Marl	490	25.7	57
18	Keuper sandstone, soft	560		
19	Diluvial loess, dry	580	23.5	
20	Gravel under 12 ft. sand cover	730		64
21	Gravel, dense	940	30.0	64
22	Sandstone, disintegrated	1120	32.0	⅙ of the allowable compression strength
23	Keuper sandstone, medium hardness	1450		
24	Sandstone, undisturbed	2450		

Formation of the Soil.—The measuring system is indicated in Fig. 3. The field under investigation might be planned over an old creek of marshy material not visible to the eye. Closed profiles are obtained measuring from point I to point II with the oscillator at I and from point II to point I with the oscillator at II. Hence the slightest lack of

homogeneity, easily missed with borings, will show up twice. The time-distance curve in Fig. 4 shows the result through cross section A-B of



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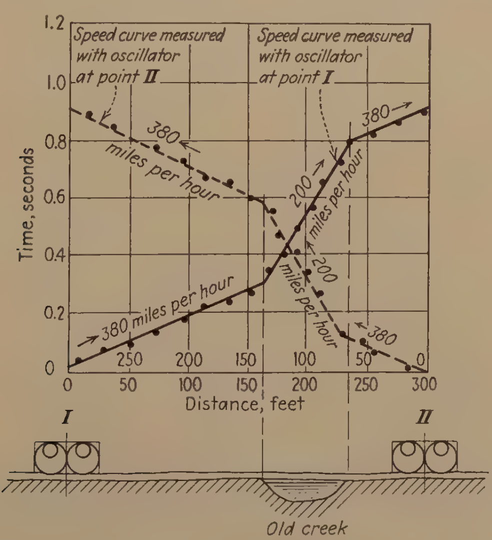

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FIG. 3.—COMPARISON OF SOIL INVESTIGATION WITH OSCILLATOR AND BORINGS. I and II indicate positions of oscillator. Dynamic method gives closed and overlapping profiles on dotted lines. With borings old creek might easily be missed.

FIG. 4.—SUBSOIL INVESTIGATION WITH PHASE SPEED CURVES ON STRATA INCLUDING AN OLD CREEK CORRESPONDING TO FIG. 3, SECTION AB.
Old creek is marshy and has low rigidity, causing a smaller phase speed.

Fig. 3. The change in speed from 380 to 200 and back to 380 miles per hour* indicates clearly the presence of the old creek. The natural

* Many speeds are so low that they can be compared with actual speeds of vehicles (airplanes, etc.), therefore the unit "miles per hour" was chosen for use in this work.

position of the soil is not disturbed nor is any damage done to the surface by this investigation.

Another important test is the determination of the succession of various beds. The length of the elastic waves depends on the frequency. Decreasing the frequency of the exciting vibration will increase the wave length, and will show the qualities of strata in greater depths. With a

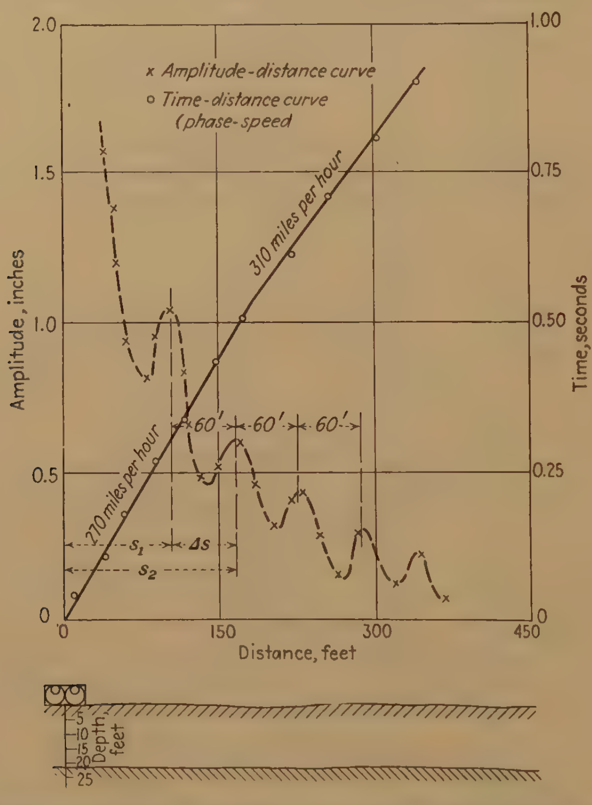

FIG. 5.—INVESTIGATION WITH AMPLITUDE-DISTANCE AND TIME-DISTANCE CURVES ON STRATIFIED SOIL.

Distance of maxima (60 ft.) and change in speed (270 to 310 miles per hr.) indicates second stratum.

frequency of 35 cycles per second strata may be investigated to a depth of about 10 ft., with a frequency of 15 cycles per second to a depth of approximately 60 ft. In Fig. 5 two speed curves are plotted; the speed curve of 270 miles per hour where the oscillator ran at high frequency indicates the upper stratum, the curve of 310 miles per hour at a lower frequency shows the second bed. This proves that the lower stratum with its higher speed is more rigid.

At the same time the existence of various strata appear in the amplitude-distance curves. Their presence may be noticed by observing the maximum and minimum points on the curves. These maxima or minima are the result of the previously mentioned interference effect and plotted likewise in Fig. 5. The distance of maxima or minima allows the determination of the depth of the bed with the help of simple formulas (see section on Theory later in the paper). In Figs. 6*a* and 6*b* the corresponding maxima are connected. Any slope of adjacent surfaces of beds (Fig. 6*c*) will distort these connection lines to approximately an ellipse (Fig. 6*b*).

In investigating these maxima it must be borne in mind that the curves indicate no resonance points; the interference maxima occur in general while exciting the soil at constant frequency (second test series), the resonance point with changing frequency (first test series).

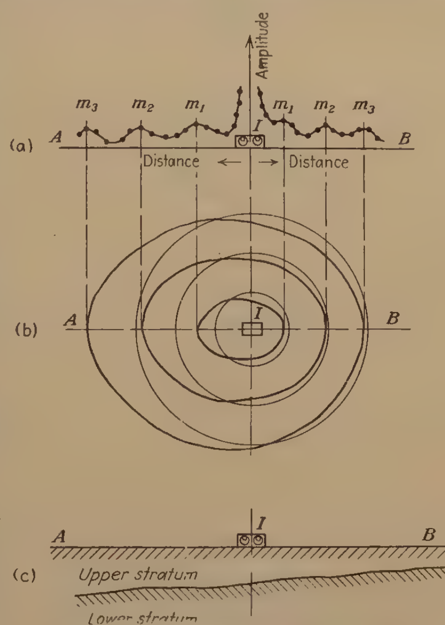

FIG. 6.—INVESTIGATION WITH AMPLITUDE-DISTANCE CURVES ON SOIL HAVING SECOND STRATUM WITH SLOPED UPPER SURFACE.

a. Amplitude-distance curves. Maxima are distorted.

b. Top view of connection lines of maximum amplitudes. Concentric circles on horizontal strata change to elliptic curves over sloped strata.

c. Cross section through strata at *AB*.

Improvement of the Soil.—Vibrating soil in its natural frequency has an effective densifying result.[5] A similar principle is often used to densify concrete mixtures. Friction between minute earth elements plays the most important part during compression. Ramming or rolling the soil affects only the surface, since the coefficient of starting friction is acting and prevents deeper penetration of the compression on account of some sort of self-locking effect. Vibrating the soil in its natural frequency suspends the minute particles in a kind of floating equilibrium. In this case sliding friction takes place, which is relatively smaller. The densifying effect with artificial vibration has reached strata more than 8 ft. deep.

The best check of any soil improvement consists in the measurement of the phase speed. The higher speed indicates greater rigidity. Fig. 7 demonstrates the difference in speed between a recent fill and cut, where the soil is still in its virgin condition.

The artificial densification of the soil is very effective in foundation work, where ground with insufficient bearing capacity is encountered. The allowable soil pressure can be raised, thus saving the expenses of a bigger superstructure.

While watching the settlement curves during the oscillation, conclusions can be drawn as to how the soil will behave under actual load conditions; i.e., under machines or traffic causing vibrations or impacts.

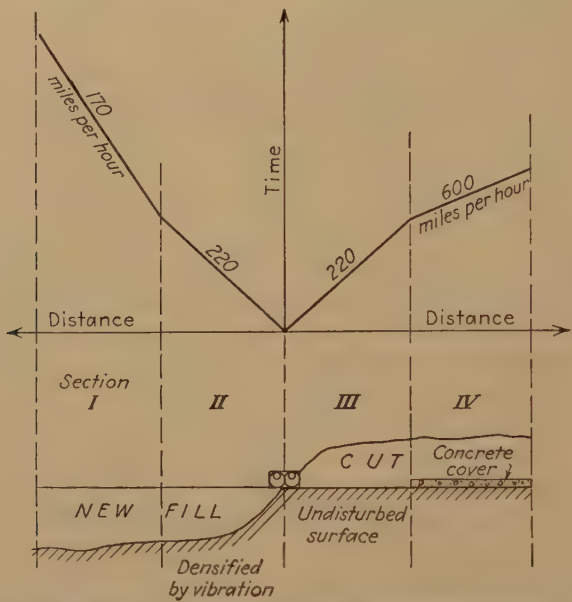

Fig. 7.—Investigation of new highway with phase-speed curves. Lowest speed indicates new fill (section I); artificial vibration densified this fill (section II) to same rigidity as undisturbed surface of cut (section III); concrete cover (section IV) improved surface again considerably. Speed ratio between various sections is 170:220 220:600 miles per hour.

Investigations on Railways and Highways

Densifying the soil in this state of suspended condition will prevent the various grains from touching and hence destroying each other. This is important when the ballast under railroad tracks has to be densified. Any destruction of the gravel would be an unfavorable effect. Vibrating a new highway fill to densify it is much more effective than any other method, especially for material with low binding qualities and low water ratio.

The required thickness of highway concrete is determinable by measuring the phase speed. Usually the concrete is more rigid than the surface, thus having a higher propagation speed. Any soil having a higher speed than the street cover is in itself strong enough and needs but a surface just thick enough to prevent penetration of loads and moisture. A big difference in speed between concrete surface and subsoil indicates

that both parts are vibrating separately, the adjacent surfaces acting as a slide plane. In this case the concrete must be thick and must be reinforced to prevent cracks through vibration. Fig. 7 (sections III and IV) shows the results of speed measurements on a highway surface with and without concrete. It proves clearly that the speed of 220 miles per hour without the cover is considerably smaller than the speed of 600 miles per hour with the concrete. Hence the effective gain in rigidity including the influence of various intermediate lower bases can be checked.

Investigation of Structures and Foundations

Bedding Value.—In each structure any resonance between critical speed of machines in the building and natural frequency of the building, the foundation or any other part of the structure, has to be strictly avoided. A decisive factor in determining the natural frequency of the structure on the soil is the dynamic bedding value of the soil, which is an elastic constant of the soil—i.e., the pressure per unit of area (pounds per square inch) required to cause an elastic vertical deflection of one inch. Hence the unit of the bedding value is pound per cubic inch. For the corresponding shear value, pressure and deflection of the load have to be measured horizontally. These values can be determined with the oscillator for various surfaces and different pressures and frequencies. The settlement curves mentioned above give the required values. Investigations have shown that with increasing area and pressure the dynamic bedding and shear values tend to reach a constant maximum (Table 4). Investigations of static bedding values showed increasing values corresponding to the enlargement of area.

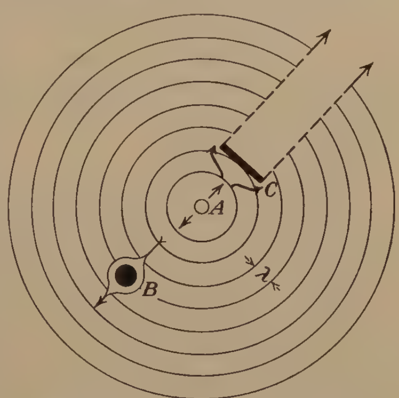

FIG. 8.—SCHEME OF SCREENING EFFECT FOR SMALL AND LARGE OBSTACLES ON WATER SURFACE WITH WAVES EXCITED BY FALLING STONE.

Waves excited at A will pass obstacle (small stick) at B nearly undisturbed, being smaller than wave length (diffraction effect). Obstacle C (flat board), however, being several times larger than wave length λ will protect area behind C effectively.

Insulation against Vibration Caused by Traffic and Earthquake Shocks.—The imitation of traffic or earthquake shocks[2] by artificial vibrations offers another possibility for studying the effect of oscillation not only in the soil and the highway surface but also in the building itself. The efficiency of insulating material can be investigated. One of the test results is the small insulating effect of trenches around the building as a protection against surface waves. The wave length of the natural vibration of the soil usually is several units greater than the width or depth of any

trench used as insulation, this being the reason why the screening effect is generally very small. In Fig. 8, the relation between wave length and protection screen is schematically demonstrated; for example, surface waves caused by a stone striking water will be diffracted around a stick that is small in width compared with the wave length. A larger obstacle —for instance, a wide, flat board—will protect the area behind it very effectively.

Results

Bearing Capacity and Dynamic Constants for Various Soils.—The most important results of various soil investigations are presented in Table 1.

A relation between formation of the soil, phase speed, natural frequency and allowable soil pressure is established. It is obvious, from the facts mentioned above, that with increasing speed the natural frequency as well as the soil capacity rises. Hence a fundamental scale for various soils is formed to predetermine the bearing capacity from dynamic constants.

Densifying Soil by Various Methods.—A second result is represented in Table 2, which indicates the effect of improvement of dams in various depths from 4 to 47 in. by different methods. In Table 2, group 1

TABLE 2.—*Relations between Various Methods of Soil Densifications*[6]

Group No.	Method of Densification	Densifying Number p at Depth of						
		4 In.	12 In.	20 In.	24 In.	31 In.	40 In.	47 In.
1	Not densified	43		25		43		47
2	Ram (middle)	52		67				
3	Ram (border)	75		81				
4	Oscillator (speed, 160 ft. per hr.)	62		64		61		80
5	Oscillator (5 min. at the same spot)	91	71		79	74	77	68

$$p = \text{densifying number} = \frac{n_0 - n}{n_0 - n_d} \times 100.$$

n = pore volume of test specimen.
n_0 = pore volume of loosest strata.
n_d = pore volume of densest strata.

indicates the densifying number for the undisturbed dam; groups 2 and 3, the effect of ramming, and groups 4 and 5, the effect of artificial vibration. No doubt remains that the effect of artificial vibration is the strongest, being fairly uniform and reaching the deepest strata of any of the four groups.

Required Thickness of Highway Surface.—Table 3 shows the relation between phase speed and street surfaces of various thicknesses. It is

TABLE 3.—*Relation between Road Surface and Phase Speed*[6]

Frequency of Oscillator, 40 cycles per second

Thickness of Concrete Plate, In.	Phase Speed, Miles per Hr.
6	460
8	540
10	620
Maximum theoretical speed in concrete...........	4500–6500

obvious that the thickest concrete plate must also show the highest speed (the theoretical speed in a uniform concrete block of infinite dimensions is about 5000 miles per hour). No improvement of the street surface can be expected if the phase speed of the bed without cover is already equal to the speed of the cover plate itself. In this case much expense for superfluous and ineffective reinforcement can be saved.

Dynamic and Static Bedding Value.—Table 4 shows the relation

TABLE 4.—*Relation between Dynamic Bedding Value and Static Pressure*[7]

Static pressure, lb. per sq. in..................	7	14	21	28	35	42
Dynamic bedding value, lb. per cu. in..........	730	1090	1270	1270	1270	1270

between dynamic bedding value and static pressure. With increasing pressure the dynamic bedding value tends to become uniform. Thus the conclusion may be drawn that for foundations covering a large area combined with higher static pressure the dynamic bedding value will tend to have a constant maximum, which will be approximately 1270 lb. per cu. in. Tests with static loads did not reveal such a tendency towards a constant static bedding value.

Further Development of Dynamic Method.—The new method of artificial soil vibration is still in development. There are and will remain always certain cases where the routine static methods have to be used. A comparison between the X-ray image of the surgeon and the picture of the soil obtained by the artificial vibration method might be used; only if the image of the X-ray exposure or the curves of the dynamic investigation indicate that something is wrong in the interior, the surgeon might take his knife and the borer his tools to make further investigations.

THEORY

Elaborate theoretical treatises are cited in the bibliography at the end of this paper, listed as to the time of their appearance.[8–13] In the following paragraphs an abstract is given.

Principle

The fundamental principle is very simple. The soil under investigation is loaded at various places by alternating forces having a sine form.

Frequency and size of these forces can be changed. Hence the soil must vibrate with forced and damped oscillations.

The oscillator for exciting the artificial vibrations consists of two disks, supported eccentrically and revolving in opposite direction (Figs. 9 and 1). All horizontal forces are neutralized. Two vertical forces alternating in a sine form remain. Their frequency depends on the speed and their size on the eccentricity of the two disks.

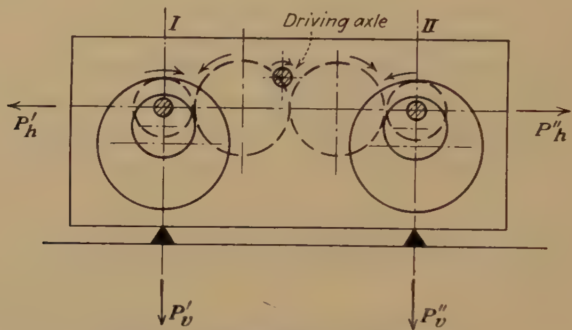

FIG. 9.—SCHEME OF OSCILLATOR.

Two masses I and II revolve in opposite directions. Horizontal forces P_h' and P_h'' neutralize each other. Two vertical forces P_v' and P_v'' alternate in a sine form. (See also Figs. 1 and 2.)

Natural Frequency of Oscillator on Soil

The relation between frequency and size of the exciting force on one side and the dynamic quality of the soil on the other must be established. Dynamic qualities of the soil can be determined by the following measurements:

1. Energy absorbed in the soil by damped vibration.
2. Amplitude of the oscillator on the soil.
3. Amplitudes of the soil on the surface at various depths and at varying distances from the oscillator.
4. Phase difference between exciting forces of the oscillator and excited vibration on or in the soil.
5. Settlement of the oscillator on the soil.

The whole vibrating system is extremely complicated. The question as to whether compression or longitudinal waves, transverse or surface waves prevail will not be considered. To simplify theoretical conditions, the oscillator may be assumed to be an elementary mass vibrating on an infinite elastic surface. The internal damping of the soil depends in general on the friction. The assumption of a damping effect (δ) proportional to speed is, however, accurate enough. The deformation of the soil may be assumed to be linear with load and settlement (law of Hoock). Furthermore, only vertical forces are considered effective.

The following three internal forces and one external force must be in equilibrium:

1. The force of inertia $= m \cdot \dfrac{d^2x}{dt^2}$

where m = vibrating mass, x = amplitude of vibration and t = time.

2. The damping force $= K \cdot \dfrac{dx}{dt}$

where K = coefficient of friction (proportional to speed).

3. The elastic force $= cx$

where c = elastic constant; i.e., the force in pounds required to deflect the system one inch.

4. The centrifugal force $P \sin \omega t$

where P = external force and ω = frequency of P.

Hence the following fundamental dynamic equation is valid:

$$m\frac{d^2x}{dt^2} + K\frac{dx}{dt} + cx = P \sin \omega t \qquad [1]$$

The solution of this differential equation is thoroughly explained in the literature indicated in the bibliography.[8,12,13]

The greatest amplitude (a) will be:

$$a = \frac{P}{m\sqrt{(\omega_0{}^2 - \omega^2)^2 + 4\delta\omega^2}} \qquad [2]$$

where ω_0 = natural frequency
and δ = damping.

The phase difference φ between exciting force and excited amplitude:

$$\tan \varphi = \frac{2\delta\omega}{\omega_0{}^2 - \omega^2} \qquad [3]^8$$

Plotting on the Y axis the amplitudes and on the X axis the frequency (amplitude-frequency curves), the maximum (the highest amplitude) will indicate, at least for smaller damping values, the so-called resonance point (Fig. 10*a*). These resonance curves determine certain elastic constants of the soil; for example, the natural frequency of the oscillator on the soil including the damping capacity of the soil beneath.

Plotting the settlement of the oscillator, the settlement-frequency curves have a similar characteristic (Fig. 10*b*). The maximum increase in settlement indicates the resonance point. At this resonance point the minute particles of the soil are in some kind of suspended equilibrium and do not touch each other. In this state the soil will settle in the densest possible formation, therefore this method can be used to densify certain soils effectively with the smallest energy input. Rolling and ramming is less effective and requires more energy.

Natural Frequency of Strata

The theoretical considerations described above cover only the natural frequency (resonance) of the exciting force on the soil. This depends on the mass and the size of the lower surface of the oscillator and its

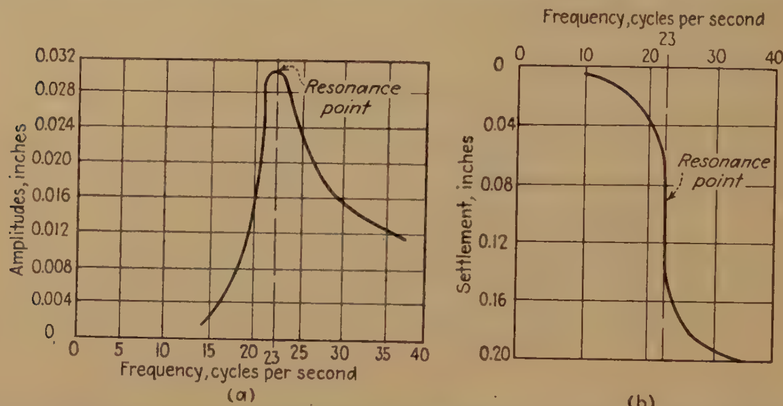

FIG. 10.—AMPLITUDE AND SETTLEMENT IN RELATION TO FREQUENCY OF EXCITING FORCE.

a. Amplitude-frequency curve. Highest amplitude indicates resonance point at 23 cycles per second.

b. Settlement-frequency curve. Greatest increase in settlement occurs while passing resonance point at 23 cycles per second.

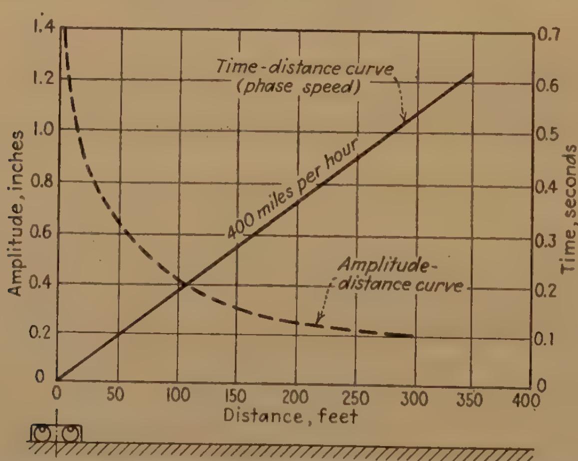

FIG. 11.—THEORETICAL AMPLITUDE-DISTANCE AND TIME-DISTANCE CURVES ON COMPLETELY HOMOGENEOUS SOIL FOR CONSTANT FREQUENCY.

Amplitudes indicate no maximum; phase-speed curve is a straight line.

centrifugal forces, and finally on the elastic constants of the soil. In other words, the "vibrating mass" includes, besides the oscillator itself, a relatively small amount of the soil, which is vibrated directly under the machine and has approximately the well-known form of a more or less

complete circle (onion shape). The elastic medium is formed by the vibrating soil, therefore depends on the elastic qualities of the soil.

A second form of vibrations consists of the oscillations of the soil, which are independent of the quality of the exciting forces. Elastic waves in the soil assume certain borders, which are determined by the various strata in the ground. The main difference between a maximum caused by the resonance effect and a maximum caused by interference of these elastic waves is the following: Resonance maxima can be excited only through periodic vibration (sine form), elastic wave maxima also occur with nonperiodic vibration excited through blasts, shots or earthquake shocks.

All elastic waves must have a characteristic propagation speed, which depends first of all on the modulus of shear of the soil. Hence the propagation speed can also be used to determine certain soil qualities.

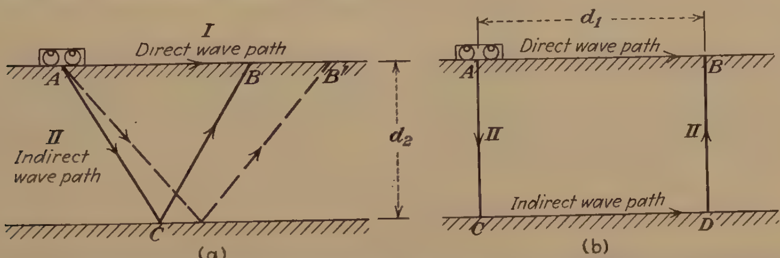

FIG. 12.—SCHEME OF DIRECT AND INDIRECT PROPAGATION WAVES FROM OSCILLATOR A TO MEASURING POINT B, B', ET CETÉRA.

a. Assuming that indirect wave is reflected at upper surface of lower bed in C, C' etc.

b. Assuming that indirect wave drops down vertically to lower bed C, travels along this surface to D and returns vertically to measuring point B. This theoretical assumption gives best results as checked by borings.

Plotting on the X axis the distance from the exciting force A to various points B on the soil and on the Y axis, the increasing phase difference between exciting and excited vibration (Fig. 11), any time-distance line indicates the propagation speed of the wave between A and B. Only on a completely homogeneous soil will this theoretical speed curve become a continuous straight line through A (Fig. 11).

A similar continuous curve can be plotted when, instead of the phase speed, the decreasing amplitudes at various points B are marked on the Y axis. In Fig. 11, a theoretical amplitude-distance curve on a completely homogeneous soil is also indicated. Any discontinuity in these two curves will show that something is included in the soil that is not homogeneous, thus forming another means of determinating certain soil qualities.

The propagation waves extend theoretically with some type of spherical form. In Fig. 12, a point B at a certain distance d_1 from the oscillator in A is assumed, both points being situated on the ground

surface. Some waves will propagate on the surface of the earth directly from A to B, some will penetrate below the ground and be reflected from the surface of the lower bed (at a depth d_2). The surface wave travels a shorter distance from the exciting force A to the assumed point B than any reflected wave (ACB or $ACDB$). The direct and the indirect wave, having both a sine form, cause an interference effect. Hence characteristic interference maxima and minima of the vibration amplitude must result when varying the distance AB. The distance of these maxima or minima depends on the difference in length of both paths, which is further dependent on the depth and the slope of the strata.

In general the time t is equal to distance d, divided by the propagation speed v:

$$t = \frac{d}{v} \tag{4}$$

Assuming a horizontal upper and lower surface of the first bed, a constant frequency of the oscillator, and the indirect path ($ACDB$), as in Fig. 12*b*, and neglecting any phase discontinuity, the difference in time (Δt) between the two paths will become:

$$\Delta t = \frac{2d_2}{v_1} + \frac{d_1}{v_2} - \frac{d_1}{v_1} \tag{5}$$

When d_1 is the distance AB,
d_2, the distance $AC = DB$,
v_1, the phase speed in the upper strata,
v_2, the phase speed in the lower strata.

The assumption of the indirect path $ACDB$ as shown in Fig. 12*b* gives the best coincidence when compared with field borings to check the results. No attempt is made to give a theoretical explanation for this phenomenon. If r is the number of order, 0-1-2-3-4 . . . , equation 5 changes in case of a maximum to:

$$2\pi r = \left(\frac{2d_2}{v_1} + \frac{d_1}{v_2} - \frac{d_1}{v_1}\right)2\pi n \tag{6}$$

where n is the frequency of the oscillator. Wherever the two waves (direct and indirect) arrive at the same time a maximum will result, and for $r = 0$ equation 6 changes into:

$$\frac{2d_2}{v_1} = d_1\left(\frac{1}{v_1} - \frac{1}{v_2}\right) \tag{7}$$

d_1, v_1 and v_2 can be measured. Thus d_2, the required depth, can be determined for this special case.

For a constant frequency n and the distance of two adjacent maxima $\Delta s = s_2 - s_1$ (see Fig. 5), formula 7 changes to:

$$\frac{1}{v_1} - \frac{1}{v_2} = \frac{1}{n\Delta s} \tag{8}$$

If one speed is known, the other can be checked, independently of the distance of the oscillator.

With these eight fundamental formulas the theoretical principle is outlined in general. An elaborate mathematical treatment is presented by A. Ramspeck,[12] dealing with the interference of elastic waves in the subsoil only.

Summary

The method of artificial vibration has been used successfully for the following soil investigations:

1. Qualification of soil for building purposes, highways or railways.
2. Dynamic constants (bedding value, natural frequency, damping) for foundations, especially when loaded with machines causing vibrations.
3. Bearing capacity and depth of upper and lower beds.
4. Densifying of soil, especially new fills.
5. Required thickness of concrete highways.
6. Checking all kinds of insulations against traffic vibrations or earthquake shocks.

The main advantages of the new method are:

1. Undisturbed soil, excluding any damage on the surface, even for investigations to considerable depth.
2. Continuous profiles forgoing the disadvantages of inadequate and expensive borings.

In several practical cases the dynamic investigation resulted in saving a considerable amount of expense. One striking actual example might be mentioned. A heavily loaded machine shop, with an area of about 15,000 sq. ft., had to be built on a piling. Densifying the soil by artificial vibration for about 10 in. at an expense of approximately 2¢ to 4¢ per square foot improved the soil so much that the pile foundation could be avoided. The required excavation of 10 in., a side product of the artificial vibration method, would have been more expensive than the densifying job has been. Hence the whole costs for piling, an amount of approximately $300,000, was saved.

This makes it quite evident that the dynamic soil-testing method is not only a theoretical or a laboratory job, but can be applied with success in actual field work.

Credit is to be given to the Degebo and Seismos, both in Berlin, Germany, from which most of the data have come.

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DISCUSSION

(*J. J. Jakosky presiding*)

L. B. Slichter,* Cambridge, Mass.—Dr. Bernhard has introduced a new concept or a new property of material, which he defines as the dynamic bedding concept. This he defines in terms of deflection per unit load; that is, in terms of deflection and pounds per square inch.

The formula for the deflection of a homogeneous elastic ground, caused by the application of an oscillatory pressure on a circular area on its surface, shows that the deflection is determined not only by the load per square inch but by the frequency of the vibration and by the diameter of the area on which the pressure is applied. Thus Dr. Bernhard's definition omits two essential quantities that enter the picture; that is, the frequency of vibration and the size of the disk over which the load is applied. Both are needed.

To explain my point, let us consider the simplest possible case of a homogeneous elastic material. In that case, the deflections depend upon the frequency, the unit pressure and the diameter of the disk that applies the load. One must obviously so formulate the definition of the "dynamic bedding" property that the definition will be valid and rational in this simplest possible case. The definition adopted by the author (p. 335), "the pressure per unit of area required to cause an elastic deflection of one inch," is thoroughly unsatisfactory. It is certainly necessary also to specify both the diameter of the disk and the frequency; otherwise the definition is ambiguous.

R. K. Bernhard.—The dynamic bedding value depends upon the frequency, the static pressure and the area over which this pressure is applied. However, experiments have shown that for larger contact areas and constant soil pressure the dynamic

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bedding value has the tendency to approach asymptotically a limiting value. These results are demonstrated in Table 4 and Fig. 13, and indicate a pronounced difference between static and dynamic bedding values.

Hence, for larger contact surfaces the dynamic bedding value no longer depends on the contact area and becomes a definite function of the static pressure only. Consequently, the dynamic bedding value represents to a certain extent a substitute of the well-known elastic constant (spring constant) of the vibrating system. The equation for the dynamic bedding value (C) is:

1. For vertical excitation:

$$C_v = 4\pi^2\omega_v^2 \frac{p}{g} \text{ (lb-in.}^{-3}\text{)}$$

where ω_v = vertical natural frequency of the oscillator on the soil (sec.$^{-1}$),
p = static soil pressure (lb-in.$^{-2}$), and
g = gravity acceleration (in-sec.$^{-2}$).

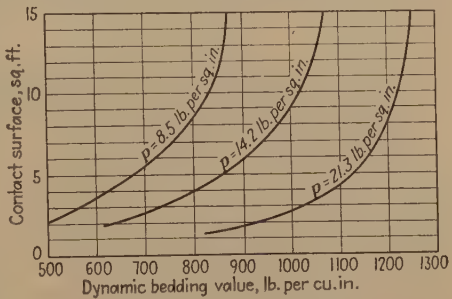

FIG. 13.—RELATION BETWEEN DYNAMIC BEDDING VALUE, CONTACT SURFACE AND STATIC SOIL PRESSURE (p).

2. For torsional excitation (dynamic shear value):

$$C_t = \frac{4\pi^2\omega_t^2 T_p^{\text{osc. mass}}}{T_p^{\text{area}}} \text{ (lb-in.}^{-3}\text{)},$$

where ω_t = torsional natural frequency of the oscillator on the soil (sec.$^{-1}$),
$T_p^{\text{osc. mass}}$ = moment of inertia of mass of the oscillator with respect to its vertical axis (lb-sec.^{2}-in.), and
T_p^{area} = polar moment of inertia of contact area (in.^{4}).

Horizontal torsional vibrations can be excited, by giving the eccentric masses of the oscillator an initial phase difference of 180°. Furthermore, both rotating axes must be vertical and rotate in the same sense.

The two formulas are derived in detail by H. Lorenz[14] and thoroughly illustrated by several practical numerical examples.

No doubt can remain that the frequency, the static pressure, and the area enter the equation. In other words, the dynamic bedding value represents a definitely determined unit and cannot be considered as ambiguous.

J. J. JAKOSKY,* Los Angeles, Calif.—I wonder if that is not a fact that cannot be mathematically stated. There is a series of variables in the ground. I think Dr. Bernhard has grouped under one term of the natural frequency, a sort of heading to carry everything.

R. K. BERNHARD.—As often in new engineering problems, the theory lags behind the practical application, and as far as I know no mathematical theory has been published as yet for some of the phenomena mentioned above. However, a physical explanation might be offered. For *static* tests it must be kept in mind that the contact area increases with the second power of length, the effective soil volume with the third power of length; since the settling increases in proportion to the effective soil volume, the settling also increases with the third power

[14] H. Lorenz: New Results on Dynamic Soil Research. *Ver deut. Ing.* (March 24, 1934) **78** (12), 384.

* President, International Geophysics, Inc.

For *dynamic* tests, however, the effective soil volume is almost independent of the static pre-stress, which represents only a relatively small part of the total load (static and dynamic). The vibrating soil ellipsoid (Fig. 14), which is a multiple of the oscillator mass, predominates and hence the contact area becomes of secondary importance.

The investigations have been grouped under two different headings; the first series of investigations uses gradually increasing frequencies, starting with slow and ending with higher frequencies and vice versa, in order to be able to plot resonance or settling curves. The second series of investigations applies constant frequencies only, in order to measure propagation velocities.

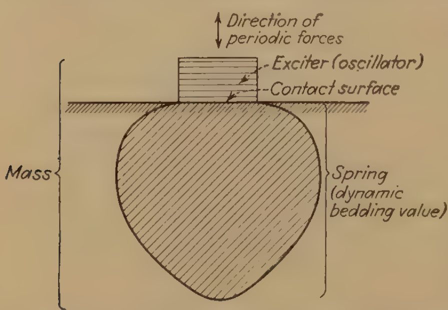

Fig. 14.—Dynamic substitute for standing vibrations of exciter on soil.

Member.—How did you know that you were measuring the corresponding phases? I mean, you have one seismograph at one point and another at another point and you get two sine curves for each. How did you know the one you measured had the peak of the second?

J. J. Jakosky.—Your wave length is also so long that the position of your seismograph would not allow you to be out of phase?

R. K. Bernhard.—Wave lengths from 9 to 100 ft. induced in various soils have been reported so far. All measurements with seismographs can be started close to the exciting force (oscillator); then the pickup units (seismometers) are moved systematically in steps of a few feet only away from the oscillator. A close check of any phase change is possible by comparing the corresponding maxima, minima, or zero points of the sine curves recorded by two adjacent pickup units.

Furthermore, a more complete outfit, say of six or more pickup units, connected with one recording oscillograph will facilitate substantially the simultaneous determination of the continuous phase increase at subsequent points.

Member.—Does your phase permit you to work on more than two-layer problems?

R. K. Bernhard.—Deeper strata can be determined by vibrations with lower frequency (slower rotational motion of the oscillator), thus increasing the wave lengths and reaching soil at greater depths, and consequently hitting more than two layers.

I. B. Crosby,* Boston, Mass.—I will confine my remarks to the use of this method in foundation studies, where I see that it may have some very interesting applications. The example of locating a buried creek bed filled with soft material might be helpful because lines of borings might miss such a feature.

There are, however, some claims made for this method that I think are exaggerated and misleading. I would like to call attention to Table 1, the first column of which gives allowable soil pressures in pounds per square inch and correlates these with the phase speed and natural frequency. In the text the allowable soil pressure is described as bearing capacity. This table is misleading and is of doubtful value in any case. The relations between dynamic characteristics and static strength are not proved in this paper, and such relations as are known from other sources cast doubt upon the claims made here. There are serious practical restrictions to the use of this table or similar dynamic studies.

* Consulting Geologist.

For example, the allowable soil pressure in sand depends, among other things, upon the way the foundation is constructed. It will be one value if the foundation is on top of the sand, but will be very much higher if the foundation is set deeply into the sand. The bearing value of sands is also dependent upon ground-water conditions that will exist in the sand when the structure has been completed. Ground-water movement in the sand may greatly reduce the bearing value, and this cannot be determined by any dynamic method.

This method cannot be applied to clay or silt because there is no true bearing value or allowable pressure for clay. Although bearing values for clay were not given in this table, the inclusion of clays there gives the impression that the method is applicable to them. A load on clay will cause settlement lasting over a period of years. This settlement is caused by slow squeezing out of the water in the pores of the clay. The amount of settlement will depend on the porosity and thickness of the clay, and the rate of settlement will depend on the permeability, its thickness, and other factors. The allowable bearing value will depend upon the amount and rate of settlement that are permissible for the structure in question. With clays and silts, static and not dynamic factors control the settlement of a load on the soil.

R. K. Bernhard.—The experimental data of the three values—wave velocity, natural frequency and allowable soil pressure—that are compared in Table 1 have been determined by dynamic and static methods simultaneously on the same soil and the same spot, respectively, and, furthermore, in exactly the same depth, where the foundations have been planned.

Since a rigid soil has in general a higher allowable bearing capacity than a soft soil (result of static tests), similarly a dense soil will yield a higher propagation velocity and higher natural frequency (result of dynamic tests).

Soils containing water in larger quantities have been investigated by the dynamic method to determine the propagation velocity only, hence are included in Table 1 to demonstrate the gradual increase of propagation velocity. The exclusion of certain values for the allowable soil pressure—for example, of clay—shows, however, that no conclusions have been drawn in this respect. Hence, the soil density is obviously a predominant factor for all three qualities and the correlation between these two dynamic and one static unit seems logical. However, no doubt can remain that neither a short static nor a short dynamic test will yield results if a considerable time (years) is required to squeeze water out of pores in soils with larger water content.

L. W. Blau,* Houston, Tex.—I have difficulty with this resonance frequency. I must think of a vibrating system when I describe a frequency. What is the system that vibrates here?

R. K. Bernhard.—The natural frequency of the vibrating system—i.e., its mass and its spring constant—may be explained in the following way: Both oscillator and underlying soil, the latter having an onion-shaped form (ellipsoid, Fig. 14), represent the mass of the vibrating system. The elastic resistance between the moving soil particles must be considered as the origin of the spring constant in the vibrating system. The amount of moving soil particles depends certainly on the contact area between oscillator and soil. Hence, natural frequencies of such a system can be compared only when determined with standardized oscillators having a contact area of the same size and the same shape.

L. W. Blau.—What is the diameter of this ellipsoid?

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* Director Geophysical Research, Humble Oil and Refining Co.

R. K. Bernhard.—As far as I know, no results with respect to the diameter of the vibrating soil ellipsoid have been published as yet. However, the maximum weight of the vibrating mass was approximately 40 T.

L. W. Blau.—You cannot have resonance unless you have a reflection of energy inside some certain continuum. Now, therefore, we will not talk about how little amplitude there is outside—there is some—but, at the resonance frequency, there is a greater difference, we will say, between inside and outside than there is at the other frequencies. I still must think of something that wobbles when I think of a frequency.

R. K. Bernhard.—Measurements have shown that the oscillator is vibrating (wobbling) on the underlying soil ellipsoid, either in vertical or horizontal direction, corresponding to the direction of the exciting forces, with its maximum displacement at the resonance peak.

M. K. Hubbfrt,* New York, N. Y.—I have been interested in the application of this sort of technique to earthquake-prevention studies. It is well known that the greatest destructiveness of earthquakes occurs on unconsolidated ground. Vibration studies of this sort give a resonant frequency of the ground. At the same time the resonant frequency of any oscillator diminishes as its mass increases. Considering the fact that an earthquake oscillates a larger and more massive block of soil than that described here, is it safe to assume that the resonant frequency of the ground would be the same in the two cases? Also, would not the resonant frequency of the ground be different before and after loading with a large building?

R. K. Bernhard.—Experiments revealed that buildings, including the vibrating soil under their foundations, have a pronounced natural frequency. This natural frequency might be excited by earthquakes as well as man-made earthquakes; i.e., by oscillators. Hence, both types of excitements, nonperiodic (natural) as well as periodic (artificial) can lead to serious damage of the structures in case earthquake shocks or induced vibrations coincide with the natural frequency of the building.

Let us assume that the oscillator is so close to the building that the moving soil particles of the ellipsoid are hitting the building foundation directly. Furthermore, the building may have the same natural frequency as the exciter. Under these conditions large amplitudes might be induced.

A. T. Ireland,† College Park, Md.—You spoke about densifying the soil by that method. I wonder if you noticed any change in the natural frequency after the soil had been densified by your method?

R. K. Bernhard.—The increase in propagation velocity is the best indication of an effective soil densification, as illustrated in Fig. 7. As far as I know this represents the only way to determine whether a densification had any result whatsoever without destroying the soil.

A. T. Ireland.—You found a certain resonant frequency, which of course, would be a resonant frequency for a certain type of wave, but you have not definitely determined which kind you are using, although probably it is the Rayleigh surface wave. Have you made any test to determine whether other types of waves would have different resonant frequencies?

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* Department of Geology, Columbia University.

† U. S. Bureau of Mines.

R. K. Bernhard.—Which type of propagation waves are excited has not yet been definitely established. However, a comparison with the blast method on exactly the same soil and same spot revealed a difference of strata depths of 4 per cent only.

E. R. Shepard,* Washington, D. C.—The speeds you measured, which you call phase speeds, apparently are only about one-third the speeds often measured by the shock method.

R. K. Bernhard.—The propagation velocity as measured by E. R. Shepard on the same soil and at the same spot with the blast method yielded 1500 ft. per second; the propagation velocity as determined by the oscillator method, 1100 ft. per second. Two tentative explanations might be given for this deviation of 33 per cent. The change from upper to lower strata (from silt-loam to rock) was rather gradual. Hence, the blast method may indicate the higher velocity in a greater depth. Or, if different types of waves are excited by either method this might be the reason for the obvious difference in velocity.

L. W. Blau.—Near the surface, the increase of velocity with depth is very great. We have measured speeds as low as 275 ft. per second at 6 in., so that agrees very well with the speeds you have measured. As a matter of fact, it is lower. I have some difficulty in thinking miles per hour unless an automobile is concerned. We always measure in feet or meters per second and I have continually to divide back or multiply just to find out what this means.

R. K. Bernhard.—The presentation of propagation velocities in miles per hour, as shown in Table 1, was chosen only to facilitate a comparison of these velocities with the more familiar speed of vehicles.

The criticisms offered by the various discussers are welcome. A close cooperation with soil specialists as well as with geophysicists is of the utmost importance. The relatively new method of induced sinusoidal vibrations is far from being complete, and needs much additional development.

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* U. S. Bureau of Public Roads; more recently, War Department.

Geophysical Investigations Concerning the Seismic Resistance of Earth Dams

By C. A. Heiland,* Member A.I.M.E.

(New York Meeting, February, 1939)

Geophysical methods are playing an ever increasing part in various engineering fields. About ten years ago, geophysical exploration was first applied in civil engineering to the study of foundations and the location of construction materials and water. Its usefulness is now being extended into the field of structural engineering.

Almost every structure has to be so designed that it will withstand any force to which it may be subjected. Such forces may result from its own weight, applied loads, pressures of various kinds, including those due to wind and water, ground subsidence and vibrations. The last two, *problems involving the nature and dynamic behavior of surface and subsurface formations, are within the scope of geophysical investigations.*

In the design of any structure, the disposition and dimensions of its members as well as their materials must be so selected that they will withstand any or all of the forces enumerated above. In other words, its "resistance" to such forces must be sufficient, allowing for a definite margin of safety. Often a static treatment of the problem leads to satisfactory results, but the static method entails serious errors when vibrations are involved, as in resonance with an impressed vibration the displacements of a structure or parts thereof may dangerously exceed the allowable values. The principle of "dynamic design"† is, therefore, avoiding or reducing resonance by suitable selection of the natural frequency of a structure, or by the provision of damping.

Methods of Investigation

Dynamic design of structures is required where ground vibrations occur naturally or are produced artificially. Earthquake regions fall in the first group; areas of industrial activity where vibrations are produced by machinery, blasting, etc., fall in the second. In either case, dynamic

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Manuscript received at the office of the Institute Nov. 29, 1938. Issued as T.P. 1054, February, 1939.

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† Sometimes referred to as "aseismic" design.

design requires two fundamental sets of data: (1) the dynamic characteristics of the *ground;* (2) the dynamic characteristics of a proposed *structure.*

Various methods are available for the determination of dynamic characteristics of ground and structure. They may be divided into two groups: (1) measurements of *free* vibrations produced by individual (controllable or noncontrollable) impulses; (2) measurements of *forced* vibrations produced by sustained periodic vibrations of variable frequency or by transients. The first group of measurements requires vibration detectors set up on the ground or structure to be tested; i.e., seismographic recording methods are applied with little, if any, attention to, or control of, the source of vibration; the records furnish natural frequency and damping. In the second group, detectors are again required, but vibrations of controllable frequency and amplitude are produced in the ground or structure to be investigated, generally by unbalanced flywheel machines or vibrators. The records thus obtained furnish a complete response curve from which not only natural frequency and damping but also velocities of propagation may be determined.

Although involving forced oscillations to some extent, the first procedure is herein referred to as the "free vibration" method. Its application to the determination of ground frequency date from the beginning of seismic observatory recording; the theoretical analysis of frequency characteristics of machines and engineering structures probably began with Stokes' investigations of vibrations and failure of railway bridges in 1849. However, systematic tests of machine vibrations were not undertaken until 1890. The second group of procedures in which the response characteristics of ground or structures are measured may be termed the "forced vibration" or "resonance" method. Its origin may be traced to the fatigue tests of metals, the earliest probably having been made by Jones and Galton in 1849. It was not until 1927 that unbalanced flywheel machines were constructed and applied to tests of completed structures; in 1932, vibrator methods were first used for soil testing.

The experimental investigation of completed structures by free and forced-vibration methods has furnished invaluable data for the design of new structures. It would obviously be desirable that such tests be made before the final design of the structure is decided upon. The only practical way of doing this is by models. As natural frequency and damping of a model structure, like that of the prototype, depend on the elastic and mechanical properties of the materials used, it is necessary to determine them first. For this a dynamic (resonance) and not a static method should be used. Model experiments to determine natural frequencies of engineering structures are of comparatively recent date. When the elastic properties of the materials to be used in a structure are known it is possible, for simple geometric arrangements, to calculate its natural frequency.

The question next arises how to determine the frequency of the ground to which a structure is coupled. The theoretical approach is rather difficult; calculations cannot be attempted without knowing the elastic properties and thicknesses of the formations involved. As, therefore, seismic determinations of these would have to precede such calculations in any event, it is easier to solve the problem by the two methods mentioned above. In the free vibration method, predominant ground frequencies are measured by vibration detectors or seismographs. It has been found by experience that the prevalent frequencies are more or less the same, regardless of method of excitation. Hence, for a given location and epicentral distance, prevalent ground frequencies may be obtained from a statistical analysis of seismic station records provided the location of a structure is (geologically) close enough to such station. If not, an analysis of frequencies in blasting records may give the desired information. More complete data can be obtained, however, by the forced vibration method, as the energy source is controllable in regard to amplitude and frequency and a complete frequency spectrum is available if desired. This method encounters one difficulty, not mentioned in the literature, that its depth penetration is generally not sufficient. Hence, all resonance frequencies of a geologic section may not be detected, which makes it desirable to duplicate a geologic section on a small scale and to measure its frequency response with miniature vibrators whose depth penetration is proportionately greater.

The preparation of a reduced-scale model necessitates in turn a determination of geometric dimensions and elastic properties of the formations composing the geologic section. This is possible by a combination of refraction and vibrator measurements.

The accompanying schematic arrangement will aid in visualizing the relationship of the two methods. On the left side are possible methods of investigation; on the right, crosses indicate to what problems they are applied or applicable. Two groups of investigations are indicated, "theoretical" and "experimental." The latter are divided into three groups: (1) "free vibration" method; (2) "forced vibration" method; and (3) combination of refraction and resonance method. The first two are applied for determining natural frequency and damping on prototype and model, as well as for measuring elastic properties of specimens used in model construction. The third is used for obtaining data for construction of models of the geologic section, and of earth-dam models (borrow pit tests). In the table, the applications of these methods are indicated for both ground and structure.

The combination of the two sets of data obtained for ground and structure makes possible determination of the response and therefore the seismic resistance of a structure.

Application of Methods to Rigid Structures

The necessity for dynamic design of structures in earthquake countries led to systematic determinations of free vibrations of ground and buildings in Japan about 10 years ago. At the same time, unbalanced flywheel machines were developed in Europe and employed a few years later for the determination of elastic properties of foundations.[1,2] Vibrations of

Methods of Vibration Tests

Type of Investigation		I. Ground		II. Structure	
		Prototype	Model	Prototype	Model
A. Theoretical	Natural frequency.............. Damping....................			×	×
B. Experimental	1. Free vibration method:				
	a. As preliminary for determining elastic properties of specimen for model construction................		×		×
	b. On prototype and model (natural frequency and damping)................	×	×	×	×
	2. Forced vibration method:				
	a. As preliminary for specimen, as above...........		×		×
	b. On prototype and model:				
	aa. Periodic vibrations..	×	×	×	×
	bb. Transient vibrations.		×		×
	3. Combination of refraction and forced vibration methods:				
	a. As preliminary, for determining elastic properties on prototype...............	×		For earth dams only	
	b. As preliminary, for determining dimensions of prototype for model experiments.	×			

structures due to traffic, blasting, machines and earthquakes were investigated by the Geophysical Institute at Goettingen.[3–5] In 1934 and 1935 the U. S. Coast and Geodetic Survey conducted extensive investigations on earthquakes in California. Numerous buildings and several dams were tested with the free and forced vibration methods; the results have been discussed in a comprehensive report.[6]

[1] References are at the end of the paper.

Application of Methods to Earth Dams

It is not difficult to believe that a rigid structure such as a building, bridge, or concrete dam would have one or more definite natural frequencies and comparatively small damping. For a structure such as an earth dam or levee, being composed largely of unconsolidated material, it might be inferred that it is not capable of vibration and would possess strong damping. The investigations reported in the present paper show that a definite natural frequency exists; measurements made with the forced vibration method on a number of borrow pits intended for construction of earth dams indicate that the damping of such unconsolidated materials is much less than would be expected.

Before constructing an earth dam in an earthquake area, it is, therefore, advisable to determine the natural frequency of the proposed structure and to investigate whether it is apt to resonate with the predominant ground frequencies in the area.

Earth dams have suffered damage from earthquakes; several instances have been reported from Japan and California. Whether such damages resulted from poor static design or actual resonance with predominant earthquake frequencies has not been established. However, since it is certain that an earth dam has a definite natural frequency and in all probability low damping, it would appear to indicate negligence in design not to investigate the seismic resistance of a proposed dam. The following pages present a general description of methods used in the attack of this problem. After a definition of seismic resistance, determinations of natural frequency and resonance of earth dams by theoretical studies and model experiments are discussed. Then comes a discussion of field determinations of elastic constants for experiments on models of dam and geologic section followed by a description of methods used to determine prevalent ground frequencies in a seismic area: (1) by forced vibration methods in the field; (2) by experiments on models of the geologic section; and (3) by an analysis of seismic station records.

Definition of Seismic Resistance

"Seismic resistance" may be defined as the *ratio of ultimate strength to stress*, preferably stated for a point in the structure most likely to fail, and referred to a ground motion of given frequency and amplitude. Evidently the seismic resistance must be dependent on the natural frequency and damping of the structure (for sustained harmonic motion). The types of strength and stresses involved depend on the mode of vibration of the structure.

In an earth dam, bending vibrations are likely to be negligible because of the high ratio of base width to height. Vertical and longi-

tudinal vibrations may be excluded from consideration as unimportant compared with the transverse or shear vibration in horizontal direction. When subjected to harmonic ground motions, its oscillations are those of an inertia-coupled system and *approximate* expressions may readily be given by assuming the structure to act as an inverted horizontal pendulum. As shown in a previous paper,[7] the amplitude of forced oscillation of such a pendulum is given by

$$x = \frac{VA\omega^2 \sin(\omega t - \varphi)}{\sqrt{(\omega_0{}^2 - \omega^2)^2 + 4\epsilon^2\omega^2}} \qquad [1]$$

where the ground motion is $a = A \sin \omega t$, φ the phase shift, ϵ a damping factor, ω_0 the natural frequency, and V the geometric magnification of the pendulum. For a structure having its center of oscillation at an elevation z_0 and height H, $V = \frac{H}{z_0}$ so that its peak amplitude, corresponding to the peak ground amplitude becomes at the elevation H:

$$x_m = \frac{HA}{z_0} \times \frac{\omega^2}{\sqrt{(\omega_0{}^2 - \omega^2)^2 + 4\epsilon^2\omega^2}} \qquad [2]$$

If the angle of shear is ψ (Fig. 1) and the shear modulus μ, the shearing stress $S = \mu\psi \approx \mu\frac{x}{H}$. By referring this to the ultimate shearing strength U, the seismic resistance R may be expressed as ratio of shearing strength to shearing stress, so that

$$R \equiv \frac{U}{S} = \frac{Uz_0}{\mu A} \frac{\sqrt{(\omega_0{}^2 - \omega^2)^2 + 4\epsilon^2\omega^2}}{\omega^2} \qquad [3]$$

The seismic resistance increases with the ratio of ultimate shearing strength to shear modulus of the material and its damping; it is not only inversely proportional to the ground acceleration but depends also on the ratio of impressed to natural frequency. A factor R less than one indicates failure at the weakest point. For an undamped structure the resistance is zero at resonance. For very rapid ground motions the term $\frac{\sqrt{(\omega_0{}^2 - \omega^2)^2 + 4\epsilon^2\omega^2}}{\omega^2}$ approaches 1; for very slow motion it approaches $\left(\frac{\omega_0}{\omega}\right)^2$. The seismic resistance R is a minimum when

$$\omega = \frac{\omega_0{}^2}{\sqrt{\omega_0{}^2 - 2\epsilon^2}}$$

In the Mercalli-Cancani-Sieberg scale, degree VIII (250 to 500 mm. sec.$^{-2}$) is recorded as "developing fissures on steep embankments and in wet ground," while degree X (1000 to 2500 mm. sec.$^{-2}$) produces "wide cracks in loose and wet soil, slides off embankments, and considerable

damage to levees and dams." As the average of the accelerations for degrees VIII and X is about 1000 mm. sec.$^{-2}$ (maximum of degree IX) is convenient to refer seismic resistance to an acceleration of this order of magnitude. The strongest intensity observed in the Long Beach earthquake was IX; accelerations up to 1000 mm. sec.$^{-2}$ were recorded[6] in the penthouse of the Hollywood Storage Co. at an epicentral distance of 24 miles, at the time of the Southern California earthquake of Oct. 2, 1933.

List of Symbols

a, ground amplitude.
A, maximum ground amplitude.
b, dam width.
c, constant.
D, model ratio of density.
E, Young's modulus.
f, frequency.
F, model ratio of frequency.
J_0, Bessel's function, order zero.
L, linear scale ratio.
H, height of dam.
l, length.
M, model ratio of elastic moduli.
m, factor.
p, proportionality factor.
R, seismic resistance.
S, shearing stress.
t, time.
x, vibration amplitude of dam.
x_m, maximum amplitude of dam.
X, vibration amplitude of dam at any point.
U, ultimate shear strength.
v_t, transverse wave velocity.
v_l, longitudinal wave velocity.
V, geometric magnification.
w, factor.
z_0, elevation of center of oscillation.
z, depth of dam, from crest.
Y_0, Bessel's function, order zero.
α, phase angle.
ϵ, damping factor.
η, coefficient of internal friction.
λ, modulus of volume elasticity.
μ, shear or rigidity modulus.
δ, density.
σ, Poisson's ratio.
φ, phase shift.
ψ, angle of shear.
ω, angular frequency of ground motion.
ω_0, angular frequency of dam.

Determinations of Frequency Characteristics of Earth Dams

Theoretical Calculations

As in the theoretical treatment of most other engineering and geophysical problems, it is necessary, in the calculation of the dynamic characteristics of earth dams, to resort to certain simplifications and to remove these step by step if feasible. The simplifications introduced here are concerned with three parameters: (1) geometric disposition; (2) physical properties; and (3) mode of vibration.

In regard to geometric arrangement the treatment is simplified by assuming that the dam extends to infinity in both directions from the section considered. The section is assumed to be an isosceles triangle of base width greater than height; the unevenness of upstream and downstream slopes and the effect of the truncated top are not considered. For the calculation of natural frequency, it is first assumed that damping is absent and that the elastic modulus remains constant throughout the

section. The next step is to consider velocity damping, constant throughout the section; damping proportional to the square of the velocity is probably unimportant. To what extent Coulomb friction should be considered in addition to velocity damping cannot be stated at this time. In further analysis, the assumption of constant elastic modulus and constant damping may be dropped and a uniform linear increase with depth from the crest be substituted for both. For a most complete analysis, differences in the effects of the sections composed of different materials and the influence of a surface layer oscillating with the dam should be considered; however, the treatment becomes so complex that model experiments will give the desired results more readily. As to mode of vibration, vertical transverse and longitudinal

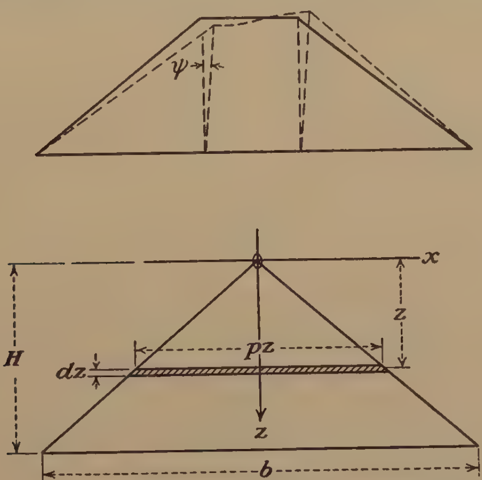

FIG. 1.—SCHEMATIC DIAGRAM OF EARTH DAM.

horizontal vibrations are not believed to be of sufficient significance from the point of view of damage to the structure and have not been considered in the present analysis. For transverse oscillations, the possibilities of bending and shearing vibrations exist. Of these, bending vibrations have been disregarded, which leaves, for the first approximation, a consideration of horizontal shearing vibrations at right angles to the dam axis for triangular section and homogeneous material.

To derive an expression for natural frequency consider (Fig. 1) a section of the dam at right angles to its axis with a system of coordinates so placed that its zero point is in the crest, the y axis horizontally along the crest, the x axis horizontal and transverse, and the z axis is downward. An expression for oscillations of the dam in the x direction may be derived from the condition of equilibrium of restoring and inertia forces acting on a thin section of thickness dz at a depth z and length pz if p is the ratio of base width b to height H.

The linear effect of the restoring force is proportional to the length of the section pz, the modulus of rigidity μ and the shear $\frac{\partial x}{\partial z}$. Therefore, the restitution is

$$pz\mu \times \frac{\partial x}{\partial z}$$

and its derivative with respect to z:

$$\frac{\partial}{\partial z}\left(pz\mu\frac{\partial x}{\partial z}\right).$$

The effect on the section and therefore the restituting force:

$$\frac{\partial}{\partial z}\left(pz\mu\frac{\partial x}{\partial z}\right) \times dz \tag{4}$$

The inertia force is the product of acceleration and (two-dimensional) mass. The latter is $pz\rho dz$ (ρ = density) so that the inertia force

$$pz\rho dz\frac{\partial^2 x}{\partial t^2} \tag{5}$$

The condition of equilibrium is therefore given by

$$\rho z\frac{\partial^2 x}{\partial t^2} = \frac{\partial}{\partial z}\left(z\mu\frac{\partial x}{\partial z}\right) = \mu\left(z\frac{\partial^2 x}{\partial z^2} + \frac{\partial x}{\partial z}\right) \tag{6}$$

Substituting $x = XZ$ so that $\ddot{X} = \frac{\partial^2 X}{\partial t^2}$, $\ddot{Z} = \frac{\partial^2 Z}{\partial z^2}$, $\ddot{X}Z = \frac{\partial^2 x}{\partial t^2}$, $X\ddot{Z} = \frac{\partial^2 x}{\partial z^2}$, and $X\dot{Z} = \frac{\partial x}{\partial z}$, eq. 6 is

$$\ddot{X}Z - \frac{\mu}{\rho}X\ddot{Z} - \frac{\mu}{\rho z}X\dot{Z} = 0 = \frac{\rho\ddot{X}}{\mu X} - \left[\frac{\ddot{Z}}{Z} + \frac{1}{z}\frac{\dot{Z}}{Z}\right] \tag{7}$$

Substituting:

$$-m^2 \equiv \frac{\ddot{Z}}{Z} + \frac{1}{z}\frac{\dot{Z}}{Z} \tag{7a}$$

eq. 7 reduces to the familiar second-order differential equation of free oscillation:

$$\ddot{X} + m^2\frac{\mu}{\rho}X = 0$$

whose solution

$$X = c_1 \cos\left(m\sqrt{\frac{\mu}{\rho}}t - \alpha\right) \tag{8}$$

To determine the coefficient m in eq. 8, eq. 7a may be written, with the substitution $w \equiv mz$, in form of a Bessel differential equation

$$w\frac{\partial^2 Z}{\partial w^2} + \frac{\partial Z}{\partial w} + wZ = 0$$

whose solution $Z = c_2 J_0(mz) + c_3 Y_0(mz)$. As $Y_0(mz)$ is ∞ for $z = 0$, $c_3 = 0$. Therefore,

$$x = cJ_0(mz) \cos\left(m\sqrt{\frac{\mu}{\rho}}t - \alpha\right)$$

which is 0 for $z = H$. If $J_0(mH) = 0$, $mH = 2.4048$. Then the amplitude of vibration

$$x = cJ_0\left(2.4048\frac{Z}{H}\right) \cos\left(\frac{2.4048}{H}\sqrt{\frac{\mu}{\rho}}t - \alpha\right) \qquad [9]$$

or the angular frequency of the dam

$$\omega_0 = \frac{2.4048}{H}\sqrt{\frac{\mu}{\rho}} \qquad [10]$$

It is interesting to note that the natural frequency of an earth dam is *not* dependent on the base width but only on the height and on the velocity of the *transverse seismic* waves. When, therefore, the speed of the transverse wave has been determined on dam material (as in a borrow pit) it is possible to calculate directly the frequency of the dam to be built of such material. If the material is to be compacted to a different density than exists in situ, allowance may have to be made for some variation in the shear modulus and density. However, the effect is not great, as the square root of the ratio of the two quantities is the controlling factor.

Substituting average values for μ and ρ as determined on dam materials in the field into eq. 10 shows that the angular frequency of a dam 100 ft. high is of the order of 15.

The effect of damping may be incorporated in eq. 6 by adding a term proportional to the coefficient of internal friction η and velocity of shearing motion $\frac{\partial}{\partial t}\left(\frac{\partial x}{\partial z}\right)$. Therefore, the linear damping resistance of the strip previously considered is $pz\eta\frac{\partial^2 x}{\partial z \partial t}$ and the effect on a strip of the thickness dz is

$$\frac{\partial}{\partial z}\left(pz\eta\frac{\partial^2 x}{\partial z \partial t}\right)dz$$

Therefore the complete equation considering damping (ρ and dz cancel) is:

$$\rho z\frac{\partial^2 x}{\partial t^2} = \mu\left(z\frac{\partial^2 x}{\partial z^2} + \frac{\partial x}{\partial z}\right) + \eta\left(z\frac{\partial^3 x}{\partial z^2 \partial t} + \frac{\partial^2 x}{\partial z \partial t}\right) \qquad [11]$$

In a paper given before the Second Congress on Large Dams, Mononobe, Takata and Matumura gave a curve showing the decrease of the resonance amplitude at the top of a dam with an increase in damping. They also showed how this curve is modified when both rigidity modulus and internal friction increase with depth; in the latter event, the amplitude at the top is greater than the amplitude calculated for constant damping.

Evaluation of eq. 11 is involved and not followed further, as another line of approach is believed to lead to the same result more rapidly. Instead of computing damping from internal friction, it may be obtained directly from the resonance curve taken on borrow-pit material with a vibrator and be substituted with sufficient accuracy into the expression for the dynamic magnification of a damped system performing forced oscillations (formula 2).

Theoretical calculations involving oscillations of earth dams can and should be verified by model experiments, of which the chief merit lies in the fact that actual conditions, involving principally irregular shapes of the object to be investigated, may be duplicated while they are difficult to analyze theoretically.

Certain definite rules must be observed here which will be discussed in the next section.

Model Experiments: Effect of Scale Reduction

In making model experiments with earth dams or any other kind of engineering structure, it is generally impossible to use the same kind of materials as in their construction and to obtain correct results as far as dynamic response is concerned. One reason is that the size of the particles would be quite out of proportion to the size of the structure in the model, so that all physical reactions depending on particle size would be changed. A second reason is that in a model any physical quantity depending on the three fundamental physical dimensions—length, mass and time—must be changed to conform to the change in scale. Two possibilities exist in the practical application of this principle: (1) to modify the physical properties of the materials in the model in such manner that the desired physical quantity is obtained in the same numerical value in both model and original; (2) to modify the physical properties of the materials in the model consistently in such manner that a constant ratio is obtained between the physical quantities observed on prototype and model. In model experiments on earth dams it is advisable to use the last procedure, because, owing to the large linear scale ratios that

must be employed, it is not possible to "weaken" the model materials sufficiently.

The most important quantity determined in model tests of seismic response is natural frequency. As the frequency f of an elastic body is given by

$$f = \frac{c}{l}\sqrt{\frac{E}{\rho}}, \tag{12}$$

where c is a constant, ρ density, l length and E an elastic modulus, it follows that the relation of the model ratio of frequency F, the model ratio of elastic moduli M, the linear-scale ratio L and density ratio D is given by

$$F^{-1} = LM^{-1/2}D^{1/2} \tag{13}$$

It follows from this equation that in order to obtain practicable frequency ratios with such scale reduction as 1:1000 or 1:2000, the elastic moduli of the model materials must be reduced materially.

Determinations of Elastic Moduli of Model Materials

It would be rather difficult, particularly for large linear-scale ratios, to find materials that would give identical frequency values for model and prototype. It is easier to test a series of materials readily available, select those giving a frequency model ratio suitable for observation, and to convert the model data by this ratio to prototype data. Again, difficulties arise in this procedure when the prototype (geologic section) is composed of formations with different elastic moduli; as soon as a material with a given model ratio has been selected for one formation, there is no longer any freedom in the selection of the others. All materials in the model have to meet the same ratios and if the constituent parts are numerous, the selection of materials with consistent model ratios becomes a task requiring considerable patience. It is possible to overcome this handicap to some extent by changing linear-scale ratios for different formations in a geologic section, or to substitute equivalent masses with single degrees of freedom of oscillation, as is done in simulating the effect of water in tank-tower vibration experiments.[8]

In order to reduce to a minimum the time required for the preparation of desirable model materials, methods for rapid elastic tests must be available. Inasmuch as in the model experiments dynamic methods are used, the same type of test should be applied in measurements of specimens. A rapid and reliable method for comparatively small specimens is the resonance method. The specimen is driven at various frequencies and its end amplitude recorded on a camera. Amplitude plotted against frequency gives a resonance curve from which natural frequency and damping may be obtained. Elastic moduli are calculated from the

dimensions of the specimen, its undamped frequency and density, the latter being determined by any of the customary methods.

Determinations of Natural Frequency and Damping on Dam Models

After a suitable material or series of materials (to represent, if desired, a small difference in elastic properties between impervious core and surrounding material) has been decided on, a mold is made of the proposed dam at a reduced scale and the dam model is cast of these materials.

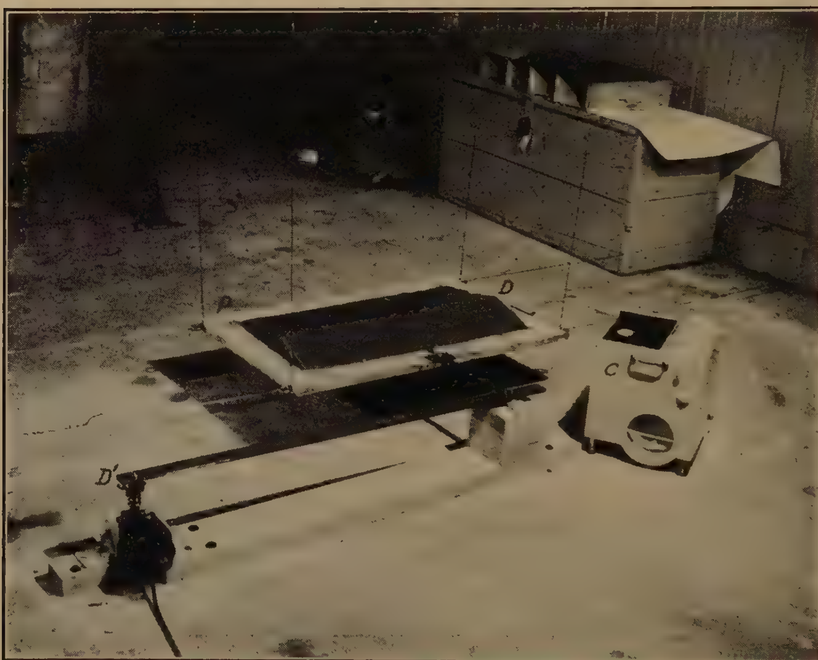

FIG. 2.—ARRANGEMENT FOR DETERMINING DYNAMIC RESPONSE OF MODEL DAMS. *D*, dam; *P*, platform; *D'*, drive; *C*, camera.

There exist two possibilities: (1) to place the dam model on a rigid shaking platform and to determine the ratio of dam-top amplitude to the amplitude of the base as function of frequency; (2) to place the model dam on a model of the geologic section and to determine the ratio of dam-top amplitude to the ground amplitude next to it as function of frequency. The difference between the two procedures is that the first gives the dam amplitude in reference to a rigid foundation, whereas the second offers the possibility of detecting a change in dam frequency due to a portion of ground or a portion of the top layer oscillating with it. The results of the first kind of test should agree very closely with the results of theoretical calculations of the frequency of the dam; the results of the second

need not necessarily agree and appear to be related to the thickness of the overburden. Fig. 2 shows the arrangement of a shaking platform for determining the response of the dam and the ratio of its amplitude at the top in reference to that of the platform. The speed of the drive *D* is varied and after stationary conditions are reached the amplitude at the top of the dam is recorded by a camera *C*. Fig. 3 shows a response curve of a dam obtained in this manner. Allowing for the scale factor, the model frequency of 2.03 agrees closely with the calculated frequency of 2.13, and the amplitude at the top of the dam is about five times that

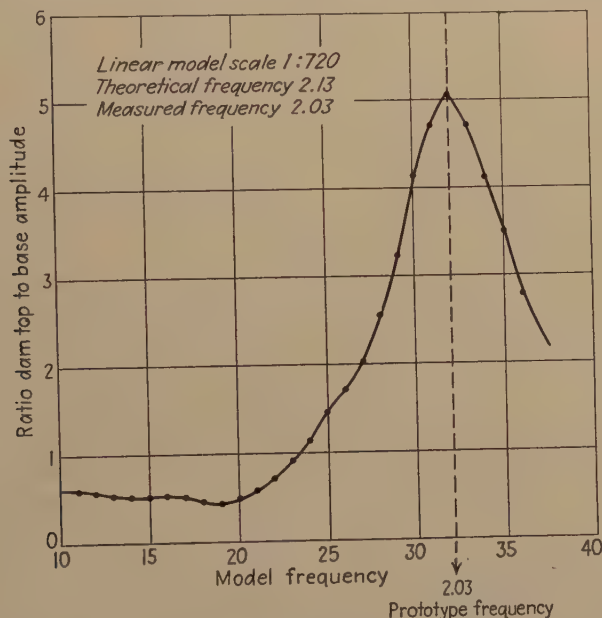

FIG. 3.—RESPONSE CURVE OF MODEL DAM, SHOWING AMPLITUDE OF TOP IN RELATION TO BASE.

of the (rigid) base at resonance. It was not possible in this case to make the damping of the material quite conformable with damping determined in the field on borrow-pit material. However, the difference may be readily allowed for in the interpretation of the model curve as the ratio of top amplitude to base amplitude at resonance is a function of damping only.

In the second test of dam response, the model dam is cemented to a model of the geologic section whose construction is based on seismic refraction and vibrator data with complete model ratio similitude (see p. 373). It may be of advantage, for reasons discussed later, to arrange the geologic model in such manner that it may be set into forced vibration in either horizontal or vertical direction.

Fig. 4 represents a somewhat schematized combination of curves obtained by placing a vibration detector on various locations on the

(model) dam and on the (model) ground to the side of it. The model ratio in this case was slightly different from the one used in the separate tests of the dam mentioned before. Three curves are shown in the figure: (1) the dam response, (2) the ground response, and (3) the ratio of dam and ground amplitude. The dam resonates at a somewhat lower frequency than for a rigid base, probably because a portion of the overburden is added to the vibrating mass and increases the effective height of the dam. It is therefore possible that various sections of an earth dam resonate with somewhat different frequencies, not only because of varying

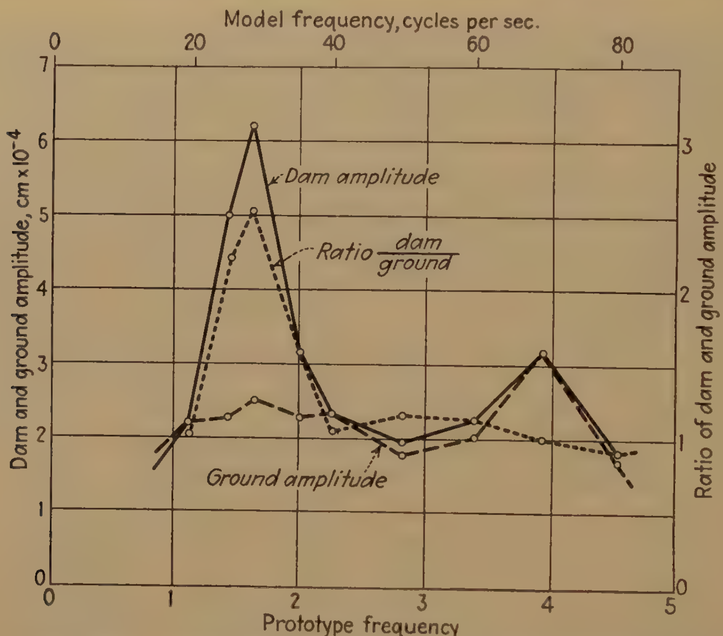

FIG. 4.—RESPONSE OF MODEL DAM (COMBINATION OF CURVES ON SEVERAL LOCATIONS) ON MODEL OF GEOLOGIC SECTION.

thicknesses of overburden but also on account of irregularities in topography. There appears a second peak in the response curve at a higher frequency; this, however, is not due to the dam, as the ratio of dam and ground amplitude does not go up with the dam amplitude. It is interpreted as a characteristic ground frequency for horizontal excitation.

FIELD MEASUREMENTS TO OBTAIN GEOLOGICAL AND PHYSICAL DATA FOR MODEL CONSTRUCTION

Geophysical measurements in the field are made in earth-dam investigations for two purposes: (1) to furnish depths and physical characteristics of geologic formations for the preparation of a model on which the response of the dam site and of the dam itself is to be tested; (2) to furnish

the physical characteristics of the materials used in the construction of the dam (by measurements in borrow pits).

Geophysical measurements may also be applied to obtain other data useful in designing earth dams; viz., seismic measurements to determine formation characteristics to aid in the calculation of the percolation slope in the dam, and electrical measurements to determine ground-water level. These applications will not be covered in this paper.

To obtain the geological and physical data mentioned, it is necessary to apply seismic refraction and vibrator methods and to combine their results.

Seismic Refraction Method

The principle of seismic refraction methods has been frequently described in the literature. It is therefore not necessary to elaborate on them here except to describe briefly the equipment and methods applied specifically in this work.

A seismic equipment consists fundamentally of three primary and two secondary component parts.[7] The three primary parts are: (1) the vibration detectors provided with a transducer to convert mechanical vibrations into electrical impulses; (2) as many amplifiers as there are vibration detectors; and (3) a camera for recording the amplified electrical impulses by oscillographs. The two secondary units are: (1) a system for communication between shot point and receiving point and for transmitting the instant of the shot to the record; (2) a device to project accurate time lines across the record. In dam investigations, shot holes are generally put down by hand augers and detectors are set up at intervals from 25 to 100 ft. As in dam investigations the problem is usually one of determining depth to bedrock under overburden, it is convenient to apply a modification of the standard technique in which the detectors are set out at distances past the high velocity intercepts. Fig. 5 shows a record obtained in such a manner. The break indicated in the fifth trace is the instant of the shot. This is followed in all traces by impulses resulting from longitudinal refracted waves having followed the shortest time path. At each station two profiles are shot in opposite directions to determine dip and other horizontal variations in the interface to be mapped. Records are evaluated by measuring arrival times of first impulses with an accuracy of $\pm 1/1000$ sec. and by plotting these times against detector distances from the shot point. For two profiles, two travel-time curves are thus obtained whose slopes represent the two apparent underlayer velocities. If the velocity of the overburden is known, its thickness may then be determined for each detector location. Overburden velocities are determined by short refraction profiles and by shooting in core holes, which frequently are available in dam exploration. As implied above, the problem is facilitated considerably because in dam

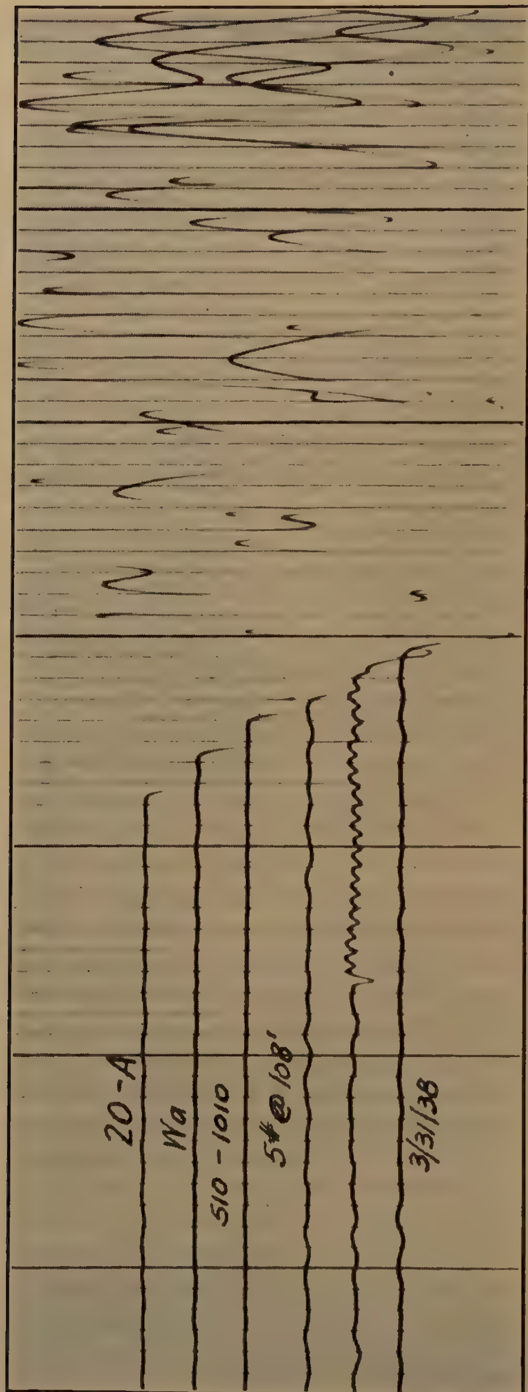

FIG. 5.—SEISMIC REFRACTION RECORD OBTAINED IN DAM INVESTIGATIONS.

exploration the geologic section can usually be represented with sufficient accuracy by two members, overburden and bedrock. From the data of a refraction survey a contour plan of the bedrock can then be drawn and a mold prepared from which the members of the sections of the geologic model may be cast. In selecting materials for this model, averages of the overburden and bedrock velocities are used as determined from the refraction survey in conjunction with the results of vibrator tests.

Vibrator Measurements

The vibrator as used in soil investigations is a machine intended to impress periodic forces of variable frequency upon the ground. It consists, in its simplest form, of two eccentrically loaded cylinders revolving in opposition on a chassis. The cylinders may be so geared that the horizontal components of the centrifugal force are cancelled and the vertical components added, or vice versa. The cylinders are driven by a motor, which generally is mounted on the top of the vibrator. Power is supplied to this motor from a motor-generator set carried in a separate truck. It is convenient to read the speed of revolution of the cylinders on a tachometer. In the vibrator shown in Fig. 6, two additional devices are added—an electromagnetic indicator intended to record the instants at which the maximum force is applied to the ground (see bottom traces in Figs. 8 and 9) and a vibration detector mounted on the vibrator frame, to carry a reference phase to the recorder. For recording the vibrations in various distances from the vibrator, the equipment used

FIG. 6.—VIBRATOR AND RECORDING EQUIPMENT SET UP ON LOCATION. *D*, DETECTORS; *V*, VIBRATOR, MG, MOTOR GENERATOR.

in refraction work may be employed, with an accurately calibrated recording channel to convert the recorded amplitudes to actual ground motions. Profiles are recorded usually in two directions, to determine possible differences in frequency and velocity characteristics.

The evaluation of frequency data obtained in vibrator measurements will be discussed later (p. 370). In reference to the subject here discussed —determination of elastic properties—the measurement of "phase speed" is important; this is the speed with which any given vibrator impulse is propagated. It is less than the speed of the longitudinal

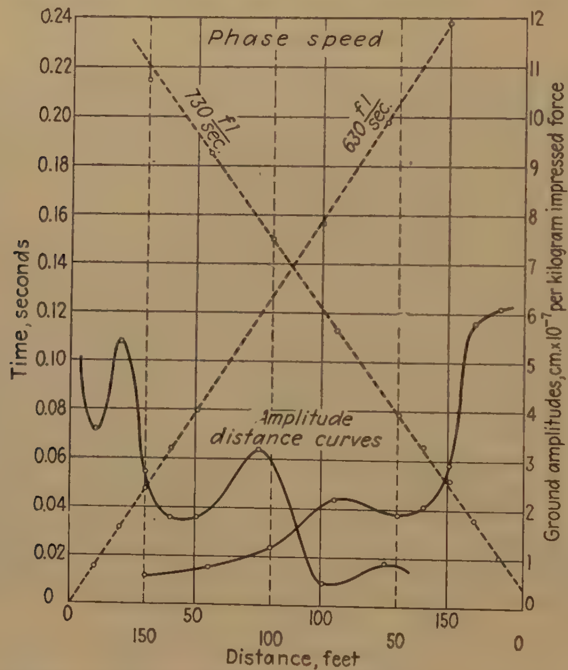

Fig. 7.—Amplitude variation with distance and phase speeds in two directions.

elastic waves; it is generally assumed that waves transmitted from a vibrator are Rayleigh waves. In any event, there is no question about it that they are transverse waves. By combination of these wave speeds with the wave speeds derived from an analysis of refraction records, a complete determination of all elastic properties of the ground is possible (see p. 369).

The phase-speed curves shown in Fig. 7 are straight throughout the profile. In longer profiles, higher velocity intercepts will appear and are due to the wave energy that has traveled through deeper high-speed media. Combination of these speeds with longitudinal wave speeds determined for the underlayer from the analysis of refraction travel-time curves makes it possible to determine elastic constants of bedrock mate-

rials. However, in dam investigations where outcrops of bedrock are frequently available on abutments, it is simpler to determine bedrock properties directly by vibrator setups on the abutments. The elastic properties of the materials used in the construction of the dam are determined by refraction and vibrator measurements on borrow pits, as mentioned before.

Calculations of Elastic Constants from Field Data for Model Construction

The propagation of elastic waves is a function of the Lamé coefficient of volume elasticity λ, the rigidity or shear modulus μ, and the density ρ. While the transverse wave speed v_t depends on the rigidity only, the longitudinal wave speed v_l is a function of both moduli. The two moduli

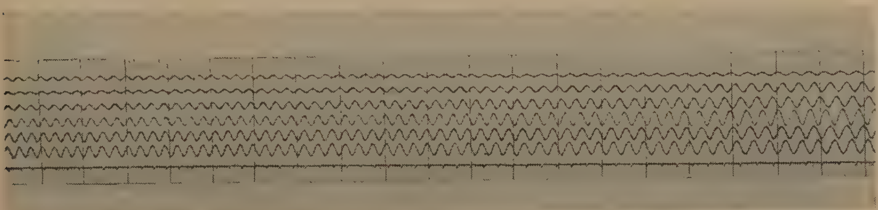

FIG. 8.—VIBRATOR RECORD MADE WITH CONTINUOUSLY DECREASING FREQUENCY OF EXCITATION.

λ and μ are each, in turn, functions of Young's modulus E and Poisson's ratio σ.

As velocities of transverse waves are observed in vibrator tests and longitudinal waves in the refraction survey, all elastic constants can be calculated provided: (1) the Rayleigh wave velocity has been converted to transverse wave velocity; and (2) density values have been determined on specimens of the formations concerned. The modulus of rigidity then follows immediately from the relation

$$\mu = v_t^2 \times \rho \qquad [14]$$

Poisson's ratio is obtained from the ratio of transverse and longitudinal wave velocities:

$$\sigma = \frac{1 - 2\left(\frac{v_t}{v_l}\right)^2}{2 - 2\left(\frac{v_t}{v_l}\right)^2} \qquad [15]$$

By combination of eqs. 14 and 15, Young's modulus E follows from

$$E = 2\mu(1 + \sigma) \qquad [16]$$

and may also be obtained directly from Poisson's ratio and the longitudinal wave speed as

$$E = \frac{\rho v_l^2(1 + \sigma)(1 - 2\sigma)}{1 - \sigma} \qquad [17]$$

In utilizing field data for model construction, it is most convenient to express all elastic constants in terms of Young's modulus, as the same quantity is most readily determined on specimens of model materials.

For the preparation of models, data on formation thicknesses and damping are required, in addition to the elastic constants. How these are obtained from seismic refraction observations and vibrator response curves has been discussed before.

Determinations of Ground Frequencies

As stated at the outset, it is necessary to have accurate information on the frequency of the ground on which an earth dam is to be placed in

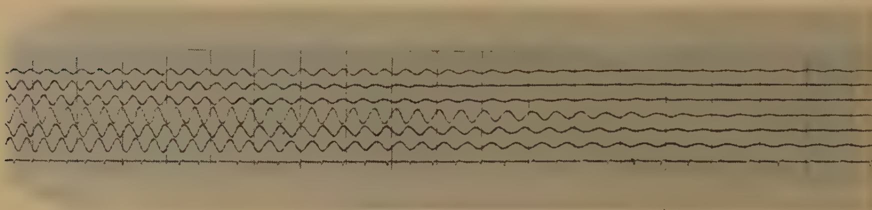

Fig. 8.—(Continued.)

order to evaluate its seismic resistance. It may be determined (1) from vibrator measurements, (2) by measuring the response of a model of the geologic section, (3) by a frequency analysis of refraction and other blasting records, (4) by a frequency analysis of earthquake records.

Determination of Ground Frequencies by Vibrator Measurements

Two methods are available for determining the frequency response of the ground. The first is more qualitative in nature and is intended to determine rapidly the frequency at which the ground resonates. Records are taken by bringing the vibrator up to maximum speed, then shutting it off and allowing it to run down. Thus, the ground is excited by a continuous frequency spectrum. Fig. 8 shows a record taken in this manner. Actually the peaking of the response curve is sharper than appears from this record after amplitudes have been reduced for increase of centrifugal force with frequency and after the calibration curve of the recording channel has been considered.

In the second method a vibrator is run with constant frequency and records are taken after equilibrium has been reached. Fig. 9 shows a record obtained in such manner. The first five traces are recorded by detectors set up away from the vibrator at the distances indicated. The sixth trace is produced by the detector on the vibrator with its amplitude cut down. The seventh trace records the instant at which maximum force is impressed on the ground.

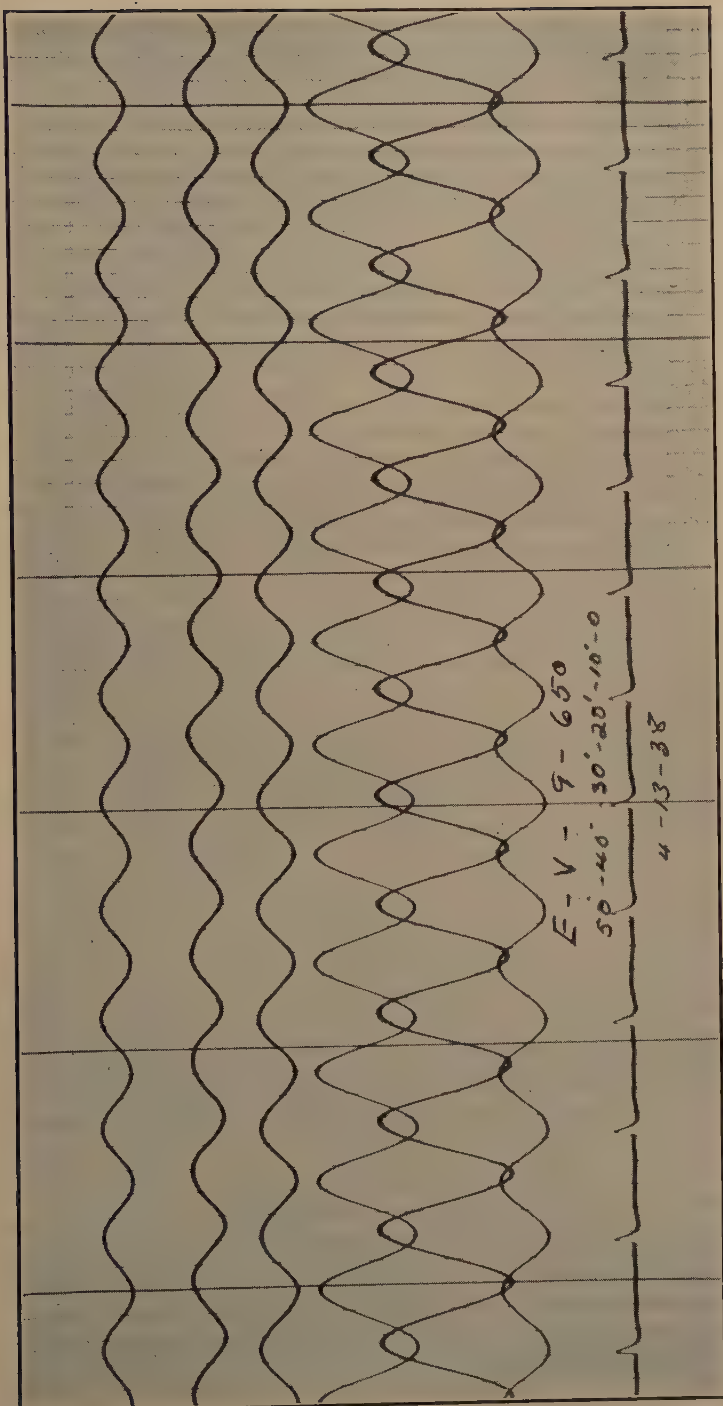

FIG. 9.—VIBRATOR RECORD MADE WITH CONSTANT FREQUENCY OF EXCITATION. SIXTH TRACE: REFERENCE DETECTOR, MOUNTED ON VIBRATOR.

A number of important data as to dynamic ground characteristics may be obtained from such records. First the recorded amplitudes must be reduced to true ground motion by means of the calibration curve of the recording channel. Second, they are reduced to uniform impressed force by means of the calibration curve of the vibrator. Amplitudes so obtained may be plotted as shown in Fig. 10. From such a diagram, three essential quantities are obtained: (1) the resonance frequency of the ground; (2) damping; (3) the decrease of amplitude with distance.

The ground response frequency is high (Fig. 10). Data published elsewhere in the literature likewise indicate high frequencies recorded under similar geologic conditions. It must be assumed that such high

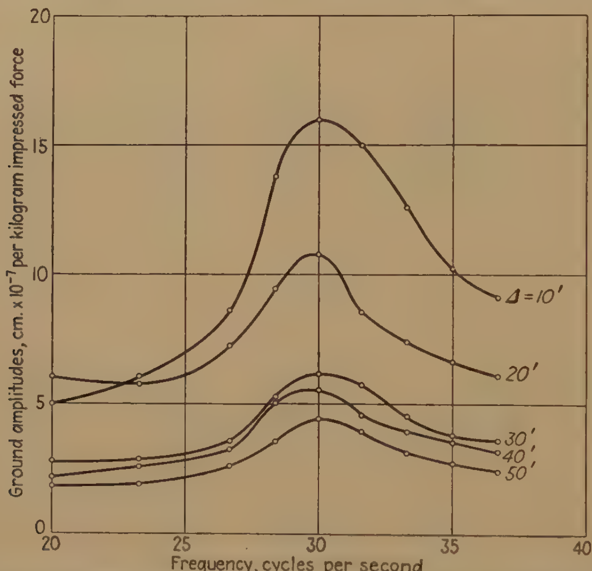

FIG. 10.—GROUND RESPONSE (VERTICAL VIBRATIONS) IN FIVE DISTANCES FROM VIBRATOR.

frequencies represent the dynamic characteristics of overburden only; further, it is noted that these frequencies are obtained for vertical excitation. If heavier vibrators and greater distances between vibrator and detectors are used, it is possible that lower frequencies would make their appearance, corresponding to greater depth penetration. Furthermore, indications are that these lower frequencies appear more readily for horizontal instead of vertical excitation. It is also significant that in most seismic-station records low frequencies are emphasized, which undoubtedly is due, in part at least, to the greater epicentral distances involved, although the characteristics of the recording instruments have a great deal to do with the suppression of the higher frequencies. In order to obtain frequencies in the lower range existing in a geologic section,

it is more advantageous to construct a geologic model on the basis of data furnished by refraction and vibrator surveys and to excite them with vibrators of proportionately greater depth penetration rather than to carry much heavier equipment in the field, which appears to be required for that purpose.

From the response curve shown in Fig. 10, the damping of the ground may be calculated. This is important when determining characteristics of borrow-pit material used in the construction of an earth dam. Damping rates determined for such materials on various locations have varied from 7 to 15 per cent, which is certainly much less than would be expected.

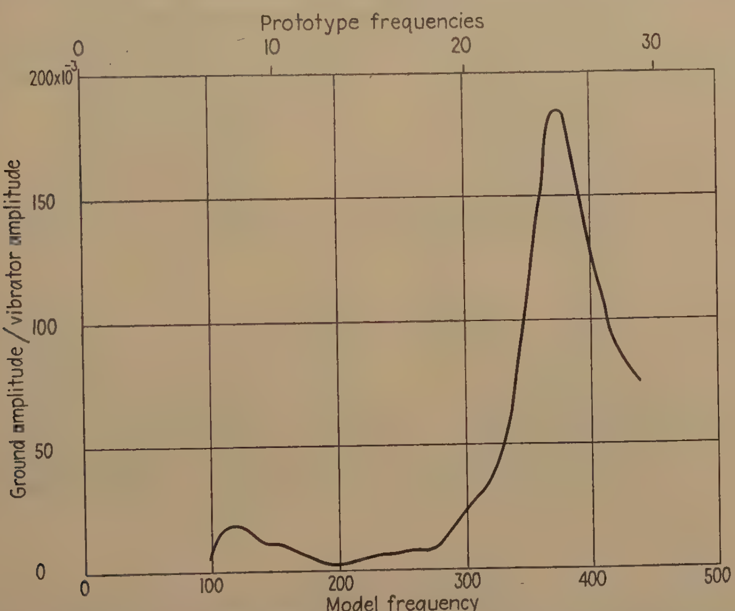

FIG. 11.—FREQUENCY RESPONSE OF GEOLOGIC MODEL FOR VERTICAL EXCITATION.

From the set of curves shown in Fig. 10, amplitude distance curves shown in Fig. 7 may be calculated. The peaks in these curves are due to interference phenomena. It is sometimes possible to calculate from them the depth to bedrock;[9] however, refraction methods are much more reliable for this purpose.

Determination of Ground Frequencies on a Model of the Geologic Section

On a model of the geologic section it is not only possible to obtain greater depth penetration but to simulate methods of excitation that can be used in the field only with difficulty. While it is not possible to measure wave speeds on a model, its response to impulses and directions of impulses more closely resembling conditions occurring in an earthquake can be determined more conveniently than is possible in the field.

When a geologic model has been constructed for this purpose it is advisable to examine it first for correct ratio similitude. This may be done by comparing its response with that obtained in the field with a vertical vibrator. Fig. 11 shows the results of such test where a miniature vibrator was placed on the surface of the model and its response measured with a miniature vibration detector. The vibrator was run through the higher frequency spectrum and the observed amplitudes

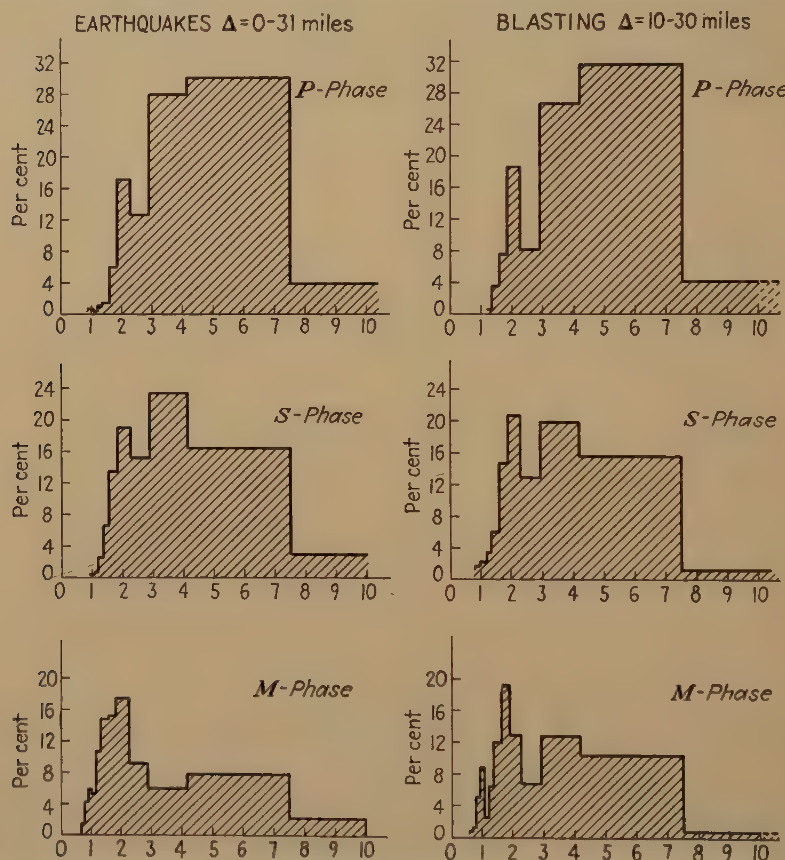

FIG. 12.—PREVALENCE OF FREQUENCIES RECORDED AT PASADENA FOR BLASTING AND EARTHQUAKES.
From data published by Gutenberg.[6]

reduced: (1) for variation of impressed force with frequency; (2) for variation of detector response from linearity. A peak in response occurs at 370 cycles. As the model ratio was 15.1, the prototype frequency is 24.5. To the north of the model station, a frequency of 23.5 was obtained; to the south the frequency was 24.0. The agreement between field and model data may be considered satisfactory, considering variations in frequency observed in the field between locations and generalizations

involved in the selection of dimensions and elastic properties to simulate the actual geologic conditions.

To simulate deeper excitation, a geologic model may be subjected to horizontal impulses. If lower frequencies are present they are likely to be brought out by that method. In the tests of a dam on a geologic model shown in Fig. 4, a resonance frequency higher than that of the model dam appeared which was not accompanied by a large amplitude ratio of dam to ground motion. This was interpreted as a resonance frequency of the ground alone for deep excitation and coincided with the prevalent frequencies recorded by seismic stations in the area investigated.

A model of the geologic section may not only be tested by applying periodic forces but transients as well. Here again it is superior to large-scale experimentation in the field; it offers, as a matter of fact, an almost unlimited number of possibilities in the investigation of dynamic (and static) phenomena.

Determination of Ground Frequencies from Analysis of Blasting and Earthquake Records

In comparing vibrator data with results obtained by an analysis of blasting and earthquake records, it must be borne in mind that the prevalent frequencies recorded by either method are likely to be a function of epicentral distance.

This is explained on the basis of the mechanism of generation and propagation of earthquake and detonation waves. It is certain that in explosions a single impulse is generated; while we know little about the actual phenomena causing an earthquake, it is likely that at least earthquakes due to faulting start with one or a series of *single* impulses. Yet when an earthquake or an explosion is recorded at some distant point, the recorded phenomenon is more or less periodic. Probably the only satisfactory explanation is that formations of different dimensions and character when traversed by the seismic energy are induced to oscillate. This has been verified, at least for surface and near-surface formations, by a number of investigations. The reaction of a subsurface section to seismic impulses can be likened to a network of series or parallel resonant filters. It is to be expected, therefore, that the geologic characteristics of the section next to a seismic station exert the strongest influence on the frequency response. This is verified by the fact that an analysis of frequencies recorded in blasting and earthquakes shows almost identical results. Differences in the mechanism of generation of seismic impulses can therefore be of but small consequence.

If the explanation given above is correct, it must be expected that the prevalent frequencies recorded for blasting or earthquakes are dependent on epicentral distance. The preliminary P and S waves penetrate to greater depths with increasing epicentral distance and are therefore likely

to induce oscillations in progressively deeper formations. Superimposed on this is an increase of wave length with distance postulated by the theory of wave propagation and is therefore particularly noticeable in the surface waves. The frequency characteristics must finally be expected to be dependent on the mode of oscillation and should therefore be somewhat different for longitudinal and transverse waves.

Accordingly, vibrator measurement made with very short distances of excitation will give generally high ground frequencies, which are not repeated in earthquake records of larger epicentral distances. Vibrator measurements of ground frequencies should, on the other hand, be relied on exclusively if a structure is likely to be affected by a *near-by* source of vibrations such as quarry blasting, machinery or heavy traffic.

In evaluating the resistance of a structure to earthquakes, ground frequencies corresponding to greater depth of excitation must be used. They are obtainable from geologic models as discussed in the preceding section or an analysis of blasting and earthquake records taken at an earthquake station in the immediate vicinity.

It goes without saying that seismic-station records can be used for an evaluation of predominant ground frequencies only if the geologic conditions are reasonably similar to those in the area to be investigated. A frequency analysis of seismograms recorded in Pasadena has been made by B. Gutenberg.[6] His values, given in periods, have been converted to frequencies and plotted for the P, S and M phases, for both earthquakes and blasting (Fig. 12). The similarity of the diagrams is remarkable.

Calculations of Seismic Resistance of Earth Dams

If the frequency characteristics of an earth dam and the predominant ground frequencies have been determined by any or all methods described, its seismic resistance can be estimated by substitution in the approximation formula (3). Values for the ultimate shearing strength may be obtained from tests of representative soil samples; for unconsolidated materials the ultimate shearing strength is approximately 1000 times smaller than the shear modulus. For damping, a mean value obtained in vibrator experiments on borrow pits is used.

For rapid estimates of the stability of an earth dam its resistance at resonance may be calculated, in which case the impressed frequency is the resonance frequency calculable from the dam frequency. For a dam 100 ft. high with natural frequency of 2 cycles per sec. and comparatively high damping (12.8 per cent critical) seismic resistance at the top for a maximum acceleration of 1000 mm. sec^{-2} would be 0.6; i.e., the dam would fail near the top in an earthquake of intensity IX. If the height of the dam were raised it would be taken out of resonance; for the same ground frequency as before, its seismic resistance would be brought up to one; i.e., the dam would be stable at the top, if the increase in height were 25 per cent. For any other damping rates the resistance

at resonance is readily calculated, as for low damping the dynamic magnification at resonance is the reciprocal of twice the relative damping. In these estimates, a factor of safety has not been allowed for specifically. It is, however, expressed indirectly by the selection of earthquake intensity for which the seismic resistance is estimated.

Acknowledgments

Most of the data contained in this paper were secured in the course of an investigation into the earthquake resistance of Hansen Dam and dam site near Los Angeles, Calif. This work was done for the Corps of Engineers of the War Department. Colonel Warren T. Hannum, Division Engineer, very kindly gave permission for the reporting of the results in this paper. A great deal of assistance was received, during the course of the work, from Major F. M. S. Johnson, Assistant to the Division Engineer, and Mr. B. F. Jakobsen, engineer attached to the Division office. Their courtesies are hereby gratefully acknowledged, as are those of Major Theodore Wyman, District Engineer in charge of the Los Angeles office. Mr. John C. Hollister, of the Heiland Research Corporation, Denver, and Mr. W. B. Greenlee gave valuable assistance in the course of the investigations reported in this paper.

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DISCUSSION

(Mark C. Malamphy presiding)

R. K. Bernhard,* State College, Pa.—Vibrator measurements as discussed by Dr. Heiland, that is, the so-called "oscillator method," have been used successfully in other places, both in this country and in Europe. The writer has presented this "oscillator method" more in detail for forced and free vibration on several occasions: with respect to seismic investigations;[10] with respect to testing materials;[11] and with respect to model testing.[12] (Each of the three papers mentioned includes a comprehensive bibliography.)

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* Professor and Head of Department of Engineering Mechanics and Materials, The Pennsylvania State College.

[10] Geophysical Study of Soil Dynamics. Page 326, this volume.

[11] Dynamic Tests by Means of Induced Vibrations. *Proc.* Amer. Soc. Test. Mat. (1937) **37,** pt. 2, 634.

[12] Dynamic Properties of Structures Determined by Models. *Mech. Eng.* (Oct. 1937) **59,** 763.

One of the numerous amplitude distance curves on soil (Fig. 13) derived by the discusser from recent tests made at Washington, D. C., shows the same characteristic wave forms as are given in Fig. 10, demonstrating obviously the effect of interference phenomena.[13] The depth of the underlying strata has been determined at the same spot, both with the oscillator method and the blast method. The resulting deviation of 4 per cent is well within the accuracy of these two and any other methods.

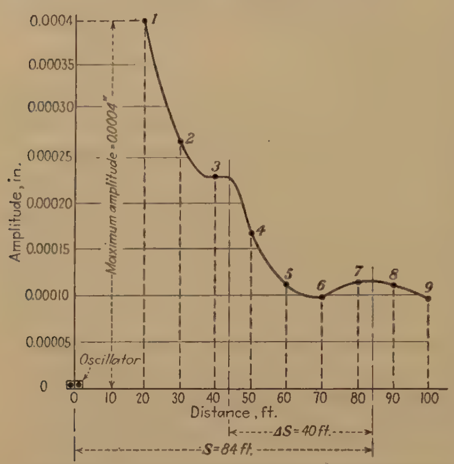

FIG. 13.—AMPLITUDE-DISTANCE CURVE ON SOIL.
Distance of maxima indicates second stratum. Depth to second stratum (rock), 22 feet.

In comparing, for example, the blast method and the oscillator method, it might be mentioned that it is difficult to build instruments that indicate correctly steep wave fronts, such as those caused by blasts. On the other hand, it is rather easy to build apparatus that will record a true sine curve such as those caused by oscillators. Furthermore, a blast endures only a brief moment, while the induced vibrations can be extended at will, so that there is ample time to adjust the recorders. Finally, tests can be reproduced with exactly the same amplitude and frequency as often as desired.

M. MUSKAT,* Pittsburgh, Pa.—Since the Rayleigh waves used for determining the transverse wave velocities must have been concentrated largely about the surface layers, how were the transverse velocities determined for the deeper beds in order to make the complete model of the basement for the dam?

M. K. HUBBERT,† New York, N. Y.—I notice that Dr. Heiland's model reservoir contained no water. Is not this a serious omission, since a reservoir filled with water would be expected to have a different dynamic behavior from one that contains no water?

C. A. HEILAND (author's reply).—The transverse wave velocities for bedrock were measured on outcrops of bedrock sandstone near the abutments of the dam (see top of page 369).

Replying to Dr. Hubbert, it is desirable to use the same model ratios for all model materials (see p. 361). When the scale reduction is considerable, as is required for a dam a mile long, it is nearly impossible to find a suitable model material for water. The omission is not considered serious, since this is a flood-control project in which the reservoir is seldom filled. Where water reservoirs have simple geometric forms, it is possible to substitute equivalent masses of one or two degrees of freedom in the model (see p. 361).

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[13] R. K. Bernhard: Highway Investigations by Means of Induced Vibrations. Pennsylvania State College *Bull.* 49 (1939).

* Gulf Research and Development Co.

† Department of Geology, Columbia University.

Measurement of Ordinary House Vibrations

By J. R. Thoenen,* Member and S. L. Windes,† Junior Member A.I.M.E.

(Washington, D.C., and College Park Meeting, October, 1937)

Abstract

The amplitudes and frequencies of vibrations of a four-story stucco building were measured by specially developed electrocapacitive seismometers. Records were made on three floor levels with as many as 10 seismometers recording simultaneously.

Sources of vibration were: (1) an electric motor with unbalanced pulley, maximum speed 1800 r.p.m., maximum centrifugal force 90 lb., (2) truck traffic (both solid and pneumatic tires) moving over inequalities in the street in front of the house, (3) falling weights dropped on the street pavement from different heights, (4) miscellaneous sources, including a man jumping on the floor of the house and slamming doors.

Tests were made to determine the natural frequencies of floor and wall panels by varying the speed of the unbalanced motor and by exciting the panels with a single impulse.

The vibrations recorded were compared to records of vibrations from quarry blasts and it was observed that the vibrations from truck traffic were comparable to those recorded from quarry blasts and in no instance was there any damage to the building.

As the tests were all of a negative character as far as damage was concerned, no index of destruction is formulated. Instead, the records are presented simply as vibration measurements obtained in an ordinary house under typical conditions.

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Correlation of Earth Resistivity with Geological Structure and Age

By R. H. Card*

(New York Meeting, February, 1937)

The geophysicist is interested greatly in the resistivities of different formations or parts of the earth's crust; sometimes he is interested in a single figure in the nature of an average, or what may be called the effective resistivity, of all the formations or parts of the crust at a given location to a certain depth. Conversely, the engineer whose work is concerned with the coordination of power and communication lines in such a manner as to avoid electrical interference between these two types of facilities is concerned very much with this effective resistivity; to a lesser extent he is interested in the resistivities of individual formations. In connection with his work the coordination engineer often estimates the effective resistivity of the earth from measurements of the mutual impedances of lines; in other instances he derives from measurements of a different type the resistivities of different layers of the crust and synthesizes these to obtain an estimate of the effective resistivity.

In connection with problems concerning the coordination of power and telephone lines, measurements from which earth resistivities can be estimated have been made at many points in the United States by the American Telephone and Telegraph Co. and its associated companies, in cooperation with various power companies and with railroad companies having electrified sections of line. Other such measurements have been made by the Joint Subcommittee on Development and Research of the Edison Electric Institute and the Bell Telephone System.

A study has been made of the correlation of these data with the ages and types of materials of the strata of the earth's crust at each of the test locations. The purpose of this study has been primarily to provide information to assist in predicting the order of magnitude of effective resistivities to be used in the preliminary consideration of problems concerned with the coordination of power and communication lines. To make the results of such a study most useful, it is essential that the geological data to be secured when such a prediction is wanted be readily obtainable. Data on the ages of strata and the types of materials composing geological structures can usually be obtained, therefore particu-

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Manuscript received at the office of the Institute March 16, 1937. Issued as T.P. 829, August, 1937.

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lar attention has been given to correlation with these characteristics of structures.

The study has also provided data that have been found useful for other purposes. For instance, they have been used in connection with investigations of the behavior of natural earth currents and of the potentials between points on the earth's surface that accompany the flow of these currents, and for estimating resistances of grounds, potential gradients in the vicinity of ground electrodes carrying currents, and the attenuation of radio signals.

Some of the more important results of this study were presented by the author in a recent paper.[1] The information given is reviewed herein, additional data are presented, and the discussion of certain features of the study is amplified.

Effective Resistivity

The effective resistivity of the earth as the term is used in coordination work is a parameter that may be substituted in a proper formula to derive the alternating current mutual impedance between two ground return circuits, the metallic sides of which are roughly parallel to one another. For a homogeneous earth structure the effective resistivity would equal the resistivity of the material composing this structure; but the earth's crust is nowhere homogeneous, therefore the effective resistivity is always of the nature of an average of the resistivities of the several strata of the crust.

For a given nonuniform earth structure, the effective resistivity is not a fixed quantity. Among the variations, one that is sometimes large is that of the frequency of the current concerned. Most of the data presented here were derived from measurements made at 60 cycles. In a few instances the frequency of the test current was 25 cycles. Where the effective resistivity given has been estimated from data on the resistivities of the different layers of a structure, a frequency of 60 cycles has been assumed.

Summary of Principal Results

The principal results of the study upon which this paper is based may be tersely summarized as follows:

1. There is a more or less consistent relation between the low-frequency effective resistivity and the age and physical characteristics of the geological formations at any given point. Formations to depths ranging from several hundred to several thousand feet must be considered. With certain exceptions increasing effective resistivity corresponds to increasing age, but this relation is very irregular. The range in resis-

[1] References are at the end of the paper.

tivities, in general, is from 2 to 30 meter-ohms* for very young structures to 1000 to 14,000 meter-ohms for the oldest structures.

2. Structures of given periods within certain large geographical regions have resistivities quite different from those of apparently similar structures of the same periods in other large regions.

3. Effective resistivities at different points within a given locality are generally fairly uniform if the structure is composed of the younger sedimentary strata or of sedimentary strata of intermediate age that have not been extensively metamorphosed. In pre-Cambrian areas, and in other areas where such metamorphosis has taken place, the effective resistivities usually are much less uniform.

4. Where volcanic rocks are present in a comparatively young structure, the effective resistivity may be considerably higher than otherwise would be expected.

5. Where local alluvial deposits overlie older rock strata, the effective resistivity may be very much lower than would be expected were the deposits not present. Soils, glacial drift, clays and other overburden cause variations in effective resistivities, but at low frequencies the effects apparently are not nearly so great as those of alluvial deposits.

6. In areas where combinations of very old and very young strata are present, as where unconsolidated sediments of Cretaceous or later age overlie pre-Cambrian rocks, the effective resistivities may range between very wide limits.

Principles of Correlation

When some of the natural processes of formation of the earth's crust are considered, it seems logical that there should be a more or less consistent relation between the effective resistivity and the age of geological structures. Such consideration indicates also, however, that this relation should be a highly irregular one and that factors other than age enter into the determination of the relative resistivities of different structures.

The correlation with age has as its basis the fact[2] that the resistivity of a particular material is determined largely by its pore volume, and the amount and composition of the waters contained in the pores. When sediments are first deposited the pore volume is comparatively great and, so far as this factor is concerned, they might be of either high or low resistivity. However, young sediments usually are comparatively soluble and the salts dissolved from them form good electrolytes; therefore usually they are of low resistivity. Exceptions to this are sands and gravels. These are relatively insoluble and where they are so situated that they do not receive conducting waters from other strata, as where they form the upper portions of structures, they may have very high resistivity. Also,

* Earth resistivities in meter-ohms may be converted to corresponding values in ohm-centimeters by multiplying by 100.

soils and other materials when located above the water table may have high resistivities because the pores may be only partly filled with water.

After deposition, sediments may undergo chemical changes or be subjected to great pressure; by these and other means the loose materials are consolidated and metamorphosed, and these changes may continue until dense crystalline structures are formed. During these processes, not only is the pore volume gradually decreased but a reduction in the amount of salts available to form low-conductivity electrolytes also may take place. Hence there is a tendency for the resistivity of sedimentary materials to increase with increasing age. The end products, the very old crystalline rocks, are not only of low pore volume but they are relatively insoluble and impervious. Consequently, they neither provide salts to form electrolytes nor receive conducting waters from other strata. Hence they are necessarily of high resistivity.

Igneous rocks when first formed may be porous, or relatively dense, therefore the resistivities of such rocks may vary widely. As time goes on they may be extensively metamorphosed, and when very old are of low pore volume and consequently of high resistivity.

Representative values of pore volumes, as given by Meinzer,[3] are 35 to 53 per cent of the total volume for clays and unconsolidated sands, 4 to 16 per cent for sandstones, shales, slates and limestones, and 0.02 to 1.85 per cent for old crystalline rocks.

Individual strata of the crust at any point cannot be considered independently. In the more previous materials, an intermingling of the impregnating waters of adjacent strata may take place; also, these waters sometimes flow long distances along the strata. Hence the resistivity of a particular stratum may be dependent not only upon its own characteristics but also upon the surrounding conditions, and strata of the same age and of apparently similar materials in different localities may have quite different resistivities.

Irregularities in the relation between age and resistivity are smoothed out to some extent so far as low-frequency effective resistivities are concerned, because the resistivities of the different parts of structures are combined and the resultant is in the nature of an average. As the frequency is increased the relative influence of the upper portions of structures becomes greater and there is a tendency for the irregularities to become more pronounced, because it is in these upper portions, such as the overburden and the weathered portions of rock strata, that large deviations from a smooth relation between age and resistivity are most frequently encountered.

Young formations are usually simple in physical structure, and, since they are relatively pervious, the flow of waters between strata should tend to make the structure appear even simpler from a resistivity standpoint. Where such formations prevail, it is to be expected that low-fre-

quency effective resistivities determined from measurements at different points within the same locality will be comparatively uniform. Very old structures are likely to be complicated by much faulting, crumpling and folding of the strata, and inclusions of igneous rocks. Very little flow of waters between the parts of such structures is to be expected; hence the resistivities of the several parts may be quite different. Also, where effective resistivities are high, the structure to depths of several thousand feet will be involved if the frequency is low; thus the chances are great for structural irregularities to affect the resistivities. Because of these factors, wide and erratic variations in effective resistivities are likely to be encountered in regions where very old rocks prevail.

Methods of Determining Earth Resistivities

The greater part of the resistivity data used in the study was derived from measurements of the mutual impedances of existing lines, usually a power-transmission line and a telephone line. In each test one of the lines concerned was energized as a ground-return circuit and the induced voltages in one or more sections of the second line were measured. The length of the energized line ranges in different cases from a few miles to as much as 50 miles or more. The sections of the second line usually range from a fraction of a mile to several miles in length.

For a second type of measurement used in other instances, either a section of an existing line or a long wire supported in a temporary manner was energized as a ground-return circuit and voltage measurements were made in several comparatively short wires, generally termed exploring wires, placed at various separations from the energized circuit and parallel to it. In making tests of this type the length of the energized wire usually ranges from about one mile where low resistivities are anticipated to several miles if high resistivities may be encountered. The exploring wires must be long enough to average out the effects of local irregularities in the earth's structure; 500 ft. is about the minimum length. If the earth's structure at the location concerned is highly irregular, the exploring wires should be at least 2000 ft. long. Lack of sensitivity of the available measuring equipment is sometimes the limiting factor and dictates the use of longer wires than otherwise would be required. The separations between the energized wire and the exploring wires usually range from around 100 ft. to 1000 ft. or more. Where high earth resistivities are anticipated, separations up to 1 or 2 miles are sometimes employed.

For either sectionalized or exploring-wire tests, the measuring methods, in principle, are relatively simple. Essentially they consist of measurements of the applied current in the energized wire and of the voltages to ground at one end of a wire in each selected section of the telephone line or at one end of each exploring wire with the distant end of each wire

grounded. The current is usually measured with an ordinary ammeter; the voltages with a voltmeter of either the thermocouple or vacuum-tube type. Sometimes both of these quantities and in addition the angles between the current and the various voltages are measured with an alternating current potentiometer.

With few exceptions the measurements made have been at a frequency of 60 cycles. For measurements at this or other low frequencies the current usually is obtained from a commercial source or from a specially provided generator. The applied current usually is from 1 or 2 amp. up to 20 or 30 amp. In a few instances measurements have been made over a range of frequencies from low values, around 25 cycles, to 1000 cycles or more. In these tests the currents of higher frequency have been obtained from an oscillator operated in conjunction with a power amplifier.

The mutual impedances derived from the current and voltage measurements are compared with values of these impedances calculated for a series of assumed values of earth resistivity. By interpolation, the values of effective earth resistivity corresponding to the measured mutual impedances are then found. Usually the resistivities indicated by the measurements in various exploring wires or subsections vary over a considerable range. Where the measurements from which the data presented herein were derived were made in consecutive subsections, the term "mean effective resistivity" is here applied to the value corresponding to the over-all mutual impedance. Where exploring wires were used, this term is applied to the weighted average of the resistivities derived from each of the several measurements.

Methods of making measurements of the types described, and of deriving from them the effective resistivities, have been discussed in published papers.[4–8] Formulas and graphs with the aid of which the mutual impedances may be calculated for given effective resistivities have also been published.[9]

Another method of measurement sometimes employed involves the exploration of the field between the end electrodes of a ground-return circuit energized with direct current. This method is similar to those frequently used in such work as geophysical prospecting, one of which has been described by Gish and Rooney.[10] These tests with direct current have provided a considerable number of data on the resistivities of individual portions of structures. In most of the tests included in this study in which such measurements have been made they have been supplementary to the a.c. tests, and the effective resistivities given have been derived from the a.c. measurements. However, a few of the effective resistivities have been derived from the d.c. tests alone. In deriving these resistivities use has been made of a recently published[11] formula for the mutual impedance of grounded wires on the surface of a two-layer earth.

General Correlation Data

The basic data for most of the cases included in the study are summarized in Fig. 1 in a manner to facilitate comparison of the resistivities derived from the results of the several tests with the geological periods and types of rocks involved in each case. When only one measurement of mutual impedance was made, the effective resistivities are indicated in this figure by isolated circles. Where measurements were made in several sections of lines or in several exploring wires, the mean effective resistivities are indicated by circles and the range of individual values by the vertical lines drawn through these circles. For any particular test the principal materials composing the structure and the geological periods of the principal strata are indicated by the brackets directly above the resistivity data. From left to right the tests are included in the figure in order of the ages of the predominant strata composing the structures, from the oldest to the youngest, so far as it has been practicable to do so.

On the right-hand side of Fig. 1 are shown the data for several tests involving structures composed of materials of two of the latest periods, the Cretaceous and Tertiary, directly underlain by pre-Cambrian strata. The upper strata are composed mostly of low-resistivity materials and the pre-Cambrian strata are of high resistivity. There is a distinct layer effect in such cases and the effective resistivities may range from the resistivity of the upper materials, where these are very thick, to values approaching the resistivity of the pre-Cambrian rocks as the thickness of the upper strata decreases. The mean effective resistivities indicated by these tests range from 24 to 1800 meter-ohms.

Disregarding the tests where these combinations of very old and very young strata are concerned, the principal data on the correlation with age that are included in Fig. 1 may be summarized as in the following tabulation, which is the result of grouping the tests in accordance with the geological periods to which the principal strata comprising the structure in each case belong and noting the ranges within which the effective resistivities determined by the greater part of the tests of each group lie. In this tabulation the various geological periods are listed in order from the oldest to the youngest.

	Meter-ohms
Pre-Cambrian and combinations of pre-Cambrian and Cambrian	1,000–14,000
Cambrian and Ordovician combinations	100–1,000
Ordovician to Devonian, inclusive, and combinations of these periods	50–600
Carboniferous, Triassic, and combinations of Carboniferous and earlier Paleozoic periods	10–300
Cretaceous, Tertiary, Quaternary and combinations of these periods	2–30

The methods followed in determining the particular strata concerned in each of the tests included in Fig. 1, and consequently in the table,

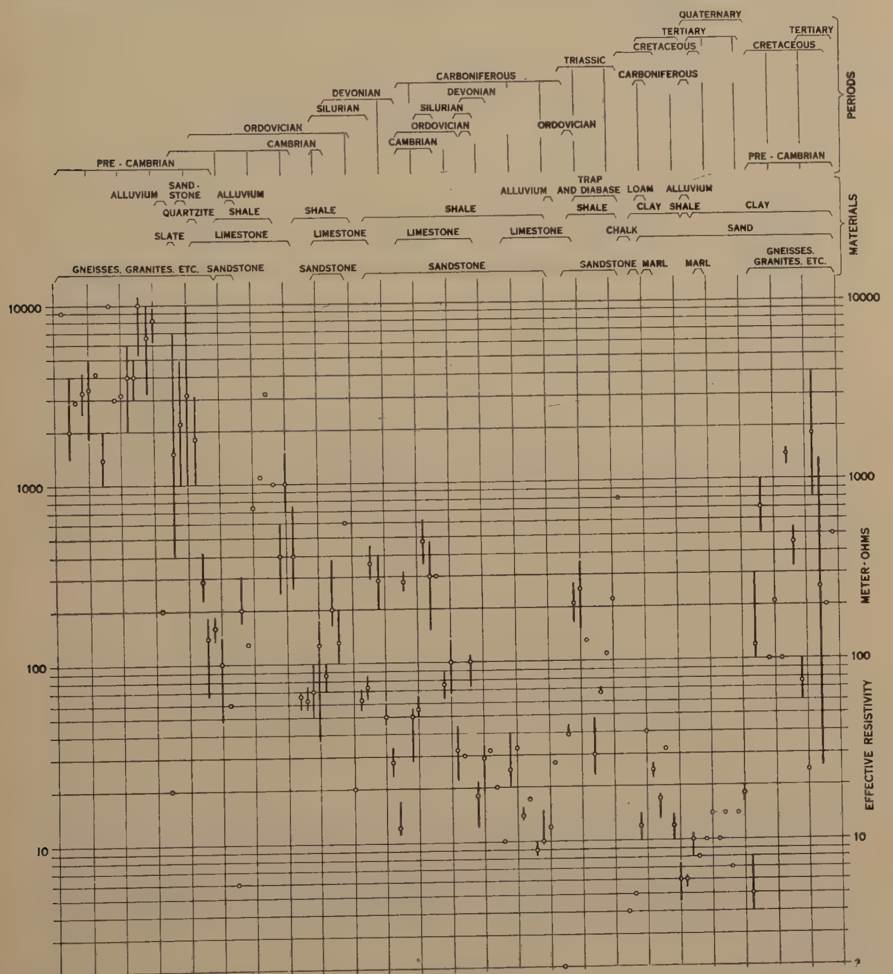

FIG. 1.—CORRELATION OF EFFECTIVE RESISTIVITIES WITH GEOLOGICAL PERIODS AND MATERIALS.

Mean resistivities indicated by circles.

For tests in which measurements were made in several subsections or with exploring wires, heavy lines indicate range of effective resistivities.

Sequence of types of materials is not necessarily their sequence in geological structures.

Overburden not indicated except when material thicknesses of alluvium were present.

"Limestone" as used in connection with Silurian and earlier formations often includes dolomites as well.

should be considered. Geological formations to a depth ranging from several hundred to several thousand feet may enter materially into the determination of the low-frequency effective resistivity of a structure.

Hence, in any given case, except where pre-Cambrian rocks alone are present, formations of several geological periods may underlie the area concerned and formations of more than one of these periods may influence the effective resistivity. For instance, at the test sites included in the second group of the summary, the upper part of the structure consists of Ordovician rocks. These are underlain by Cambrian strata, which, in turn, lie on the pre-Cambrian base. The question arises whether the test results were influenced by the Ordovician strata alone, by both the Ordovician and Cambrian, or by the Ordovician, Cambrian and pre-Cambrian. In these tests, it is probable that both the Ordovician and Cambrian strata were involved and that the effect of the pre-Cambrian was not appreciable. The tests in the other groups have been treated in a similar manner.

A discussion of the method of approach to this problem of the depth to which the strata should be considered in any particular case where the effective resistivity is known and the correlation with the structure is to be studied has been published.[1] This depth varies with the effective resistivity at the location concerned and with the frequency of the test current. Representative depths used in this study for a frequency of 60 cycles are as follows:

Effective Resistivity, Meter-ohms	Depth (Approximate), Ft.	Effective Resistivity, Meter-ohms	Depth (Approximate), Ft.
10	600	1000	3300
100	1500	3000	5000

Because of the comparative scarcity of areas within which the strata of the uppermost period are of great depth, formations of two or more periods are involved at most of the test locations, therefore it is not practicable to give representative ranges of resistivity for each individual period.

While Fig. 1 shows in striking manner the tendency for effective resistivities to decrease with decreasing age of structure, it also emphasizes the irregularity of this relation. It does not seem unreasonable that this irregularity should exist when the fact is considered that each geological period covers a wide range in age, that the strata of any given period are composed of many different types of materials, and that the impregnating waters of the strata of one period may be affected by neighboring strata of other periods.

Where strata of the same period have been deposited in widely separated areas, these factors have a greater opportunity to cause variations in resistivities. The test results confirm this; they indicate that the effective resistivities of structures of given periods within certain large

geographical regions are markedly different from those of structures of the same periods in other large regions. Within any one of such regions, excluding areas where igneous and highly metamorphosed sedimentary rocks are involved, the effective resistivities of structures of the same period are encompassed within a comparatively narrow band (Fig. 2). As shown by Fig. 2 and other figures to be discussed later, the effective resistivities in the Paleozoic areas of the eastern Appalachian Mountains and of central Tennessee are higher by ratios of 5 or 10 to 1 than those determined for structures of corresponding periods farther to the west.

Not only the variations between the effective resistivities at more or less widely separated localities are of interest; but also the variations

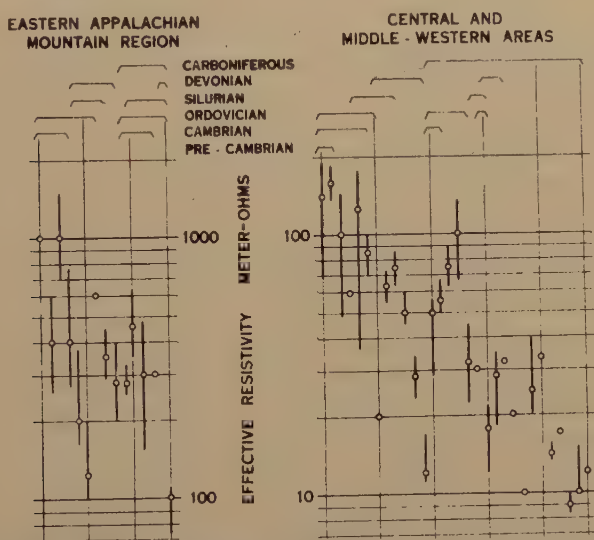

FIG. 2.—CORRELATION OF EFFECTIVE RESISTIVITIES AND GEOLOGICAL PERIODS. SELECTED TESTS IN PALEOZOIC AREAS.

indicated by measurements of the mutual impedances of different combinations of line sections or exploring wires in the same locality. In each of the tests in areas of the older rocks, the pre-Cambrian and combinations of pre-Cambrian and Cambrian, in connection with which measurements were made in several consecutive sections of line or several exploring wires, the variations in resistivities were very erratic and in several instances were quite wide, in one test nearly 20 to 1. Such variations are in accordance with the expectations for the behavior of these older structures as outlined above.

Considering the tests involving structures composed principally of strata of Cambrian or later periods, it will be noted from Fig. 1 that the range of variation indicated for any particular test is comparatively narrow; in general, 2 or 3 to 1. The structures in most of these cases

are composed of fairly uniform layers, horizontal or inclined at low angles, but even where these simpler structures are involved there may be considerable variation in the stratification at different points along the test lines. In some of the tests involving strata of intermediate age, the Cambrian, Ordovician, Silurian and Devonian, the structures are complicated by faulting, folding and tilting of the strata. The uniformity of the results shown by individual tests where such structural variations are encountered is perhaps explained, at least in part, by the tendency for the resistivities of the several parts of structures composed of the more pervious materials to merge, owing to the intermingling of the impregnating waters.

Resistivities of Types of Materials

The foregoing discussion has been confined almost entirely to the relation between the effective resistivities of structures and the periods to which the component parts of these structures are assigned. The resistivities of the various types of materials must also receive consideration.

Where unconsolidated or semiconsolidated materials such as clays, sands, marls and chalks predominate, the effective resistivities range from 2 to 32 meter-ohms; in one exceptional case a value of 100 meter-ohms was indicated. The resistivities of sands and gravels forming the upper portions of structures at such points sometimes have very high resistivities; ranging up to several thousand meter-ohms. In general, the higher of the values just given were found at locations where the upper strata were of this nature. Recent alluvial deposits along the courses of streams appear to be generally of low resistivity, as indicated by several effective resistivity determinations. In one instance d.c. measurements of the resistivity of alluvial deposits indicated a value of about 3 meter-ohms, except for a comparatively thin top layer, which had a resistivity of around 2000 meter-ohms. Comparatively few data have been obtained on the resistivities of soils, glacial deposits, clays and other surface materials, but in the tests included in this study the effects of such overburden have not been such as to throw the effective resistivities outside the ranges given in the above tabulation. This overburden is usually comparatively thin and apparently of fairly high resistivity compared with that of the underlying structure. Where this is true, the current penetration is such that the low-frequency effective resistivity is largely determined by the underlying strata.

The range in effective resistivities where consolidated sedimentary rocks such as shales, slates, limestones, dolomites and sandstones prevail is from 10 to 1800 meter-ohms. These rocks are generally found in combinations, so that the characteristics of the individual types cannot be separately determined from the data available. The indications are that the resistivities of the individual rock strata range from around

20 meter-ohms for some of the younger Paleozoic shales, limestones, and sandstones to some 3000 to 4000 meter-ohms for the Cambrian dolomites of eastern and central Tennessee. Effective resistivities where gneisses, schists, granites, quartzites and other dense, hard rocks prevail range from about 1000 to 14,000 meter-ohms. Direct current measurements indicate that the separate resistivities of the latter type of rocks may range from values of the order of 1000 meter-ohms to at least as high as 16,000 meter-ohms.

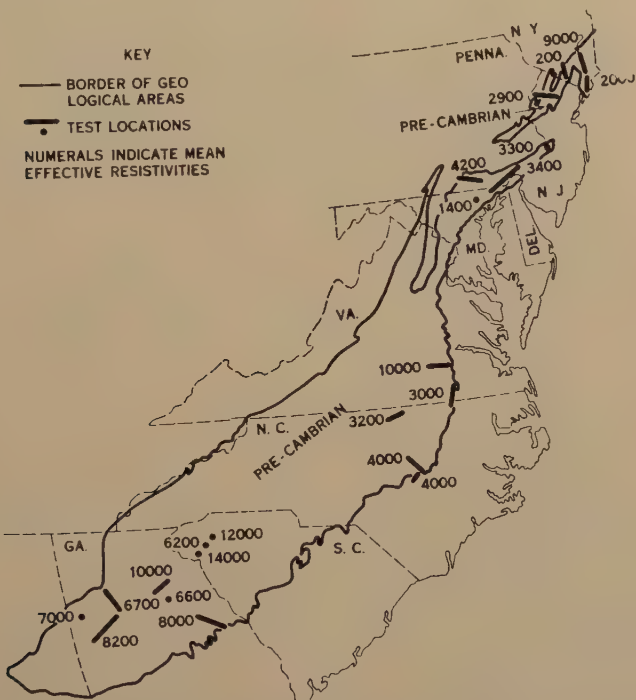

Fig. 3.—Areal geology and effective resistivities—pre-Cambrian areas.

Correlation Data for Specific Areas

To give a general picture of the resistivity characteristics of different areas within which tests have been made, Figs. 3 to 7, inclusive, are presented. The figures show the geological periods of the upper strata as they would appear were the overlying mantle of soil, glacial drift, local alluvial deposits, etc., removed. The small scale has necessitated the omission within the areas shown of many small areas of other periods. For each test the maps show the location, the mean effective resistivity and, roughly, the extent of the test section.

To emphasize the characteristics of the different areas, they have been divided into three groups: (1) the pre-Cambrian, (2) the Paleozoic

FIG. 4.—AREAL GEOLOGY AND EFFECTIVE RESISTIVITIES—APPALACHIAN PALEOZOIC AREAS.

FIG. 5.—AREAL GEOLOGY AND EFFECTIVE RESISTIVITIES—CENTRAL AND MIDDLE WESTERN PALEOZOIC AREAS.

and (3) the Mesozoic and Cenozoic. The principal Paleozoic areas are further subdivided into two subgroups which, for convenience, are denoted, respectively, as the Appalachian and the Central and Middle-Western areas. The principal Mesozoic and Cenozoic areas, denoted as the Eastern and coastal plain areas, are shown on a single map. In addi-

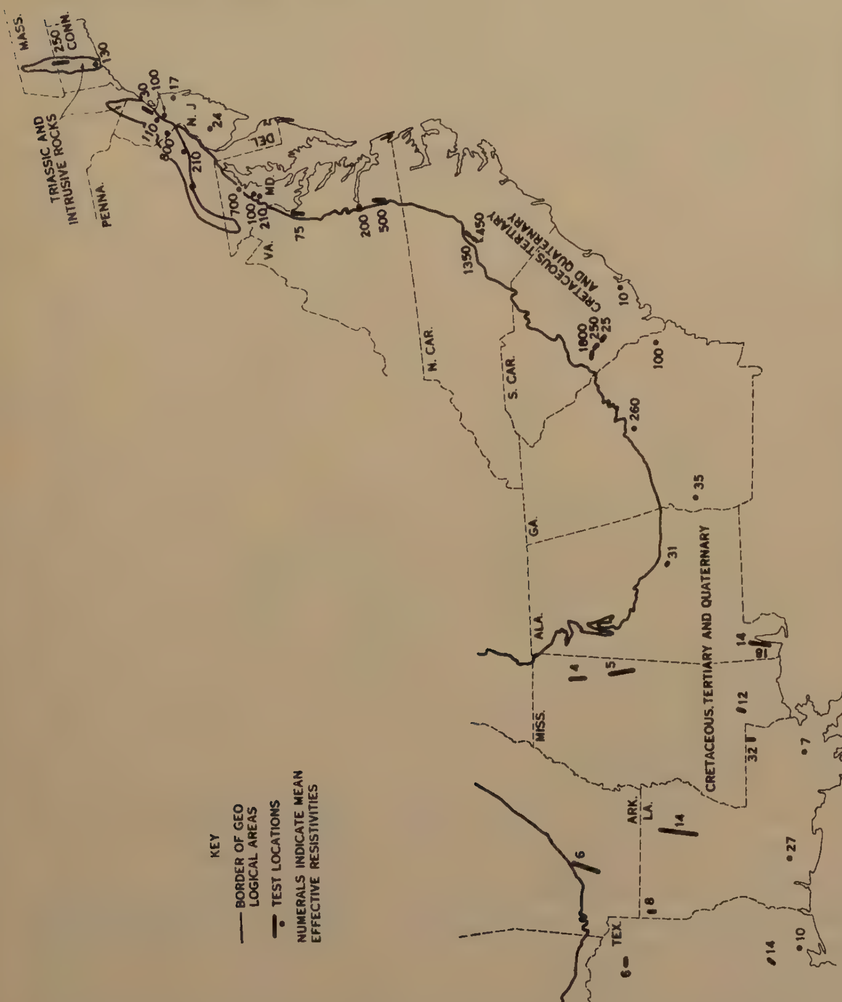

FIG. 6.—AREAL GEOLOGY AND EFFECTIVE RESISTIVITIES—EASTERN AND COASTAL PLAIN MESOZOIC AND CENOZOIC AREAS.

tion a number of tests in Paleozoic, Mesozoic and Cenozoic areas of the West are combined into one figure.

Within the pre-Cambrian areas just west of the Atlantic coastal plain (Fig. 3), the test results indicate mean effective resistivities ranging from 1400 to 14,000 meter-ohms. The only exception is one test that indicated a value of 200 meter-ohms. At the site of this test the rocks are partly

covered with alluvium. Within the southern portion of these areas, in South Carolina and Georgia, a number of direct-current measurements have been made. Here there is a highly complex array of pre-Cambrian schists, gneisses, granites and other dense, crystalline rocks, together with large areas of intrusive Carboniferous granite. A thick layer of clay covers almost the entire region. The direct-current measurements indicate that throughout most of this region the resistivities of both the pre-Cambrian rocks and the later intrusions range from 7000 to 15,000 meter-ohms. Only two measurements have indicated lower values:

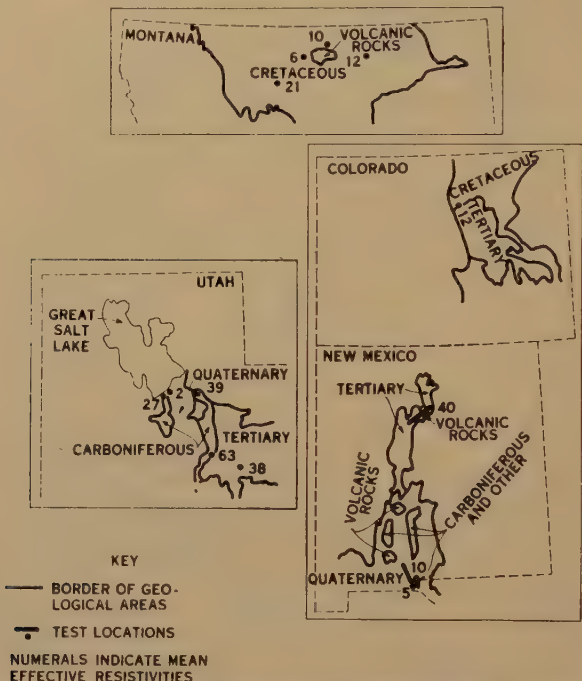

FIG. 7.—AREAL GEOLOGY AND EFFECTIVE RESISTIVITIES—WESTERN PALEOZOIC, MESOZOIC AND CENOZOIC AREAS.

around 3000 meter-ohms. The clays, possibly together with the upper, weathered portions of the rock strata, have resistivities ranging generally from 400 to 2000 meter-ohms.

A wide variety of structures is represented in the Paleozoic areas shown in Figs. 4, 5 and 7; also, the effective resistivities vary widely, the mean values ranging from 2 to 3200 meter-ohms. Values above 1800 meter-ohms have been indicated only in central Wisconsin, where pre-Cambrian rocks lie under a thin covering of Cambrian sandstone and 100 ft. or less of glacial drift, and in southeastern New York, where rocks of the earlier Paleozoic periods have been partly or wholly metamorphosed and pre-Cambrian rocks are also involved. At the site of the one test

showing the extremely low value of 2 meter-ohms, which was made in the salt flats bordering Great Salt Lake, Carboniferous strata are covered to an unknown depth by recent alluvial deposits. A second test, not shown on the maps, made in northeastern New York at a location where Ordovician and Cambrian rocks are covered by recent alluvial deposits, likewise indicated a low resistivity (6 meter-ohms).

The remaining tests of the Paleozoic group may be divided into two classifications: (1) those in the Appalachian areas, which indicated values from 67 to 1800 meter-ohms, and (2) those in the Central and Middle Western and Western areas, which indicated values from 9 to 160 meter-ohms. The line of demarcation between these areas of higher resistivity and those of lower resistivity appears to be rather sharp. The higher resistivities are found in the simple Devonian structures of the Catskill Mountains; in the eastern Appalachians from Pennsylvania to Alabama, where the structures are complicated by extensive folding, tilting and faulting of the strata; and again in the relatively simple structures of the central Tennessee arch, where the effective resistivities range from 80 meter-ohms for the thicker Carboniferous structures to a maximum of 1800 meter-ohms for Ordovician and Cambrian combinations. Immediately to the west, the lower resistivities of the central and midwestern areas are encountered, ranging from 9 to 30 meter-ohms for thick Carboniferous structures to 100 to 160 meter-ohms for the Ordovician and Cambrian. The presence of glacial drift doubtless affected to some extent the results of the tests in Michigan and the northwestern portion of Fig. 5. At the remaining test sites, glacial deposits are either nonexistent or very thin. Resistivities in the Carboniferous areas of Utah are also low (Fig. 7). Only meager data on the separate resistivities of the different strata in these areas have been obtained.

Two distinctly different types of structure are of concern in the Mesozoic and Cenozoic areas of Fig. 6: (1) those of the Triassic areas shown in the northern portion of the figure, and (2) the younger areas of the Atlantic and Gulf coastal plains.

In the Triassic areas, the structures are highly complicated. The strata have been tilted and extensively faulted, and there are many volcanic intrusions and extrusions. Shales and sandstones form the greater portion of the structures. In some instances layers of the sedimentary rocks several hundred feet thick lying adjacent to the volcanic sheets were highly altered by the heat of the molten lava. As would be expected in regions of this degree of complication, the measurements at the different test sites indicated widely different resistivities, the range in mean values being from 30 to 800 meter-ohms.

The sediments of the Atlantic coastal plain, of Cretaceous, Tertiary and Quaternary periods, lie directly on pre-Cambrian rocks. The sediments are generally very thin along the western edge of the plain and the

effective resistivities there range from 75 to 1800 meter-ohms. Toward the coast line the thickness gradually increases to 2000 ft. or more, and the effective resistivities decrease to values of 10 to 100 meter-ohms. The upper portion of the sedimentary formations seems to consist generally of gravels and sands of high resistivity—up to several thousand meter-ohms. This upper part in some localities is thin; in others it is several hundred feet thick. Underlying this high-resistivity layer are low-resistivity sands, clays, marls and other such materials.

At the test sites on the Gulf coastal plain, the Mesozoic and Cenozoic strata, composed generally of unconsolidated sediments, are very thick, in most instances more than 2000 ft. The effective resistivities are uniformly low, most of the mean values lying between 4 and 18 meter-ohms. Low resistivities, from 10 to 40 meter-ohms, were also indicated in the Western Mesozoic and Cenozoic areas of Fig. 7. Few data on the resistivities of the separate parts of the structures in these areas have been obtained. One test in Louisiana indicated a top layer about 1300 ft. thick, having a resistivity of 10 to 50 meter-ohms, overlying an unknown thickness of sediments having a resistivity of about one meter-ohm.

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DISCUSSION

(L. W. Blau presiding)

G. Wascheck,* New York, N. Y.—We are actively engaged in inductive coordination work, and this attempt at a classification of the geological features of the country on the basis of resistivity has been very encouraging. It is a stupendous problem on such a large scale, but we feel it has been a big step forward to a practical solution of our problems.

I should like to call attention to the relatively great depths that are necessary to be investigated, particularly in high-resistivity areas. There, even at 60 cycles, which is the usual power frequency, the current penetration is fairly large and the lower depths may be influential in determining the effective resistivity. Nevertheless, our experience in high-resistivity areas has brought out two facts. The first, which may be considered an advantage, is that the measurements usually are fairly easy to get because the mutual resistance is rather high. The extreme sensitivity of the instruments is not required and the extraneous voltages are a smaller proportion of the voltages to be measured. This is particularly important at the wider spacing of electrodes. We generally employ the Wenner method, with equally spaced electrodes, supplemented at times by the partitioning method. For normal explorations, electrode spacings up to over 3000 ft. are used and in some special cases spacings of 1 to 3 miles were employed.

The second phenomenon encountered in high-resistivity areas is the presence of large irregularities that appear in such regions. This is in contrast with a more or less uniformity of structure in low-resistivity areas. In this regard, I might say that whereas the problems are similar in nature, there are some differences in details between the work of determining earth resistivity for inductive coordination work and the measurement of surface potentials by geophysicists in exploring for oil, gold or veins of different kinds. That is, we are interested in the problem from a macroscopic point of view. When the geophysicist comes to an irregularity as determined by his surface measurements, he usually stops and begins to investigate in some detail the nature of this irregularity. It may be what he is looking for. We, however, are interested in the over-all resistivity to be used in connection with a given communication exposure, which may extend over 1 mile, 2 miles or 25 miles. We look for the small irregularities to iron out—as they usually do over a greater length—so that, paradoxically, it might be said that we look for a uniform heterogeneity. This permits an over-all average resistivity and structure to be outlined for a given area.

In our work we have developed instruments to our special needs, although we have followed the pioneer work of the geophysicists in their early work with direct-current testing, and we are grateful to know of their work and experience that has led the way in our investigations, which we have adopted or modified as the special nature of our problem demands.

J. L. Adler,† New York, N. Y.—Mr. Card pointed out that he had (Fig. 6) a station, a little way north of Savannah, Georgia, showing a resistivity of 100, which he said was the highest in that portion of the coastal plane. T. C. Cook, in a recent bulletin, reports an inlier of Triassic hills containing diabase and covered by 2000 ft. of sediment, which was struck in a well drilled not very far from the locality of that high resistivity.

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* Bell Telephone Laboratories.

† Pathfinder Exploration Co.

A. C. Lane,* Cambridge, Mass.—I would like to know how far Mr. Card tried to correlate these things with the amount of water and salt water contained in the formations. Of course, the older strata are more compact, and naturally have less water.

R. H. Card.—We have done nothing along the line mentioned by Dr. Lane. With regard to Mr. Adler's remarks, we merely have the one test showing the effective resistivity near Savannah. We have made no surveys of variation of resistivity with depth at that point.

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* Tufts College.

Interpretation of Earth-resistivity Curves

By G. F. Tagg*

(New York Meeting, February, 1937)

In an earlier paper[1], the author described a method of interpreting earth-resistivity curves, based on the theoretical investigation of a single horizontal underlying stratum. If the four-electrode system of Wenner is employed, the resistivity in a homogeneous soil is given by the formula

$$\rho = 2\pi a R \qquad [1]$$

where a is the electrode separation and R is the resistance of the earth between the two inner electrodes. If the earth is not perfectly homogeneous, the application of this formula gives an apparent specific resistance varying with the electrode separation.

In the special case of the single horizontal underlying stratum, there is a surface layer of thickness h and resistivity ρ_1 and a lower layer, extending to infinity, of resistivity ρ_2. It has been shown by Lancaster-Jones[2] that the apparent resistivity obtained by an application of equation 1 is related to the true resistivities, the depth h and the electrode separation a by an expression of the form

$$\frac{\rho_a}{\rho_1} = 1 + 4F \qquad [2]$$

where ρ_a is the apparent resistivity and F is a function (actually an infinite series), of the ratio h/a, and a reflection coefficient $k = (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$. For given values of h/a and k, the right-hand side of equation 2 can be calculated, and curves drawn showing the relationship between ρ_a/ρ_1, h/a and k. Such a set of curves for the case when $\rho_1 > \rho_2$ is given in Fig. 1.

When ρ_1 is less than ρ_2 it is more convenient to use conductivity instead of resistivity. Hence, if σ_a = the apparent conductivity = $1/\rho_a$ and σ_1 = the conductivity of the upper layer = $1/\rho_1$, it follows that

$$\frac{\sigma_a}{\sigma_1} = \frac{\rho_1}{\rho_a} = \frac{1}{1 + 4F} \qquad [3]$$

Manuscript received at the office of the Institute March 16, 1936. Issued as T.P. 755, February, 1937.

* Chartered Electrical Engineer, Osterley, Middlesex, England.

[1] References are on page 403.

A set of curves, generally similar to those of Fig. 1, can be calculated from this formula giving the values of σ_a/σ_1 for various values of h/a and k.

The method of interpretation previously suggested by the author in which these master curves could be used, can be summarized as follows:

1. Determine the surface resistivity ρ_1 by a series of careful measurements at small electrode intervals.

2. Determine the apparent resistivity for a number of values of electrode interval.

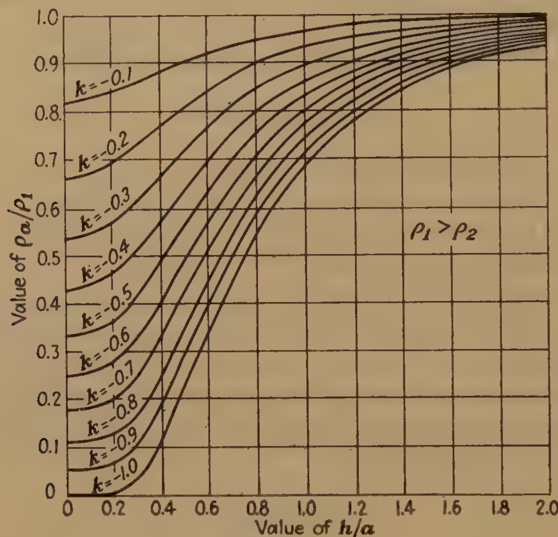

FIG. 1.

3. Plot a curve of apparent resistivity against electrode separation.

4. From this curve read off the values of the apparent resistivity ρ_a for various values of the electrode separation a, and for each value of a determine the value of ρ_a/ρ_1, if ρ_a is less than ρ_1 or the value of σ_a/σ_1, if ρ_a is greater than ρ_1.

5. From the master curves (Fig. 1, etc.), read off, for each value of a, a series of corresponding values of h/a and k, and from these calculate a corresponding series of values of h and k.

6. Plot curves, for each value of electrode interval, of h against k. These curves should all intersect in a point giving the true values of h and k.

This method is theoretically sound and, if it can be applied, should give accurate results. It must, however, be realized that the theory is based on the assumption that each layer is of a uniform resistivity, and that the interface between the two layers is parallel to the surface.

Another method of interpretation, based on theoretical considerations, has been devised by Dr. Roman[3]. In this method, the observed curves are superimposed on theoretical curves, and it is endeavored to find a theoretical curve that fits closely to the observed one.

The method described here can also be applied to three-layer problems, in which the experimental curve is of the type *ABCD* in Fig. 2. It has been shown[4] that a curve of this type may be built up from two curves of the two-layer type *EFBH* and *KCD* in Fig. 2. This method can be applied to each of these curves to determine the depths to the two interfaces, provided the value of the surface resistivity ρ_1 can be obtained in each case. For the curve *KCD* this is the equivalent resistivity of the two upper layers, and is not an easy quantity to determine.

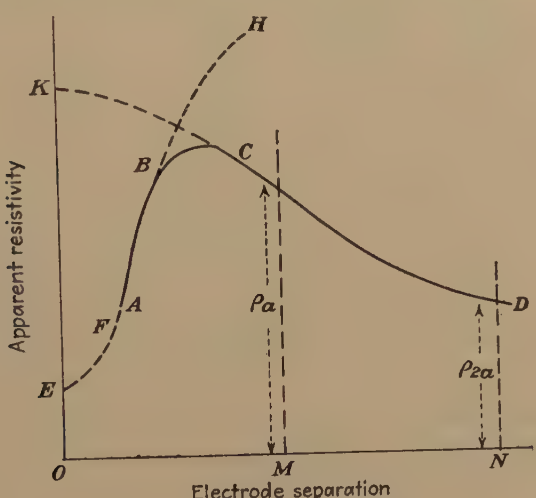

FIG. 2.

A method of successive approximation has been described by Pirson[5], but this becomes very laborious in some cases, and it is sometimes very difficult to obtain satisfactory results.

It is thus clear that the big difficulty in the application of the method, as at present described, lies in the determination of the surface resistivity or the equivalent surface resistivity. If it could be arranged so that the surface resistivity were no longer necessary, this difficulty would disappear. This can be done in the following manner (originally described in *Mining Magazine*, Sept., 1935):

Suppose the apparent resistivity at a known electrode interval a is ρ_a. Then from equation 2,

$$\frac{\rho_a}{\rho_1} = 1 + 4F(a) \qquad [4]$$

where $F(a)$ is a function of h/a and k. Next, suppose the apparent resistivity at another interval na is ρ_{na}, where n is any number, then

$$\frac{\rho_{na}}{\rho_1} = 1 + 4F(na) \tag{5}$$

where $F(na)$ is a function of h/na and k.

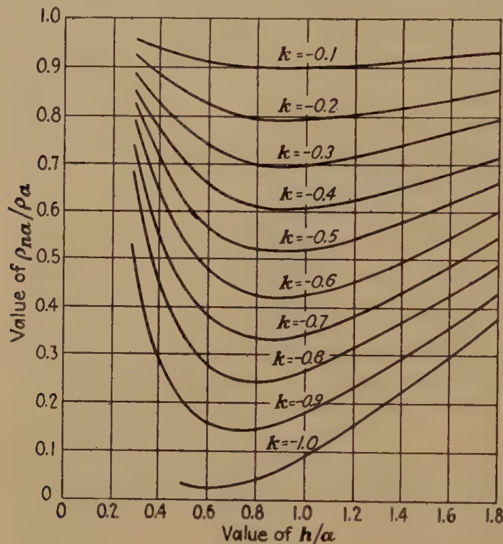

FIG. 3.

Consider first the case when ρ_1 is greater than ρ_2; i.e., the upper layer is of the higher resistivity. Dividing equation 5 by equation 4 gives:

$$\frac{\rho_{na}}{\rho_a} = \frac{1 + 4F(na)}{1 + 4F(a)} \tag{6}$$

Secondly, when ρ_1 is less than ρ_2 (the upper layer has the lower resistivity), dividing equation 4 by equation 5 gives:

$$\frac{\rho_a}{\rho_{na}} = \frac{\sigma_{na}}{\sigma_a} = \frac{1 + 4F(a)}{1 + 4F(na)} \tag{7}$$

For any value of n, the values of ρ_{na}/ρ_a or σ_{na}/σ_a can be calculated for any values of h/a and k and sets of master curves can be produced in this way. Such a set of curves is given in Fig. 3 for a value of n equal to 3, for the case when ρ_1 is greater than ρ_2. Similar sets of curves have been calculated by the author for a number of values of n, and for the two cases, ρ_1 greater than ρ_2 and ρ_1 less than ρ_2. The necessary figures for plotting these curves are given in the accompanying tables.

The method of using these master curves for the determination of h can be explained by reference to Fig. 2. Consider the second part of the

curve CD. The apparent resistivity at an electrode interval OM can be read off as ρ_a and similarly the apparent resistivity ρ_{2a} at an electrode interval $ON = 2OM$. The ratio ρ_{2a}/ρ_a can then be calculated. On referring to the set of master curves for $n = 2.0$, ρ_1 greater than ρ_2, a series of corresponding values of h/a and k can be read off. Since a is known, these can be converted into a series of corresponding values of h and k. By taking other values of a and/or n, further sets of corresponding values of h and k can be obtained, and on plotting curves of h against k, these curves should all intersect in a point, giving the true values of h and k. Having the sets of master curves, the interpretation is fairly simple, and it is no longer necessary to determine the surface resistivity, but the calculations are based on the actual measured values of the apparent resistivity.

The author has used this method successfully many times. Tables from which master curves can be plotted follow on pages 404 to 407.

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DISCUSSION

(C. A. Heiland presiding)

D. A. Keys,* Montreal, Que. Canada.—About 5 yr. ago, Prof. L. V. King worked out the complete theory of the two-layer problem and also the case where the resistivity varies with depth according to some definite law. That paper was published in the *Proceedings* of the Royal Society.[6] He mentions at the end of the paper the method by which the curves obtained from his formulas may be used. He does specify that it is from the final slope of the curve at large electrode separations that the depth is obtained. He gives in that paper a rather simple method of determining the depth from such resistivity curves.

M. Muskat,† Pittsburgh, Pa.—In relation to Professor King's work, I would also say that although I cannot state the exact relation of the resistivity curves to depth, it is a rather direct conclusion from the general theory that the asymptotic behavior of the potential variation or of the apparent resistivity for any system of multiple layers is always determined by the character of the deepest layer. This is the basis for the

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* Department of Physics, McGill University.

[6] L. V. King: *Proc.* Royal Soc. (1933) **139-A**, 237–277.

† Gulf Research and Development Co.

TABLE 1.—*Values of ρ_{na}/ρ_a for Various Values of h/a*

k	Value of ρ_{na}/ρ_a when h/a is											
	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.25	1.4	1.6	1.8
	n = 1.1; ρ_1 greater than ρ_2											
−0.1	0.992	0.990	0.988	0.988	0.990	0.990	0.991	0.992	0.996	0.996	0.997	0.998
−0.2	0.985	0.980	0.977	0.977	0.979	0.979	0.982	0.985	0.990	0.991	0.993	0.994
−0.3	0.980	0.967	0.963	0.963	0.968	0.970	0.973	0.978	0.983	0.988	0.991	0.989
−0.4	0.973	0.959	0.952	0.952	0.958	0.956	0.964	0.970	0.979	0.983	0.988	0.986
−0.5	0.964	0.943	0.931	0.937	0.945	0.948	0.952	0.959	0.972	0.979	0.983	0.982
−0.6	0.955	0.921	0.915	0.918	0.930	0.936	0.945	0.952	0.966	0.973	0.980	0.980
−0.7	0.929	0.891	0.896	0.901	0.919	0.925	0.931	0.944	0.962	0.968	0.980	0.979
−0.8	0.919	0.869	0.870	0.883	0.902	0.912	0.924	0.935	0.957	0.963	0.977	0.976
−0.9	0.849	0.827	0.807	0.849	0.882	0.895	0.913	0.924	0.948	0.957	0.972	0.973
−1.0	0.708	0.730	0.780	0.817	0.862	0.876	0.862	0.915	0.943	0.953	0.968	0.969
	n = 1.2; ρ_1 greater than ρ_2											
−0.1	0.980	0.982	0.978	0.980	0.980	0.981	0.984	0.985	0.990	0.991	0.994	0.995
−0.2	0.973	0.967	0.959	0.959	0.960	0.963	0.966	0.970	0.978	0.982	0.986	0.986
−0.3	0.959	0.945	0.933	0.936	0.939	0.943	0.949	0.953	0.966	0.972	0.979	0.980
−0.4	0.947	0.926	0.912	0.911	0.911	0.921	0.930	0.940	0.956	0.964	0.971	0.972
−0.5	0.922	0.897	0.883	0.884	0.893	0.903	0.911	0.921	0.935	0.953	0.966	0.969
−0.6	0.915	0.864	0.843	0.853	0.867	0.876	0.892	0.906	0.933	0.946	0.957	0.962
−0.7	0.891	0.829	0.808	0.818	0.835	0.848	0.865	0.887	0.922	0.935	0.952	0.960
−0.8	0.825	0.773	0.755	0.779	0.796	0.818	0.847	0.873	0.913	0.928	0.946	0.954
−0.9	0.717	0.696	0.676	0.747	0.772	0.778	0.822	0.853	0.899	0.916	0.939	0.948
−1.0	0.312	0.475	0.587	0.661	0.709	0.756	0.796	0.831	0.888	0.908	0.933	0.941
	n = 1.3; ρ_1 greater than ρ_2											
−0.1	0.983	0.976	0.971	0.972	0.972	0.973	0.976	0.974	0.984	0.986	0.990	0.991
−0.2	0.965	0.951	0.943	0.941	0.941	0.946	0.940	0.955	0.967	0.967	0.978	0.981
−0.3	0.942	0.924	0.909	0.907	0.911	0.917	0.925	0.932	0.948	0.956	0.967	0.972
−0.4	0.932	0.896	0.881	0.877	0.878	0.887	0.896	0.907	0.935	0.945	0.957	0.961
−0.5	0.902	0.856	0.838	0.834	0.843	0.857	0.871	0.883	0.916	0.930	0.945	0.954
−0.6	0.890	0.818	0.794	0.793	0.804	0.822	0.842	0.860	0.899	0.916	0.935	0.943
−0.7	0.854	0.762	0.752	0.748	0.764	0.785	0.804	0.831	0.881	0.902	0.924	0.939
−0.8	0.790	0.689	0.672	0.707	0.713	0.741	0.777	0.806	0.866	0.889	0.914	0.929
−0.9	0.647	0.581	0.574	0.643	0.671	0.706	0.745	0.776	0.850	0.874	0.903	0.920
−1.0	0.169	0.308	0.450	0.544	0.595	0.645	0.706	0.746	0.828	0.858	0.895	0.911
	n = 1.4; ρ_1 greater than ρ_2											
−0.1	0.978	0.969	0.965	0.962	0.964	0.964	0.968	0.967	0.978	0.985	0.986	0.988
−0.2	0.958	0.941	0.929	0.923	0.928	0.929	0.935	0.941	0.956	0.962	0.971	0.974
−0.3	0.933	0.908	0.888	0.881	0.889	0.893	0.902	0.911	0.931	0.943	0.954	0.963
−0.4	0.916	0.875	0.851	0.841	0.848	0.856	0.868	0.878	0.911	0.924	0.941	0.950
−0.5	0.883	0.826	0.793	0.792	0.804	0.816	0.833	0.848	0.887	0.905	0.926	0.939
−0.6	0.868	0.783	0.748	0.736	0.758	0.771	0.797	0.818	0.864	0.896	0.911	0.927
−0.7	0.825	0.713	0.692	0.684	0.706	0.728	0.753	0.781	0.842	0.868	0.898	0.915
−0.8	0.754	0.633	0.606	0.612	0.647	0.676	0.710	0.746	0.819	0.849	0.884	0.903
−0.9	0.608	0.505	0.493	0.542	0.602	0.632	0.675	0.710	0.799	0.830	0.868	0.891
−1.0		0.417	0.340	0.423	0.509	0.557	0.617	0.670	0.771	0.812	0.855	0.879

well-known fact that to find out what is at great depths one must make the potential measurements farther and farther away from the current electrodes. This is a general conclusion regardless of the number of layers, and together with the apparent resistivity itself. The slope of the resistivity curves at distant points is also directly determined by the conductivity of the deepest layers.

TABLE 1.—(*Continued*)

k	Value of ρ_{na}/ρ_a when h/a is										
	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.10	1.25	1.8
	$n = 1.5$; ρ_1 greater than ρ_2										
−0.1	0.977	0.965	0.959	0.957	0.957	0.958	0.961	0.959	0.967	0.970	0.984
−0.2	0.954	0.932	0.913	0.913	0.912	0.915	0.921	0.925	0.931	0.940	0.969
−0.3	0.932	0.896	0.873	0.866	0.866	0.872	0.881	0.889	0.899	0.917	0.957
−0.4	0.908	0.861	0.826	0.816	0.811	0.818	0.839	0.849	0.865	0.889	0.944
−0.5	0.870	0.806	0.765	0.761	0.764	0.775	0.796	0.810	0.831	0.856	0.927
−0.6	0.854	0.753	0.701	0.701	0.703	0.722	0.752	0.772	0.797	0.831	0.908
−0.7	0.808	0.676	0.630	0.640	0.642	0.669	0.703	0.733	0.765	0.801	0.893
−0.8	0.739	0.613	0.558	0.556	0.587	0.615	0.654	0.687	0.729	0.777	0.877
−0.9	0.622	0.511	0.422	0.460	0.494	0.544	0.608	0.630	0.675	0.737	0.860
−1.0	0.225	0.192	0.265	0.350	0.431	0.489	0.544	0.582	0.636	0.707	0.848

k	Value of ρ_{na}/ρ_a when h/a is											
	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.25	1.8
	$n = 2.0$; ρ_1 greater than ρ_2											
−0.1	0.985	0.969	0.952	0.939	0.934	0.927	0.929	0.931	0.930	0.938	0.955	0.966
−0.2	0.968	0.941	0.905	0.881	0.866	0.858	0.859	0.851	0.869	0.875	0.888	0.931
−0.3	0.956	0.910	0.857	0.818	0.797	0.787	0.787	0.793	0.804	0.814	0.833	0.896
−0.4	0.950	0.888	0.806	0.758	0.725	0.711	0.714	0.719	0.738	0.750	0.776	0.864
−0.5	0.943	0.868	0.743	0.684	0.649	0.630	0.642	0.651	0.674	0.690	0.722	0.831
−0.6	0.936	0.824	0.687	0.639	0.563	0.550	0.565	0.574	0.608	0.631	0.672	0.800
−0.7	0.924	0.775	0.598	0.512	0.474	0.463	0.492	0.502	0.542	0.566	0.619	0.772
−0.8	0.926	0.725	0.499	0.415	0.376	0.389	0.402	0.437	0.473	0.507	0.562	0.738
−0.9	0.850	0.598	0.342	0.260	0.253	0.276	0.314	0.348	0.416	0.438	0.504	0.706
−1.0		0.017	0.067	0.075	0.104	0.162	0.221	0.279	0.332	0.384	0.447	0.677
	$n = 2.5$, ρ_1 greater than ρ_2											
−0.1	0.982	0.971	0.946	0.930	0.919	0.911	0.907	0.909	0.909	0.915	0.921	0.945
−0.2	0.971	0.935	0.895	0.859	0.841	0.828	0.822	0.820	0.827	0.832	0.845	0.889
−0.3	0.958	0.902	0.839	0.791	0.760	0.744	0.735	0.737	0.743	0.751	0.770	0.838
−0.4	0.950	0.874	0.789	0.722	0.684	0.662	0.649	0.648	0.661	0.669	0.697	0.788
−0.5	0.943	0.846	0.739	0.639	0.597	0.569	0.557	0.561	0.580	0.592	0.627	0.738
−0.6	0.928	0.805	0.668	0.564	0.510	0.479	0.468	0.473	0.508	0.511	0.557	0.691
−0.7	0.924	0.730	0.577	0.468	0.409	0.429	0.381	0.387	0.424	0.436	0.488	0.645
−0.8	0.924	0.700	0.493	0.356	0.314	0.294	0.298	0.321	0.338	0.373	0.420	0.599
−0.9	0.881	0.578	0.329	0.211	0.187	0.185	0.198	0.222	0.257	0.287	0.364	0.538
−1.0			0.050	0.035	0.047	0.061	0.092	0.126	0.176	0.220	0.285	0.498

k	Value of ρ_{na}/ρ_a when h/a is										
	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.25	1.8
	$n = 3.0$; ρ_1 greater than ρ_2										
−0.1	0.962	0.942	0.937	0.912	0.903	0.896	0.897	0.891	0.897	0.903	0.928
−0.2	0.924	0.890	0.851	0.826	0.813	0.799	0.797	0.795	0.800	0.809	0.858
−0.3	0.890	0.833	0.778	0.742	0.719	0.706	0.702	0.702	0.709	0.722	0.789
−0.4	0.862	0.784	0.713	0.658	0.634	0.616	0.608	0.610	0.618	0.631	0.713
−0.5	0.829	0.787	0.638	0.565	0.542	0.518	0.517	0.515	0.528	0.548	0.662
−0.6	0.799	0.658	0.552	0.481	0.447	0.425	0.423	0.426	0.440	0.466	0.601
−0.7	0.746	0.571	0.457	0.383	0.350	0.332	0.333	0.340	0.359	0.389	0.542
−0.8	0.684	0.481	0.352	0.277	0.257	0.245	0.246	0.265	0.291	0.324	0.483
−0.9	0.528	0.329	0.209	0.157	0.148	0.143	0.154	0.177	0.200	0.240	0.429
−1.0			0.031	0.023	0.036	0.044	0.058	0.085	0.118	0.172	0.376

TABLE 2.—*Values of σ_{na}/σ_a for Various Values of h/a*

k	Value of σ_{na}/σ_a when h/a is												
	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.25	1.4	1.6	1.8
	n = 1.2; ρ_1 less than ρ_2												
0.1	0.993	0.986	0.982	0.981	0.981	0.981	0.981	0.983	0.986	0.990	0.991	0.994	0.993
0.2	0.986	0.973	0.967	0.965	0.964	0.965	0.965	0.968	0.970	0.978	0.983	0.988	0.987
0.3	0.973	0.961	0.954	0.950	0.948	0.949	0.950	0.954	0.959	0.969	0.976	0.979	0.979
0.4	0.961	0.948	0.939	0.933	0.933	0.934	0.934	0.939	0.945	0.959	0.965	0.970	0.973
0.5	0.951	0.930	0.919	0.920	0.917	0.920	0.920	0.926	0.933	0.948	0.956	0 964	0.968
0.6	0.934	0.914	0.903	0.898	0.902	0.903	0.908	0.914	0.919	0.935	0.944	0.956	0.962
0.7	0.917	0.894	0.885	0.884	0.888	0.886	0.890	0.898	0.905	0.926	0.938	0.949	0.954
0.8	0.891	0.880	0.874	0.869	0.871	0.879	0.884	0.893	0.898	0.916	0.930	0.940	0.947
0.9	0.862	0.856	0.855	0.854	0.856	0.860	0.865	0.874	0.883	0.907	0.918	0.932	0.940
1.0	0.809	0.824	0.829	0.831	0.838	0.845	0.850	0.854	0.867	0.895	0.908	0.925	0.933
	n = 1.3; ρ_1 less than ρ_2												
0.1	0.990	0.982	0.976	0.974	0.972	0.975	0.975	0.976	0.977	0.986	0.986	0.989	0.990
0.2	0.981	0.964	0.955	0.950	0.949	0.952	0.952	0.970	0.951	0.966	0.971	0.979	0.981
0.3	0.964	0.947	0.935	0.929	0.924	0.930	0.929	0.933	0.936	0.951	0.960	0.967	0.970
0.4	0.947	0.929	0.915	0.906	0.905	0.908	0.908	0.912	0.917	0.937	0.946	0.956	0.961
0.5	0.944	0.908	0.888	0.883	0.886	0.891	0.888	0.895	0.901	0.920	0.933	0.944	0.951
0.6	0.914	0.883	0.869	0.860	0.858	0.867	0.869	0.876	0.883	0.904	0.916	0.933	0.943
0.7	0.893	0.859	0.840	0.842	0.840	0.851	0.848	0.854	0.862	0.889	0.906	0.924	0.932
0.8	0.862	0.839	0.827	0.820	0.820	0.836	0.836	0.845	0.853	0.876	0.892	0.912	0.922
0.9	0.810	0.803	0.800	0.795	0.799	0.812	0.818	0.820	0.830	0.861	0.877	0.900	0.911
1.0	0.754	0.755	0.767	0.765	0.771	0.789	0.789	0.800	0.804	0.844	0.865	0.888	0.902
	n = 1.4; ρ_1 less than ρ_2												
0.1	0.988	0.978	0.972	0.967	0.964	0.966	0.967	0.969	0.971	0.979	0.981	0.986	0.986
0.2	0.977	0.957	0.943	0.938	0.936	0.936	0.936	0.957	0.944	0.956	0.962	0.970	0.974
0.3	0.956	0.933	0.921	0.912	0.906	0.907	0.909	0.914	0.919	0.936	0.945	0.956	0.962
0.4	0.937	0.911	0.897	0.867	0.878	0.880	0.881	0.889	0.894	0.916	0.927	0.942	0.948
0.5	0.930	0.887	0.865	0.854	0.853	0.855	0.859	0.863	0.873	0.896	0.910	0.926	0.936
0.6	0.892	0.857	0.834	0.828	0.822	0.830	0.826	0.840	0.850	0.876	0.891	0.910	0.922
0.7	0.863	0.829	0.805	0.800	0.799	0.805	0.808	0.815	0.825	0.856	0.875	0.898	0.911
0.8	0.824	0.797	0.788	0.774	0.772	0.780	0.790	0.801	0.811	0.838	0.858	0.882	0.897
0.9	0.773	0.750	0.748	0.746	0.748	0.755	0.763	0.773	0.785	0.819	0.840	0.866	0.885
1.0	0.700	0.700	0.711	0.709	0.715	0.729	0.735	0.746	0.755	0.799	0.822	0.849	0.869

k	Value of σ_{na}/σ_a when h/a is											
	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.25	1.8
	n = 1.5; ρ_1 less than ρ_2											
0.1	0.991	0.976	0.968	0.961	0.961	0.957	0.959	0.962	0.966	0.968	0.971	0.982
0.2	0.974	0.951	0.937	0.928	0.924	0.923	0.924	0.945	0.934	0.940	0.948	0.967
0.3	0.959	0.926	0.903	0.894	0.890	0.886	0.891	0.896	0.899	0.909	0.920	0.955
0.4	0.939	0.899	0.873	0.857	0.858	0.852	0.858	0.866	0.871	0.882	0.896	0.936
0.5	0.923	0.870	0.847	0.831	0.828	0.822	0.830	0.837	0.846	0.852	0.873	0.920
0.6	0.889	0.841	0.813	0.804	0.798	0.795	0.802	0.809	0.820	0.832	0.851	0.907
0.7	0.849	0.807	0.782	0.770	0.770	0.764	0.773	0.782	0.794	0.806	0.825	0.891
0.8	0.791	0.769	9.746	0.740	0.738	0.741	0.748	0.756	0.767	0.781	0.803	0.877
0.9	0.716	0.725	0.718	0.719	0.707	0.710	0.725	0.729	0.742	0.754	0.779	0.857
1.0	0.631	0.667	0.659	0.668	0.672	0.676	0.685	0.700	0.708	0.727	0.753	0.834

TABLE 2.—(*Continued*)

k	Value of σ_{na}/σ_a when h/a is											
	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.25	1.8
	$n = 2.0$; ρ_1 less than ρ_2											
0.1	0.981	0.971	0.953	0.944	0.938	0.933	0.934	0.933	0.937	0.940	0.946	0.967
0.2	0.957	0.932	0.907	0.890	0.880	0.874	0.874	0.892	0.881	0.887	0.896	0.916
0.3	0.931	0.896	0.862	0.840	0.829	0.818	0.821	0.820	0.829	0.838	0.849	0.899
0.4	0.899	0.857	0.818	0.795	0.780	0.765	0.772	0.756	0.782	0.788	0.806	0.866
0.5	0.880	0.824	0.771	0.748	0.734	0.723	0.726	0.725	0.730	0.748	0.767	0.836
0.6	0.818	0.772	0.727	0.703	0.689	0.664	0.684	0.683	0.697	0.707	0.729	0.806
0.7	0.765	0.716	0.677	0.659	0.646	0.638	0.644	0.646	0.658	0.669	0.673	0.778
0.8	0.700	0.640	0.627	0.610	0.601	0.591	0.602	0.607	0.620	0.632	0.655	0.750
0.9	0.619	0.557	0.570	0.561	0.557	0.561	0.561	0.568	0.581	0.597	0.621	0.718
1.0	0.514	0.446	0.504	0.496	0.508	0.509	0.518	0.525	0.542	0.554	0.575	0.686
	$n = 2.5$; ρ_1 less than ρ_2											
0.1	0.983	0.965	0.943	0.932	0.927	0.918	0.915	0.914	0.917	0.916	0.925	0.946
0.2	0.955	0.921	0.891	0.869	0.853	0.840	0.841	0.854	0.844	0.848	0.857	0.898
0.3	0.931	0.881	0.840	0.810	0.789	0.779	0.772	0.771	0.779	0.781	0.796	0.847
0.4	0.896	0.834	0.787	0.753	0.731	0.717	0.709	0.709	0.720	0.721	0.741	0.802
0.5	0.857	0.789	0.738	0.696	0.677	0.664	0.656	0.657	0.666	0.670	0.690	0.761
0.6	0.805	0.729	0.676	0.642	0.623	0.612	0.599	0.607	0.616	0.623	0.642	0.721
0.7	0.734	0.661	0.612	0.587	0.572	0.560	0.557	0.558	0.571	0.575	0.598	0.686
0.8	0.645	0.576	0.537	0.531	0.514	0.509	0.509	0.513	0.527	0.535	0.555	0.645
0.9	0.517	0.477	0.462	0.471	0.460	0.468	0.469	0.473	0.480	0.491	0.513	0.606
1.0	0.350	0.366	0.376	0.404	0.395	0.407	0.413	0.421	0.435	0.445	0.471	0.565

k	Value of σ_{na}/σ_a when h/a is										
	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.25	1.8
	$n = 3.0$; ρ_1 less than ρ_2										
0.1	0.968	0.940	0.924	0.916	0.910	0.905	0.902	0.901	0.902	0.906	0.928
0.2	0.926	0.884	0.856	0.838	0.823	0.819	0.832	0.816	0.818	0.827	0.865
0.3	0.889	0.827	0.793	0.767	0.752	0.742	0.741	0.742	0.743	0.752	0.805
0.4	0.845	0.768	0.731	0.702	0.686	0.674	0.675	0.670	0.673	0.685	0.750
0.5	0.803	0.712	0.672	0.638	0.626	0.614	0.614	0.612	0.617	0.629	0.699
0.6	0.748	0.646	0.605	0.580	0.568	0.559	0.557	0.558	0.563	0.578	0.652
0.7	0.686	0.575	0.541	0.522	0.511	0.503	0.505	0.505	0.512	0.527	0.608
0.8	0.608	0.496	0.471	0.463	0.453	0.449	0.455	0.459	0.465	0.483	0.567
0.9	0.519	0.411	0.394	0.404	0.400	0.400	0.415	0.417	0.423	0.440	0.524
1.0	0.421	0.318	0.313	0.339	0.334	0.342	0.355	0.363	0.371	0.391	0.480

L. GILCHRIST,* Toronto, Ont.—In addition to Professor King's paper, everyone should know Dr. Muskat's papers, Dr. L. B. Slichter's, and the mathematical side by Prof. A. F. Stevenson, of the University of Toronto. There is a very beautiful application of the same thing as Tagg's method by Peters and Bardeen,[7] using line electrodes.

M. MUSKAT.—It is of interest too that the actual theoretical expressions for the apparent resistivities when using line electrodes are considerably simpler than with point electrodes. For at least when the deepest layer is of zero or infinite conductivity the resistivity will be expressible in exponential series, which not only converge very rapidly, but may also be summed in simple closed form in many cases of practical interest, whereas with point electrodes the series involves the less familiar Hankel functions.

* University of Toronto.

[7] Univ. of Wisconsin, Eng. Expt. Sta. *Bull.* 71 (1930).

A New Method of Depth Determination in Earth-resistivity Measurements

By I. E. Rosenzweig*

(New York Meeting, February, 1938)

Geophysical prospecting by earth-resistivity methods is frequently applied to investigation of structural problems in geology.

Fig. 1 indicates a scheme of the general arrangement used in these methods of measurements. The battery (or generator) E is connected with two "current electrodes" I_1 and I_2. The current of intensity J, supplied by this battery, passes through the earth between these elec-

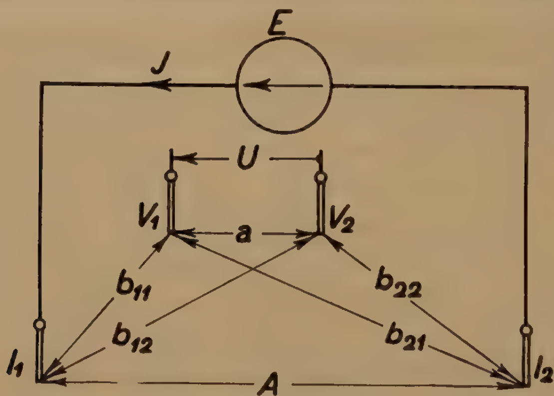

Fig. 1.—General arrangement of apparatus.

trodes. A potential difference U will be measured between two other electrodes V_1 and V_2, called "potential electrodes." According to the configuration of electrodes (Fig. 1), the resistivity of a homogeneous soil results from the formula

$$\rho = 2\pi c a \frac{U}{J} \qquad [1]$$

where

$$c = \frac{1}{\frac{a}{b_{11}} - \frac{a}{b_{12}} - \left(\frac{a}{b_{21}} - \frac{a}{b_{22}}\right)} \qquad [2]$$

Manuscript received at the office of the Institute Nov. 26, 1937. Issued as T.P. 931, August, 1938.

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If there is perfect homogeneity of the earth, a constant value of ρ may be obtained from equation 1 no matter what the variation in electrode separations. On the contrary, if the soil is not homogeneous, the application of formula 1 gives an "apparent" value of the resistivity ρ_a, which varies with the distances between electrodes. For this reason the surveys are made by varying the electrode separation a and retaining always the same mutual configuration of individual electrodes. It means that the positions of electrodes I_1, I_2, V_1 and V_2 will be changed during the measurements only in such a manner that the ratios $\alpha = \frac{A}{a}$, $\beta_{11} = \frac{b_{11}}{a}$, $\beta_{12} = \frac{b_{12}}{a}$, $\beta_{21} = \frac{b_{21}}{a}$ and $\beta_{22} = \frac{b_{22}}{a}$ will have always perfectly constant values.*

These measurements carried out as described enable us to design diagrams that show the relation between the value of ρ_a and the electrode separation a by varying the last. On the basis of these diagrams a geological interpretation of the measurements can be found; especially when the explored area is characterized by horizontal layers of different resistivities, a direct relationship exists between these diagrams and the depths and resistivities of the layers encountered.

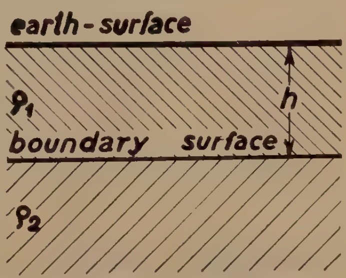

FIG. 2.—TWO-LAYER SYSTEM.

As is known, many general theoretical solutions have been proposed for determining the relation between ρ_a and a for different arrangements of horizontal layers.[1–4] These solutions cannot be directly used for practical calculations. The practical problem is reversed. We intend to compute the depths and resistivities of different layers on the basis of a given diagram of apparent resistivity ρ_a. So far, this problem has not been solved by a general formula applicable for any number of different layers. Therefore it is necessary to make some hypothetical assumptions, to simplify the problem.

Many methods theoretically well founded, for computing depth,[5–8] have been based on the assumption of a two-layer system, composed of a homogeneous surface layer of thickness h and resistivity ρ_1 and a lower layer, also homogeneous, of a resistivity ρ_2 extending to infinity (Fig. 2).

* The values of these ratios characterize the various resistivity methods in use. For example, in Wenner's method $\beta_{11} = \beta_{22} = 1$, $\beta_{12} = \beta_{21} = 2$, $\alpha = 3$; in the central electrode method $\beta_{11} = 1$, $\beta_{12} = 2$, $\beta_{21} = \infty$, $\beta_{22} = \infty$, $\alpha = \infty$.

In some methods, for example, in the central electrode method, it is not necessary to change, by varying a, the distances between electrodes. That is permitted only when the electrode separations in question have a practically infinite value.

[1] References are at the end of the paper.

The apparent resistivity ρ_a is given, under these conditions, by the general formula

$$\rho_a = \rho_1[1 + 2c\Phi(k, X)] \qquad [3]$$

where

$$\Phi(k, X) = \sum_{n=1}^{\infty} k^n \left[\frac{1}{\sqrt{\beta_{11}^2 + 4n^2X^2}} - \frac{1}{\sqrt{\beta_{12}^2 + 4n^2X^2}} - \left(\frac{1}{\sqrt{\beta_{21}^2 + 4n^2X^2}} - \frac{1}{\sqrt{\beta_{22}^2 + 4n^2X^2}} \right) \right] \qquad [4]$$

In these equations c indicates the value, defined by formula 2, and β_{11}, β_{12}, β_{21} and β_{22} the ratios $\frac{b_{11}}{a}$, $\frac{b_{12}}{a}$, $\frac{b_{21}}{a}$ and $\frac{b_{22}}{a}$, which characterize the applied configuration of electrodes.

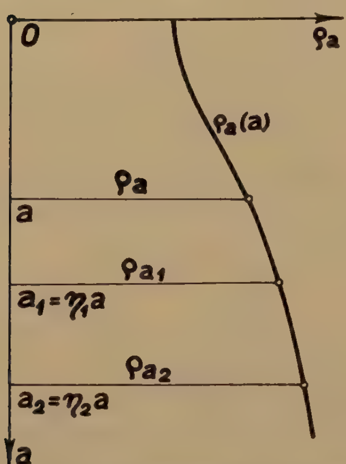

FIG. 3.—RESISTIVITY VALUES CORRESPONDING TO ELECTRODE SEPARATIONS.

k is the "reflection factor" defined by the formula

$$k = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \qquad [5]$$

X is the "relative depth" expressed by the equation

$$X = \frac{h}{a} \qquad [6]$$

Equations 3 and 4 were derived by means of the known theory of electrical images.

In this paper the author suggests a new method of calculation of the depth, which also is based on the assumptions and formulas explained above. The new method is based on the following considerations: Three resistivity values ρ_a, ρ_{a_1} and ρ_{a_2} have been determined from the apparent-resistivity curve, corresponding to the three electrode separations a, $a_1 = \eta_1 a$ and $a_2 = \eta_2 a$ (Fig. 3). The factors η_1 and η_2 have thereby constant values, assumed in advance.

Taking into consideration the hypothesis of a system of two layers, the value ρ_a can be obtained directly from formula 3 and the values ρ_{a_1} and ρ_{a_2} from the expressions:

$$\rho_{a_1} = \rho_1\left[1 + 2c\Phi\left(k, \frac{X}{\eta_1}\right)\right] \qquad [7]$$

and

$$\rho_{a_2} = \rho_1\left[1 + 2c\Phi\left(k, \frac{X}{\eta_2}\right)\right] \tag{8}$$

Dividing equations 7 and 8 by equation 3 gives:

$$x_1 = \frac{\rho_{a_1}}{\rho_a} = \frac{1 + 2c\Phi\left(k, \frac{X}{\eta_1}\right)}{1 + 2c\Phi(k, X)} \tag{9}$$

and

$$x_2 = \frac{\rho_{a_2}}{\rho_a} = \frac{1 + 2c\Phi\left(k, \frac{X}{\eta_2}\right)}{1 + 2c\Phi(k, X)} \tag{10}$$

Formulas 9 and 10 form a system of two simultaneous equations. The quantities $x_1 = \frac{\rho_{a_1}}{\rho_a}$ and $x_2 = \frac{\rho_{a_2}}{\rho_a}$ appearing in these equations are

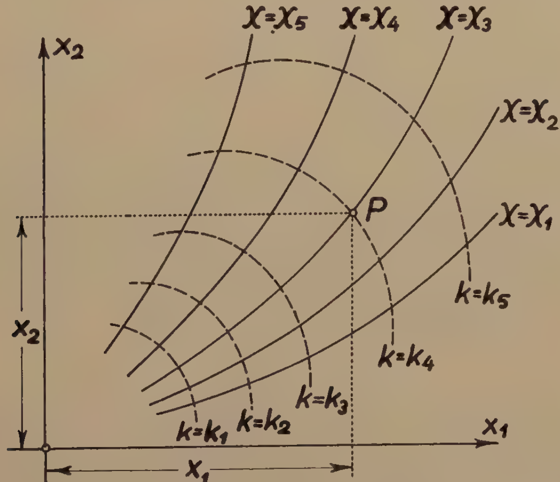

FIG. 4.—MASTER DIAGRAM.

known from measurements, thus only two values are unknown; namely, the reflection factor k and the relative depth X. These two unknown values may be determined by solution of equations 9 and 10. The system of these equations, however, being very complicated, cannot be solved exactly.

In the present paper, a new method of solution will be given, consisting of the application of a master diagram, which makes possible direct determination of the values of k and X by given quantities ρ_a, ρ_{a_1} and ρ_{a_2}. In the simplest form this diagram could be determined as follows.

Assuming various values for k and X, calculate from formulas 9 and 10 the quantities x_1 and x_2. Then tabulate the results of these calculations, which give the corresponding values of x_1, x_2, k and X. On the basis of this table, draw two sets of curves. The first one shows the relationships between x_1 and x_2 by given constant reflection factors k, and the second one shows the similar relationships, but by given constant values of relative depth X. A master diagram, obtainable in such a way, a scheme of which is shown in Fig. 4, forms a suitable basis for further computations.

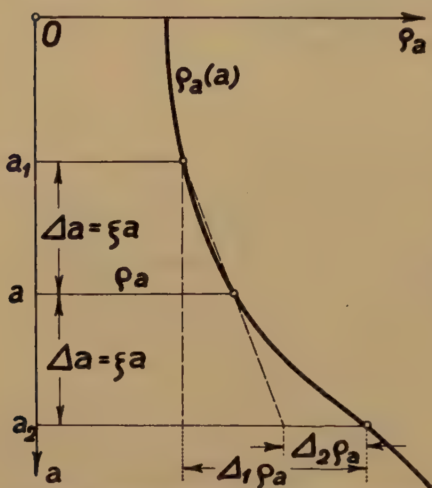

FIG. 5.—INTERPRETATION OF THE DIFFERENCE RATIOS.

When the quantities ρ_a, ρ_{a_1} and ρ_{a_2} have been determined by measurements, the values of x_1 and x_2 can be calculated by means of the formulas:

$$\left.\begin{aligned} x_1 &= \frac{\rho_{a_1}}{\rho_a} \\ x_2 &= \frac{\rho_{a_2}}{\rho_a} \end{aligned}\right\} \qquad [11]$$

These values of x_1 and x_2 determine in the master diagram (Fig. 4) a point P, which indicates directly the corresponding values of k and X. These last values form the desired solution of the simultaneous equations 9 and 10. Using the known value of X, the depth h of the boundary surface of both layers in question (Fig. 2) can be determined from the formula

$$h = \mathrm{X}a \qquad [12]$$

For practical reasons it is useful to construct some different type of master diagram.

First, in order to simplify the matter, take the following values instead of factors η_1 and η_2:

$$\left.\begin{aligned} \eta_1 &= 1 - \xi \\ \eta_2 &= 1 + \xi \end{aligned}\right\} \qquad [13]$$

where $0 < \xi < 1$.

It is obvious from Fig. 5 that thereby:

$$a - a_1 = a_2 - a = \Delta a = \xi a \qquad [14]$$

Second, bring into equations 9 and 10, instead of the variable quantities x_1 and x_2, new variables Δ_1 and Δ_2 defined by the formulas

$$\Delta_1 = \frac{\rho_{a_2} - \rho_{a_1}}{\rho_a} = x_2 - x_1 \qquad [15]$$

and

$$\Delta_2 = \frac{(\rho_{a_2} - \rho_a) - (\rho_a - \rho_{a_1})}{\rho_a} = \frac{\rho_{a_2} + \rho_{a_1}}{\rho_a} - 2 = x_2 + x_1 - 2 \quad [16]$$

The graphic interpretation of these new variables, hereafter called difference ratios of the first and second order, appears in Fig. 5.

The purpose of the transformation of coordinates of the master diagram, taken by means of formulas 15 and 16, is to increase the legibility of the master diagram.

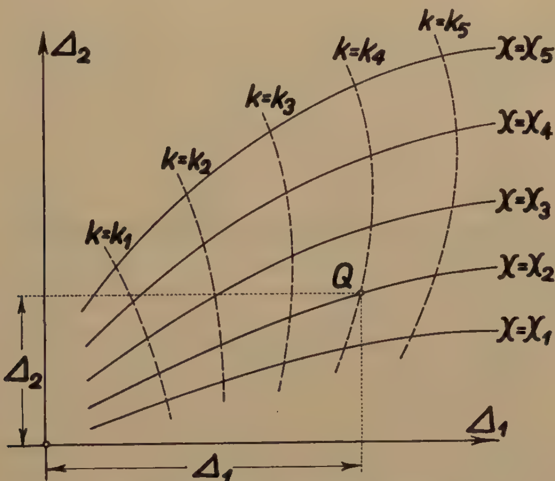

FIG. 6.—SECOND MASTER DIAGRAM.

Introducing the quantities Δ_1 and Δ_2 in equations 9 and 10 gives:

$$\Delta_1 = \frac{\Phi\left(k, \frac{X}{1+\xi}\right) - \Phi\left(k, \frac{X}{1-\xi}\right)}{\frac{1}{2c} + \Phi(k, X)} \quad [17]$$

$$\Delta_2 = \frac{\Phi\left(k, \frac{X}{1+\xi}\right) + \Phi\left(k, \frac{X}{1-\xi}\right)}{\frac{1}{2c} + \Phi(k, X)} - 2 \quad [18]$$

Now, using equations 17 and 18, a new master diagram could be plotted, as explained before. The difference consists only in the application of quantities Δ_1 and Δ_2 instead of x_1 and x_2. Fig. 6 shows a scheme of such a diagram.

The most interesting, from the practical point of view, is the drafting of diagrams for computing the results of measurements made by Wenner's method.[9] In this method, a configuration of electrodes is used as indicated in Fig. 7. The coefficients β_{11}, β_{12}, β_{21} and β_{22} have therefore,

as mentioned previously, the following values:

$$\beta_{11} = \beta_{22} = 1, \qquad \beta_{12} = \beta_{21} = 2$$

Accordingly, the function Φ determined by equation 4 become

$$\Phi(k, \mathrm{X}) = 2\sum_{n=1}^{\infty} k^n \left[\frac{1}{\sqrt{1 + 4n^2\mathrm{X}^2}} - \frac{1}{2\sqrt{1 + n^2\mathrm{X}^2}} \right] \qquad [19]$$

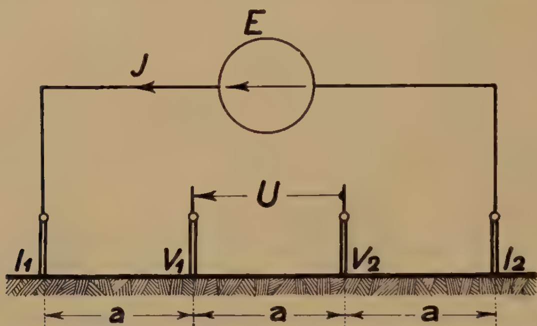

FIG. 7.—CONFIGURATION OF ELECTRODES, WENNER'S METHOD.

Now, putting this expression for Φ in equations 17 and 18, we obtain the formulas for Δ_1 and Δ_2 in the Wenner method:

$$\Delta_1 = \frac{1}{\frac{1}{4} + \sum_{n-1}^{\infty} k^n \left[\frac{1}{\sqrt{1 + 4n^2\mathrm{X}^2}} - \frac{1}{2\sqrt{1 + n^2\mathrm{X}^2}} \right]} \cdot \sum_{n=1}^{\infty} k^n \left[\frac{1}{\sqrt{1 + 4n^2\left(\frac{\mathrm{X}}{1+\xi}\right)^2}} - \frac{1}{\sqrt{1 + 4n^2\left(\frac{\mathrm{X}}{1-\xi}\right)^2}} - \frac{1}{2\sqrt{1 + n^2\left(\frac{\mathrm{X}}{1+\xi}\right)^2}} + \frac{1}{2\sqrt{1 + n^2\left(\frac{\mathrm{X}}{1-\xi}\right)^2}} \right] \qquad [20]$$

$$\Delta_2 = \frac{1}{\frac{1}{4} + \sum_{n=1}^{\infty} k^n \left[\frac{1}{\sqrt{1 + 4n^2\mathrm{X}^2}} - \frac{1}{2\sqrt{1 + n^2\mathrm{X}^2}} \right]} \cdot \sum_{n=1}^{\infty} k^n \left[\frac{1}{\sqrt{1 + 4n^2\left(\frac{\mathrm{X}}{1+\xi}\right)^2}} + \frac{1}{\sqrt{1 + 4n^2\left(\frac{\mathrm{X}}{1-\xi}\right)^2}} - \frac{1}{2\sqrt{1 + n^2\left(\frac{\mathrm{X}}{1+\xi}\right)^2}} - \frac{1}{2\sqrt{1 + n^2\left(\frac{\mathrm{X}}{1-\xi}\right)^2}} \right] - 2 \qquad [21]$$

On the basis of these formulas, a diagram (Fig. 8) has been computed numerically.* In this, ξ has been taken as equal to 0.2, because this value appears favorable for practical applications.

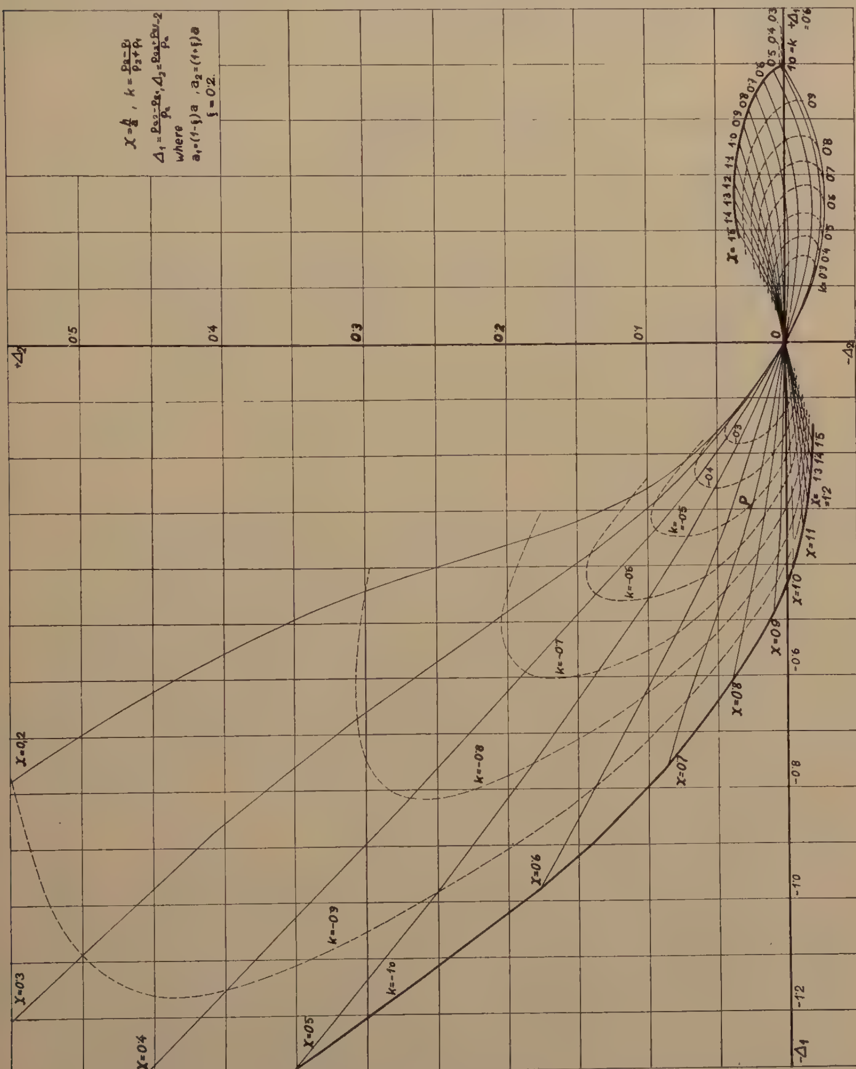

FIG. 8.—MASTER DIAGRAM FOR WENNER METHOD.

The following example shows the manner of using this diagram; also the degree of accuracy and convenience of using it. Fig. 9 shows the curve of apparent resistivity† used. The method of computation is:

1. Fix a certain value of a as the basis for calculation; for example, $a = 50m$.

* For the computation, tables given by I. Roman[10] were used.

† Earth-resistivity measurements executed in the Carpathian foreland.

2. Assuming $\xi = 0.2$, compute the value $\Delta a = \xi a = 0 \cdot 2 \times 50 = 10m$, and the values $a_1 = a - \Delta a = 50 - 10 = 40m$ and $a_2 = a + \Delta a = 50 + 10 = 60m$.

3. From the curve of apparent resistivity (Fig. 9) determine the quantities:

$$\rho_a = 39.1\,\Omega m$$
$$\rho_{a_1} = 45.4\,\Omega m$$
$$\rho_{a_2} = 33.8\,\Omega m$$

which corresponds to the values of a, a_1 and a_2 obtained above.

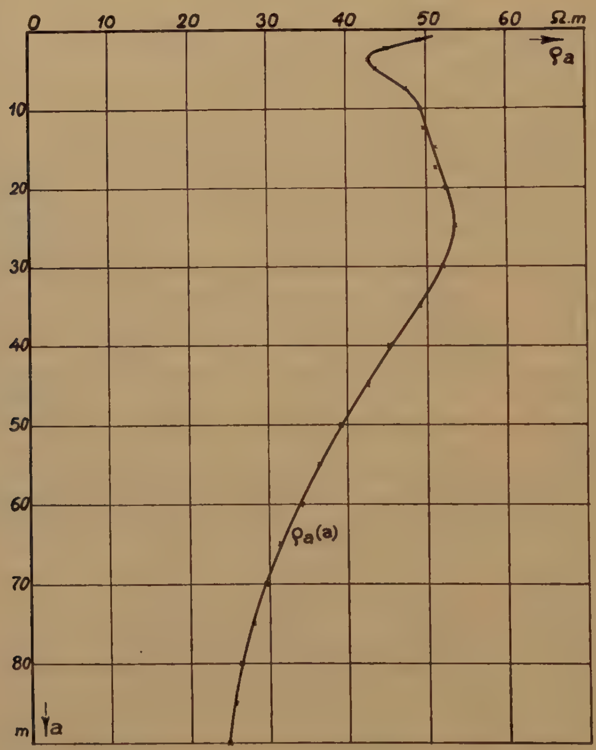

FIG. 9.—APPARENT RESISTIVITY USED IN COMPUTATIONS.
Diagram published by permission of the Pionier Oil Co., Lwów, Poland.

4. Employing formulas 15 and 16, compute the values of the difference ratios:

$$\Delta_1 = \frac{33.8 - 45.4}{39.1} = -0.297$$

$$\Delta_2 = \frac{33.8 + 45.4 - 2\cdot 39.1}{39.1} = +0.026$$

5. Determine point P on the master diagram (Fig. 8) having coordinates equal to Δ_1 and Δ_2.

6. From this diagram read off directly the values of the reflection factor k and of the relative depth X, which corresponds to the point P, obtaining thereby:

$$k = -0.51 \qquad \text{and} \qquad X = 0.70$$

7. From formula 12, calculate the desired depth of the boundary surface of both layers, and obtain

$$h = 0.70 \times 50 = 35m$$

An analogous calculation carried out on the basis of $a = 40m$, $a = 60m$ and $a = 70m$ has led to following results:

$$a = 40m, \quad k = -0.53, \quad X = 0.90, \quad h = 36.0\ m$$
$$a = 60m, \quad k = -0.54, \quad X = 0.62, \quad h = 37.2\ m$$
$$a = 70m, \quad k = -0.50, \quad X = 0.52, \quad h = 36.4\ m$$

Taking the average value of these quantities gives:

$$h = 36.1\ m \qquad \text{and} \qquad k = -0.52$$

It must be admitted that in the given case the calculation indicates an excellent conformity of the obtained results. The highest divergence of the individual values of h and k amounts to only 7 per cent.*

This method of interpretation of the measurements of earth resistivity makes it possible to compute the depth when geological conditions are sufficiently close to the theoretically assumed case of a system of two layers. The same method can often be successfully applied when conditions do not satisfy this assumption; for example, when there are three or more layers.

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* In order to obtain a sound comparison, an independent calculation of the depth was carried out by the method of G. F. Tagg.[6] The results of this calculation

$$(h = 33\ m, k = -0.57)$$

agree fairly well with the results given above.

New Theory of Apparent Resistivity of Horizontally Stratified Soils

By I. E. Rosenzweig*

(New York Meeting, February, 1939)

The problem considered in this paper is as follows: An arbitrary horizontally stratified area is given. The electrical properties of this area are characterized by a function $\rho(z)$ (Fig. 1), which shows the relationship between resistivity ρ of the soil and the depth z. For such general case the apparent resistivity ρ^* will be computed, by earth-resistivity measurements carried out by any given configuration of electrodes.

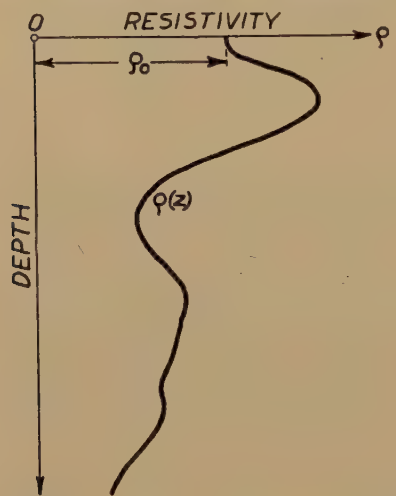

Fig. 1.—Graph of function $\rho(z)$.

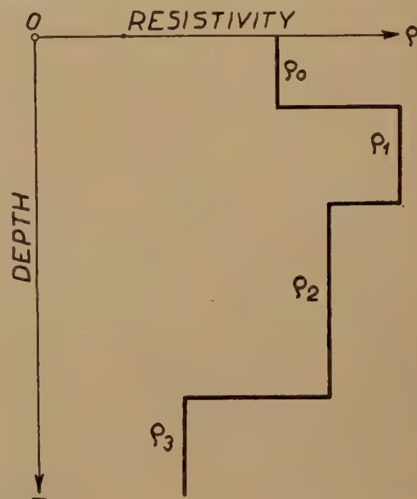

Fig. 2.—Graph of $\rho(z)$ for n homogeneous layers.

General solutions of this problem appear to be very important for interpretation of field results of resistivity measurements, concerning structural investigations. Such solutions have been given by many authors for the special case of a finite number of homogeneous layers (Fig. 2)[1–4] as well as for the more general case of continuously ranging resistivities.[5–8]

Manuscript received at the office of the Institute Dec. 13, 1938. Issued as T.P. 1102, September, 1939.

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[1] References are at the end of the paper.

The practical importance of the new method of solution that is suggested in this paper lies in the convenience of this method for approximative numerical calculations, especially when the $\rho(z)$ curve is not expressed by a formula, but given only in the form of a table or a diagram.

The formulas given in this paper make possible the determination of the apparent resistivity for every kind of variation ρ with depth z increasing especially for variations characterized by continuous $\rho(z)$ functions. These formulas are derived with the aid of a new method of determining

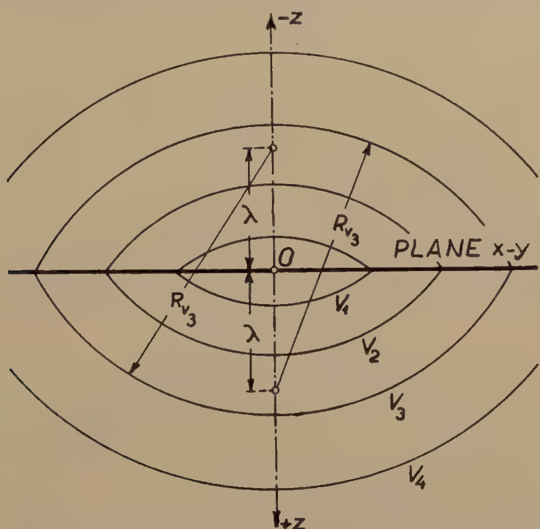

FIG. 3.—EQUIPOTENTIAL SURFACES OF A LENS-SOURCE.

electrical potentials called "the iteration of lens-sources" and by application of the theory of linear integral equations of Volterra.

LENS-SOURCES

Usually, the well-known method of electrical images is applied for solving theoretical problems connected with earth-resistivity surveys. However, for a general solution of such problems it is more useful to apply, instead of electrical mirror images, the so-called "lens-sources," which are defined as follows:

A lens-source, appearing on plane x-y and having its center in point O of coordinates, causes in a homogeneous infinite medium of resistivity ρ the potential:

$$V = \frac{i\rho}{4\pi\sqrt{r^2 + (\lambda + |z|)^2}} \qquad [1]$$

in which

i denotes *strength* of the source*

λ, the *linear parameter* of the source†

* Of positive or negative value.

† Always of positive value.

r, the radius vector measured on the x-y plane:

$$r = \sqrt{x^2 + y^2} \qquad [2]$$

Formula 1 leads to equipotential surfaces, which form a system of lenses composed by pairs of spherical calottes placed symmetrically in relation to plane x-y (Fig. 3).

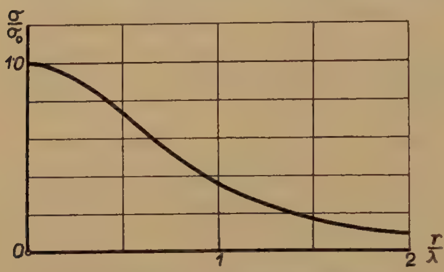

FIG. 4.—GRAPH OF CURRENT DENSITY σ.

When calculating the surface divergence of the potential gradient we obtain the current density σ of the lens-source* in any point of the x-y plane, as

$$\sigma = \frac{i\lambda}{2\pi\sqrt{(r^2 + \lambda^2)^3}} \qquad [3]$$

Using an optical analogy, we can say that σ (a graph of which is shown on Fig. 4) represents the function of illumination appearing on the x-y plane when an isotropic light source of $\frac{i}{2\pi}$ candles is situated on the z axis in distance λ from point 0.

APPLICATION OF LENS-SOURCES

In order to explain the method of using the lens-sources for solving earth-resistivity problems, we will consider the simplest case of a conducting medium that is not homogeneous in the whole space. Such a medium is formed by two "half-spaces" A and B (Fig. 5) of different resistivities ρ_A and ρ_B with a plane α-α as a boundary surface between them. In this medium we will investigate the electrical potential developed by a single point source of current of intensity J situated in an arbitrary point of half-space A.

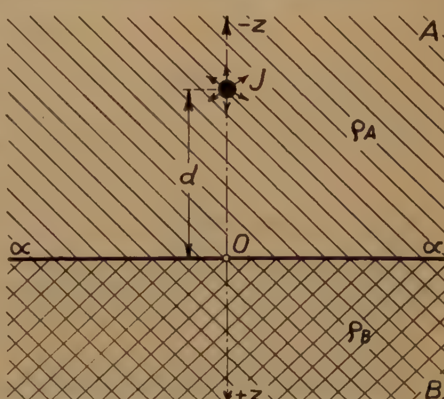

FIG. 5.—TWO "HALF-SPACES."

Assume in the given arrangement, α-α to be the plane axes of x and y, and source J to be placed on the negative part of z axis.

Like the electrical image method, the method of lens-sources is applied in order to "homogenize" the examined medium; i.e., to eliminate seemingly the unhomogeneities of resistivity. That is, using this method, we consider instead of the real arrangement shown on Fig. 5 a homogeneous medium of resistivity ρ_A extending infinitely in all direc-

* Expressed in amperes per unit of area.

tions, in which there acts the real point source J and a fictitious lens-source appearing on the α-α plane. The lens-source, the center of which is placed in O of coordinates, is characterized by the *strength*

$$i = Jk \qquad [4]$$

where k denotes the "reflection coefficient"* of plane α-α defined by equation

$$k = \frac{\rho_B - \rho_A}{\rho_B + \rho_A} \qquad [5]$$

and by the *linear parameter* λ equal to the absolute value d of distance between J and the boundary plane.

The primary source causes in the homogenized medium a potential

$$V_1 = \frac{J\rho_A}{4\pi R} \qquad [6]$$

where R is the distance of a point with coordinates x, y and z from source J

$$R = \sqrt{r^2 + (d + z)^2} \qquad [7]$$

The lens-source causes a secondary potential V_2, which can be written according to eq. 1, in the form

$$V_2 = \frac{Jk\rho_A}{4\pi\sqrt{r^2 + (d + |z|)^2}} \qquad [8]$$

Hence, we obtain the resulting potential

$$V = V_1 + V_2 = \frac{J\rho_A}{4\pi}\left(\frac{1}{R} + \frac{k}{\sqrt{r^2 + (d + |z|)^2}}\right) \qquad [9]$$

This formula gives strictly the same results as the equations

$$V_A = \frac{J\rho_A}{4\pi}\left(\frac{1}{R} + \frac{k}{\sqrt{r^2 + (z - d)^2}}\right)$$

and

$$V_B = \frac{J\rho_B}{4\pi R}(1 - k)$$

(V_A and V_B potential values in individual "half-spaces" A and B) obtained by the use of the electrical image method.† Compared with these last expressions, our formula 9 has however the important advantage of determining the potential in the whole space by a single closen formula

* We use for this coefficient the same term that is used in the theory of electrical images.

† Given, for example, in the paper of Ehrenburg and Watson.[4]

instead of two equations which correspond with the two half-spaces of the arrangement.

Method of Iterations

We will now consider a system that consists of an arbitrary, finite or infinite number of homogeneous parts of different resistivities . . . ρ_{a-1}, ρ_a, ρ_{a+1} . . . * separated from each other by a series of parallel planes. We assume that in layer O (i.e., of resistivity ρ_o) there is acting a point source of intensity J.

We will fix in this arrangement a system of coordinates with the z axis passing through source J and directed orthogonally to the boundary planes.

When the method of electrical images is applied for determining the potential in such arrangements, the manner of computing becomes very difficult and indistinct. So it is almost impossible to establish any general principles or formulas that would lead systematically to all the mirror images necessary for calculation of the potential in more complicated cases. On the contrary, the method of lens-sources makes the determination of such principles easy.

We can formulate the following general principles of iteration of lens-sources:

1. The field of the real source J causes in all boundary planes lens-sources of first order. On the boundary plane a (between layers of resistivities ρ_a and ρ_{a+1}), which is characterized by the reflection coefficient

$$k_a = \frac{\rho_{a+1} - \rho_a}{\rho_{a+1} + \rho_a} \qquad [10]$$

appears quite analogously as in the case of the two half-spaces, a lens-source of the strength

$$\left.\begin{aligned} i_a^{(1)} &= +Jk_a \text{ for } a \geqq 0 \\ \text{or} \quad i_a^{(1)} &= -Jk_a \text{ for } a < 0 \end{aligned}\right\} \qquad [11]$$

and of linear parameter $\lambda_a^{(1)}$ equal to the absolute value d_a of distance between source J and boundary plane a.

2. The field of a lens-source that is placed on an arbitrary plane develops in all other boundary planes lens-sources denoted as those of higher orders. The lens-source of nth order characterized by strength $i_a^{(n)}$ and linear parameter $\lambda_a^{(n)}$ situated on plane a causes in this way on the plane b a source of the order of $n + 1$ of the strength,

$$\left.\begin{aligned} i_{ba}^{(n+1)} &= +i_a^{(n)}k_b \text{ for } b > a \\ \text{or} \quad i_{ba}^{(n+1)} &= -i_a^{(n)}k_b \text{ for } b < a \end{aligned}\right\} \qquad [12]$$

* The index number a can run through positive as well as through negative values.

and of linear parameter

$$\lambda_{ba}^{(n+1)} = d_{ab} + \lambda_a^{(n)} \qquad [13]$$

where d_{ab} is the absolute value of distance between planes a and b.

The centers of the lens-sources of all orders are placed along the z axis of coordinates.

Using these principles, we can determine by turns all lens-sources that are necessary for calculation of the potential in arbitrary multilayer systems. Superposing the potentials of the primary source and of all lens-sources, calculated on assumption of a homogeneous medium of resistivity ρ_0, we obtain the function of the potential desired.

Specific Reflection Coefficient

Now, having established the general principles of the method of procedure, we shall determine the potential in an arbitrary horizontally stratified soil with continuous variation of resistivity characterized by the function $\rho(z)$ (Fig. 1) and with a point source of current J acting on the earth's surface.

The earth's surface (i.e., the plane of $z = 0$) forms in such case a singular discontinuity of resistivity with a reflection coefficient $k_0 = -1$. In order to eliminate this discontinuity and to obtain a medium with only continuous variation of ρ, we apply as an auxiliary the method of electrical images, and complete the real arrangement by a mirror picture obtained by reflection of the whole soil and the current source J in the plane of the earth's surface, considered as a perfect electrical mirror. In this way we obtain a $\rho(z)$ curve that is symmetrical in relation to $z = 0$ (Fig. 6) and a current source of doubled intensity $2J$.

In the arrangement obtained in such a manner we compute now the quantities of reflection coefficients. We observe for this purpose in an arbitrary depth z* a small interval Δz. The resistivity ρ changes in this interval from value $\rho(z)$ to $\rho(z + \Delta z)$. According to formula 10, we can assume that the examined depth interval Δz corresponds with a reflection coefficient

$$k_{\Delta z} = \frac{\rho(z + \Delta z) - \rho(z)}{\rho(z + \Delta z) + \rho(z)} \qquad [14]$$

Dividing this value by Δz and determining the limes value of ratio obtained for $\Delta z \rightarrow 0$, gives the *specific reflection coefficient;* i.e., the reflection coefficient per unit of depth interval, as:

$$\kappa(z) = \lim_{\Delta z \rightarrow 0} \frac{k_{\Delta z}}{\Delta z} \qquad [15]$$

* The positive part of the z axis is assumed to be directed vertically downward, so that we can express the depths as positive values of z.

Putting formula 14 into 15, we obtain

$$\kappa(z) = \lim_{\Delta z \to 0} \frac{1}{\rho(z + \Delta z) + \rho(z)} \times \frac{\rho(z + \Delta z) - \rho(z)}{\Delta z}$$

or

$$\kappa(z) = \frac{1}{2\rho(z)} \times \frac{d}{dz}\rho(z) = \frac{1}{2}\frac{d}{dz} \ln \rho(z) \qquad [16]$$

Functions of Individual Lens-sources

Applying the first principle of iteration of lens-sources, we can determine the sources of the first order in the arrangement examined, illus-

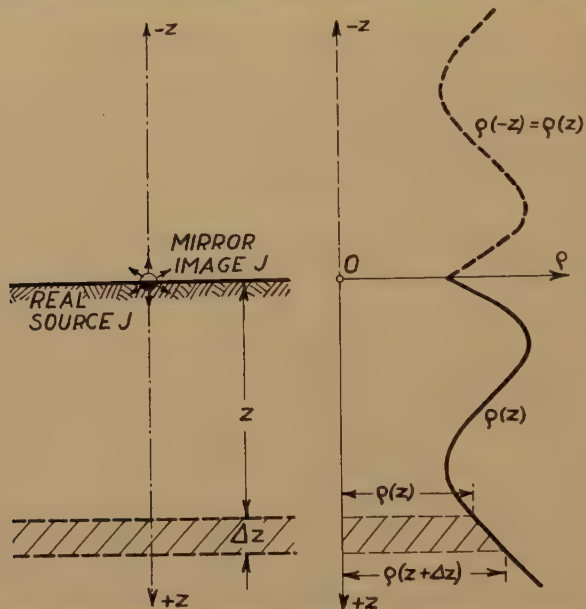

Fig. 6.—Point source in horizontal stratified soil.

trated on Fig. 6. Since the variation of resistivity ρ is continuous, these sources will be distributed continuously in the space, having their centers placed along the z axis of coordinates.

The *strength* of these sources of the first order can be characterized by the function

$$\Theta_1(z) = \frac{1}{J} \lim_{\Delta z \to 0} \frac{i_{\Delta z}^{(1)}}{\Delta z} \qquad [17]$$

which expresses the relation between the depth z and the relative density of the strength (i.e., the strength per unit of primary current and unit of the depth interval). In regard to formulas 11 and 15, this function may be written directly as

$$\Theta_1(z) \doteq 2\kappa(z) \qquad [18]$$

Following the symmetry of the arrangement, this function is symmetrical in relation to the origin, so it is $\Theta_1(-z) = \Theta_1(z)$.

The linear parameter of the lens-sources of the first order characterized by function $\Theta_1(z)$ is equal to depth z, or, generally speaking, for both positive and negative z values, to the absolute value $|z|$.

On the basis of the known distribution of the lens-sources of the first order, we are able to find, by means of the second principle of iteration, the lens-sources of the second and higher orders.

All the lens-sources of the first order, placed between the earth surface $z = 0$ and the plane of a given z (i.e., the sources characterized by $\Theta_1(\zeta)$ for $0 \leqq \zeta \leqq z$) cause in an infinitely small depth interval in depth z a source of the second order with a linear parameter equal to z, the strength of which can be characterized by the function $\Theta_2(z)$ of a significance analogous to that of the function $\Theta_1(z)$.

The lens-source of the strength $\Theta_1(\zeta)d\zeta$, acting in a layer of an elementary thickness $d\zeta$ and of the depth $\zeta(0 \leqq \zeta \leqq z)$, causes, according to formula 12, in the layer of thickness dz and depth z a lens-source of second order of strength $\kappa(z)dz\Theta_1(\zeta)d\zeta$. The linear parameter of this source, determined by eq. 13, is independent of ζ and equal to $\zeta + (z - \zeta) = z$.

Thus we obtain by integration the function

$$\Theta_2(z) = \kappa(z)\int_0^z \Theta_1(\zeta)d\zeta \tag{19}$$

or, in regard to equation 18,

$$\Theta_2(z) = 2\kappa(z)\int_0^z \kappa(\zeta)d\zeta \tag{20}$$

Besides, the lens-sources of first order, situated upon the earth surface or below the plane of given z, i.e., the sources $\Theta_1(\zeta)$ for $\zeta < 0$ or $\zeta > z$, cause in depth z sources of the second order the linear parameter of which run through all values $\lambda > z$. In order to express these sources we apply the function

$$\vartheta_2(z,\lambda) = \frac{1}{J}\lim_{\substack{\Delta z \to 0 \\ \Delta\lambda \to 0}} \frac{i_{\Delta z,\Delta\lambda}{}^{(2)}}{\Delta z.\Delta\lambda} \tag{21}$$

This expression characterizes the relative strength (i.e., the strength per unit of primary current J) per unit of depth interval and unit interval of linear parameter* as function of depth z and of linear parameter λ.

In order to find function $\vartheta_2(z,\lambda)$ we consider in depth $\zeta > z$ an elementary lens-source $\Theta_1(\zeta)d\zeta$. This source causes according to the second principle of iteration (formulas 12 and 13) in layer dz of depth z an infin-

* Expressed in amperes per ampere of primary current and per square of length unit.

itesimal lens-source of the second order of strength $-\kappa(z) \cdot dz\Theta_1(\zeta)d\zeta$ and of linear parameter $\lambda = 2\zeta - z$. Similarly, an elementary source $\Theta_1(\zeta)d\zeta$, placed at a distance ζ* upon the earth surface causes in the given layer dz a lens-source of infinitesimal strength $\kappa(z)dz\Theta_1(\zeta)d\zeta$ and of linear parameter $\lambda = 2\zeta + z$.

The addition of both these sources, carried out for similar values of λ and then dividing by dz and by $d\lambda = 2d\zeta$† leads directly to following formula for $\vartheta_2(z,\lambda)$:

$$\vartheta_2(z,\lambda) = \frac{1}{2}\kappa(z)\left[\Theta_1\left(\frac{\lambda - z}{2}\right) - \Theta_1\left(\frac{\lambda + z}{2}\right)\right] \tag{22}$$

This function differs from 0 only for values $\lambda > z$.

Both functions $\Theta_2(z)$ and $\vartheta_2(z,\lambda)$, which characterize the lens-sources of the second order, are, like $\Theta_1(z)$, symmetrical in relation to point $z = 0$.

In the same way as the sources of the second order, the sources of all higher orders can be characterized by functions $\Theta_n(z)$ and $\vartheta_n(z,\lambda)$ of like significance as the functions $\Theta_2(z)$ and $\vartheta_2(z,\lambda)$.

The function $\Theta_n(z)$ is determined by a recurrence formula of form analogous to that of equation 19:

$$\Theta_n(z) = \kappa(z)\int_0^z \Theta_{n-1}(\zeta)d\zeta \tag{23}$$

The linear parameter of the lens-sources, characterized by this function, is, as in cases discussed before, equal to z.

The function $\vartheta_n(z,\lambda)$ can be divided into two parts $\vartheta_n'(z,\lambda)$ and $\vartheta_n''(z,\lambda)$ according to the equation

$$\vartheta_n(z,\lambda) = \vartheta_n'(z,\lambda) + \vartheta_n''(z,\lambda) \tag{24}$$

the first part caused by sources of $n - 1$ order given by means of function $\Theta_{n-1}(z)$ and the second part developed by sources of the $n - 1$ given by means of $\vartheta_{n-1}(z,\lambda)$.

The first of these components we determine with the aid of a formula built up like that of equation 22:

$$\vartheta_n'(z,\lambda) = \frac{1}{2}\kappa(z)\left[\Theta_{n-1}\left(\frac{\lambda - z}{2}\right) - \Theta_{n-1}\left(\frac{\lambda + z}{2}\right)\right] \tag{25}$$

The second component of $\vartheta_n(z,\lambda)$ can be calculated by means of a recurrence formula, which can be derived by means of following considerations:

An elementary lens-source of the $n - 1$ order of strength $\vartheta_{n-1}(\zeta,\lambda + \zeta - z)d\zeta d\lambda$ placed in depth $\zeta(0 < \zeta < z)$ and characterized by linear parameter of value $\lambda + \zeta - z$ causes, following formulas 12 and 13 of the second principle of iterations, in depth z in a layer of thick-

* Considered as an absolute, i.e., positive, value.

† $d\lambda$ computed for z = constant.

ness dz a lens-source of nth order of infinitesimal strength $\kappa(z)dz \cdot \vartheta_{n-1}(\zeta,\lambda + \zeta - z)d\zeta d\lambda$ and of linear parameter λ.

Similarly, each elementary lens-source of the $n - 1$ order of intensity $\vartheta_{n-1}(\zeta,\lambda - \zeta - z)d\zeta d\lambda$ situated in depth $\zeta\left(0 < \zeta < \frac{\lambda - z}{2}\right)$ and having the linear parameter $\lambda - \zeta - z$ causes in a layer dz in depth z an nth order source of strength $\kappa(z)dz \cdot \vartheta_{n-1}(\zeta,\lambda - \zeta - z)d\zeta d\lambda$ of linear parameter λ. The infinitesimal sources of the $n - 1$ order $\vartheta_{n-1}(\zeta,\lambda - \zeta + z)d\zeta d\lambda$ placed in depth $\zeta\left(z < \zeta < \frac{\lambda + z}{2}\right)$ develop analogously in depth z in the layer dz a lens-source of nth order and of linear parameter λ, the intensity of which is $-\kappa(z)dz \cdot \vartheta_{n-1}(\zeta,\lambda - \zeta + z)d\zeta d\lambda$.

Integrating all the elementary lens-sources of parameter obtained in such way in layer dz of depth z, and dividing the resulting function by dz and by $d\lambda$ we get:

$$\vartheta_n''(z,\lambda) = \kappa(z)\Big[\int_0^z \vartheta_{n-1}(\zeta,\lambda + \zeta - z)d\zeta + \\ + \int_0^{\frac{\lambda - z}{2}} \vartheta_{n-1}(\zeta,\lambda - \zeta - z)d\zeta - \int_z^{\frac{\lambda + z}{2}} \vartheta_{n-1}(\zeta,\lambda - \zeta + z)d\zeta\Big]$$

This formula can be a bit simplified, when we replace in the third term the variable ζ by $\zeta + z$. We obtain in this way:

$$\vartheta_n''(z,\lambda) = \kappa(z)\Big[\int_0^z \vartheta_{n-1}(\zeta,\lambda + \zeta - z)dz + \\ + \int_0^{\frac{\lambda - z}{2}} \{\vartheta_{n-1}(\zeta,\lambda - \zeta - z) - \vartheta_{n-1}(\zeta + z,\lambda - \zeta)\}d\zeta\Big] \quad [26]$$

Hence, we obtain

$$\vartheta_n(z,\lambda) = \kappa(z)\Big[\frac{1}{2}\Theta_{n-1}\left(\frac{\lambda - z}{2}\right) - \frac{1}{2}\Theta_{n-1}\left(\frac{\lambda + z}{2}\right) + \int_0^z \vartheta_{n-1}(\zeta,\lambda + \zeta - z)d\zeta + \\ + \int_0^{\frac{\lambda - z}{2}} \{\vartheta_{n-1}(\zeta,\lambda - \zeta - z) - \vartheta_{n-1}(\zeta + z,\lambda - \zeta)\}d\zeta\Big] \quad [27]$$

As for $\vartheta_2(z,\lambda)$, for sources of all other orders also, the linear parameter can run only through values $\lambda > z$.

Resulting Functions of Lens-sources

Formulas 23 and 27 render it possible to determine by turns all functions $\Theta_n(z)$ and $\vartheta_n(z,\lambda)$. Using these functions, which represent mathematically the lens-sources of all orders, we obtain the resulting lens-sources defined by the infinite series:

$$\Theta(z) = \Theta_1(z) + \Theta_2(z) + \cdots + \Theta_n(z) + \cdots = \sum_{n-1}^{\infty} \Theta_n(z) \quad [28]$$

and

$$\vartheta(z,\lambda) = \vartheta_2(z,\lambda) + \vartheta_3(z,\lambda) + \cdots + \vartheta_n(z,\lambda) + \cdots = \sum_{n=2}^{\infty} \vartheta_n(z,\lambda) \quad [29]$$

In order to obtain a closen expression for $\Theta(z)$ and $\vartheta(z,\lambda)$ we will transform these series into integral equations. To obtain such equation for $\Theta(z)$ write formula 28 in the form

$$\Theta(z) = \Theta_1(z) + \Phi[\Theta_1(z)] + \Phi[\Theta_2(z)] + \cdots \quad [30]$$

where Φ denotes symbolically the linear function that appears in equation 23

$$\Phi[f(z)] = \kappa(z)\int_0^z f(\zeta)d\zeta \quad [31]$$

Formula 30 can be expressed as

$$\Theta(z) = \Theta_1(z) + \Phi[\Theta_1(z) + \Theta_2(z) + \cdots]$$

and thus we obtain, regarding equation 28, the formula

$$\Theta(z) = \Theta_1(z) + \Phi[\Theta(z)] \quad [32]$$

Substituting expression 31 for Φ and taking into consideration formula 18 for $\Theta_1(z)$, we arrive at last at the equation

$$\Theta(z) = \kappa(z)\left[2 + \int_0^z \Theta(\zeta)d\zeta\right] \quad [33]$$

which represents a very simple form of a Volterra integral equation[9] and has the general solution

$$\Theta(z) = 2\kappa(z) \cdot e^{\int_0^z \kappa(\zeta)d\zeta} \quad [34]$$

This solution can be easily determined when we transform equation 33 into a linear differential equation. The proof of the correctness of this solution can be made by putting function 34 into the given equation 33.

Taking into consideration formula 16 for the specific reflection coefficient $\kappa(z)$ we obtain the possibility of further simplification of eq. 34, getting finally

$$\Theta(z) = 2\kappa(z)\sqrt{\frac{\rho(z)}{\rho_0}} \quad [35]$$

An analogous method leads to an integral equation for $\vartheta(z,\lambda)$; namely, formula 29 can be written as

$$\vartheta(z,\lambda) = \Omega[\Theta_1(z)] + \Psi[\vartheta_2(z,\lambda)] + \Omega[\Theta_2(z)] + \Psi[\vartheta_3(z,\lambda)] + \Omega[\Theta_3(z)] + \cdots \quad [36]$$

where Ω and Ψ signify the two linear operators that appear in formulas 25 and 26:

$$\Omega[F(z)] = \frac{1}{2}\kappa(z)\left[F\left(\frac{\lambda - z}{2}\right) - F\left(\frac{\lambda + z}{2}\right)\right] \quad [37]$$

$$\Psi[f(z,\lambda)] = \kappa(z)\left[\int_0^z f(\zeta,\lambda + \zeta - z)d\zeta + \int_0^{\frac{\lambda - z}{2}} \{f(\zeta,\lambda - \zeta - z) - f(\zeta + z,\lambda - \zeta)\}d\zeta\right] \quad [38]$$

A transformation of eq. 36 gives

$$\vartheta(z,\lambda) = \Omega[\Theta_1(z) + \Theta_2(z) + \cdots] + \Psi[\vartheta_2(z,\lambda) + \vartheta_3(z,\lambda) + \cdots]$$

Hence, from formulas 28 and 29,

$$\vartheta(z,\lambda) = \Omega[\Theta(z)] + \Psi[\vartheta(z,\lambda)] \quad [39]$$

Substituting in this equation expressions 37 and 38 for the operators Ω and Ψ gives, for $\vartheta(z,\lambda)$, the following integral equation of Volterra type:

$$\vartheta(z,\lambda) = \kappa(z)\left[\frac{1}{2}\Theta\left(\frac{\lambda - z}{2}\right) - \frac{1}{2}\Theta\left(\frac{\lambda + z}{2}\right) + \int_0^z \vartheta(\zeta,\lambda + \zeta - z)d\zeta + \right.$$
$$\left. + \int_0^{\frac{\lambda - z}{2}} \{\vartheta(\zeta,\lambda - \zeta - z) - \vartheta(\zeta + z,\lambda - \zeta)\}d\zeta\right] \quad [40]$$

Like all the component functions $\vartheta_n(z,\lambda)$, the resulting function $\vartheta(z,\lambda)$ is determined and differs from 0 only for $\lambda > z$.

The theory of Volterra equations leads to many methods of solving equation 40. Thus this equation can be transformed in special cases into a differential equation.* Another kind of solution forms the method of iterations, given by Volterra and Neumann.[9] This method gives the expressions

$$\vartheta(z,\lambda) = \vartheta^{(1)}(z,\lambda) + \vartheta^{(2)}(z,\lambda) + \cdots \vartheta^{(n)}(z,\lambda) + \cdots \sum_{n=1}^{\infty}\vartheta^{(n)}(z,\lambda) \quad [41]$$

where

$$\vartheta^{(1)}(z,\lambda) = \frac{1}{2}\kappa(z)\left[\Theta\left(\frac{\lambda - z}{2}\right) - \Theta\left(\frac{\lambda + z}{2}\right)\right] \quad [42]$$

and

$$\vartheta^{(n)}(z,\lambda) = \kappa(z)\left[\int_0^z \vartheta^{(n-1)}(\zeta,\lambda + \zeta - z)d\zeta + \right.$$
$$\left. + \int_0^{\frac{\lambda - z}{2}} \{\vartheta^{(n-1)}(\zeta,\lambda - \zeta - z) - \vartheta^{(n-1)}(\zeta + z,\lambda - \zeta)\}d\zeta\right] \quad [43]$$

* In such way, for example, equation 33 has been solved.

At last, equation 40 can be solved approximatively, if we consider, instead of this equation, a corresponding system of algebraic equations, obtained by replacing the integrals by sums of a finite number of terms. For our purposes the method of differential equations (applicable only for special cases) and the method of Volterra-Neumann has only a limited practical importance. Thus for practical applications the last-mentioned method of an approximative solution appears to be most useful. The accuracy obtained by means of this method is sufficient because the $\rho(z)$ functions are practically given only with finite accuracy, usually in the form of tables or diagrams.

In order to get the approximative solution of equation 40, we assume a certain sufficiently small depth interval h and put into formula 40 $z = nh$, $\lambda = (n + 2m)h$, $\zeta = ih$, replacing simultaneously the integrals by corresponding sums. In this way the following equation is obtained:

$$\vartheta(nh,[n+2m]h) = \kappa(nh)\Bigg[\tfrac{1}{2}\Theta(mh) - \tfrac{1}{2}\Theta([n+m]h) + h\sum_{i=1}^{n-1}\vartheta(ih,[i+2m]h) + \\ + h\sum_{i=1}^{i=m}\{\vartheta(ih,[2m-i]h) - \vartheta([i+n]\,h,[2m+n-i]h)\}\Bigg]^{*} \qquad [44]$$

Using this formula we can determine by turns the values of $\vartheta(nh,[h + 2m]h)$ for all values of m and n.

Potential on Earth Surface

Observe a certain point P of the earth's surface, placed at distance r from the acting point source J. The potential $V(r)$ of this point can be considered as composed by three parts:

$$V(r) = V_J(r) + V_\Theta(r) + V_\vartheta(r) \qquad [45]$$

The first component of $V(r)$ forms the potential that would be developed by the given point source J, if the soil were exactly homogeneous (of resistivity ρ_0). This component is expressed by the formula

$$V_J(r) = \frac{J\rho_0}{2\pi r} \qquad [46]$$

The second part of the potential is caused by the lens-sources, given by means of function $\Theta(z)$.

Both elementary lens-sources of this kind, of the strength $J\Theta(z)dz$† and of the linear parameter z, placed at distance z below and upon the

---

* The application of more accurate formulas for approximative numerical integration (the formula of Simpson, for example) would lead to equations more difficult to use, but of a higher degree of approximation.

† See formula 17.

earth's surface, develop in point P, according to formula 1, the elementary potential

$$dV_\Theta(r) = 2\rho_0 \times \frac{J\Theta(z)dz}{4\pi\sqrt{r^2 + 4z^2}}$$

Thus we have

$$V_\Theta(r) = \frac{J\rho_0}{2\pi}\int_0^\infty \frac{\Theta(z)dz}{\sqrt{r^2 + 4z^2}} \tag{47}$$

The third component $V_\vartheta(r)$ is developed by the lens-sources that are characterized by the function $\vartheta(z,\lambda)$.

The pair of infinitesimal lens-sources of intensity $J\vartheta(\zeta,2z - \zeta)d\zeta dz$ and of parameter $\lambda = 2z - \zeta$, situated at distance $\zeta(0 < \zeta < z)$ upon and below the earth's surface, cause in P the elementary potential

$$dV_\vartheta(r) = 2\rho_0 \times \frac{J\vartheta(\zeta,2z - \zeta)d\zeta d(2z)^*}{4\pi\sqrt{r^2 + 4z^2}}$$

Hence, it is

$$V_\vartheta(r) = \frac{J\rho_0}{2\pi}\cdot\int_0^\infty \frac{2dz}{\sqrt{r^2 + 4z^2}}\int_0^z \vartheta(\zeta,2z - \zeta)d\zeta \tag{48}$$

By addition of the three component potentials given by equations 46, 47 and 48 we obtain:

$$V(r) = \frac{J\rho_0}{2\pi}\left\{\frac{1}{r} + \int_0^\infty \frac{dz}{\sqrt{r^2 + 4z^2}}\left[\Theta(z) + 2\int_0^z \vartheta(\zeta,2z - \zeta)d\zeta\right]\right\} \tag{49}$$

Substituting $$F(z) = \Theta(z) + 2\int_0^z \vartheta(\zeta,2z - \zeta)d\zeta \tag{50}$$

gives

$$V(r) = \frac{J\rho_0}{2\pi}\left\{\frac{1}{r} + \int_0^\infty \frac{F(z)dz}{\sqrt{r^2 + 4z^2}}\right\} \tag{51}$$

Applying for computations the approximative formula 44 for $\vartheta(z,\lambda)$, it is convenient to use for the calculation of the potential also formulas of the same degree of approximation. To obtain these formulas, replace the integrals of equations 50 and 51 by corresponding sums. By that method we obtain

$$V(r) = \frac{J\rho_0}{2\pi}\left\{\frac{1}{r} + h\sum_{n=1}^{M} \frac{F(nh)}{\sqrt{r^2 + 4n^2h^2}}\right\} \tag{52}$$

---

* The value $d(2z)$ represents here the differential $d\lambda = d(2z - \zeta)$ for ζ = constant.

where M denotes a sufficiently large figure, and $F(nh)$ is expressed as

$$F(nh) = \Theta(nh) + 2h\sum_{i=1}^{n}\vartheta(ih,[2n - i]h) \qquad [53]$$

In practical applications to earth-resistivity measurements, we deal usually not with single point sources and with potentials of single points, but with arrangements like that shown on Fig. 7. In this two current electrodes I_1 and I_2 are applied to carry the current into the earth, and between two potential electrodes V_1 and V_2 is measured the potential difference U.

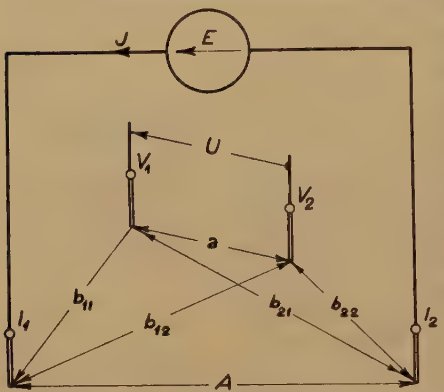

FIG. 7.—ARRANGEMENT OF MEASUREMENTS.

The two current electrodes we consider as two point sources with the same intensity J but with the opposite sign (+ and −). Using the designations given on Fig. 7, we obtain for voltage U between electrodes V_1 and V_2 the formula

$$U = V(b_{11}) + V(b_{22}) - V(b_{12}) - V(b_{21}) \qquad [54]$$

where $V(r)$ ($r = b_{11}, b_{12}, b_{21}$ and b_{22}) denotes the function given by formula 51 or 52.

For configuration of electrodes, given by Wenner[10] ($b_{11} = B_{22} = a$, $b_{12} = b_{21} = 2a$) we get thus

$$U = 2[V(a) - V(2a)] \qquad [55]$$

APPARENT RESISTIVITY

The quantity denoted as "apparent resistivity" by earth-resistivity measurements is defined as follows:

The apparent resistivity ρ^* measured by a given arrangement of electrodes is equal to the resistivity of a homogeneous soil, which gives the same potential difference U between the potential electrodes, when an identical configuration of electrodes and the same current intensity is used for measurements.

With this definition, we obtain the value of apparent resistivity $\rho_s{}^*(r)$ corresponding to an arrangement of a single current electrode expressed by the equation

$$\frac{J\rho_s{}^*(r)}{2\pi r} = \frac{J\rho_0}{2\pi}\left[\frac{1}{r} + \int_0^\infty \frac{F(z)dz}{\sqrt{r^2 + 4z^2}}\right]$$

Hence

$$\rho_s^*(r) = \rho_0\left[1 + r\int_0^\infty \frac{F(z)dz}{\sqrt{r^2 + 4z^2}}\right] \qquad [56]$$

Or, approximatively,

$$\rho_s^*(r) = \rho_0\left[1 + rh\sum_{n=1}^{M} \frac{F(nh)}{\sqrt{r^2 + 4n^2h^2}}\right] \qquad [57]$$

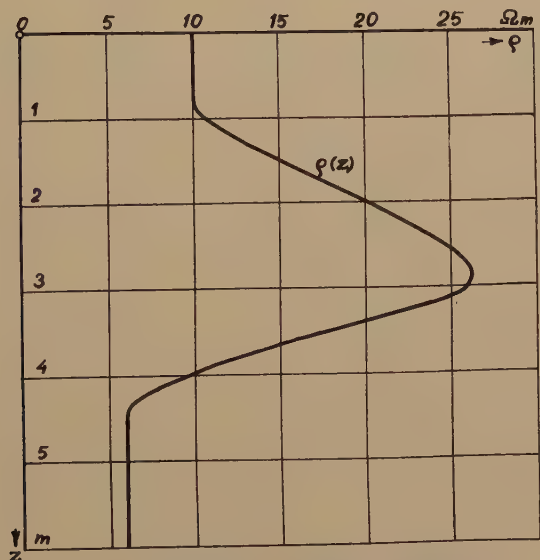

FIG. 8.—EXAMPLE I, GIVEN $\rho(z)$ CURVE.

On the basis of this formula can be obtained the apparent resistivity for an arbitrary four-electrode arrangement (Fig. 7):

$$\rho^*(b_{11},b_{12},b_{21},b_{22}) = \frac{\frac{\rho_s^*(b_{11})}{b_{11}} + \frac{\rho_s^*(b_{22})}{b_{22}} - \frac{\rho_s^*(b_{12})}{b_{12}} - \frac{\rho_s^*(b_{21})}{b_{21}}}{\frac{1}{b_{11}} + \frac{1}{b_{22}} - \frac{1}{b_{12}} - \frac{1}{b_{21}}} \qquad [58]$$

for Wenner's arrangement is thus:

$$\rho^*(a) = 2\rho_s^*(a) - \rho_s^*(2a) \qquad [59]$$

NUMERICAL EXAMPLES

By means of the new method given in this paper, two examples of numerical calculations of apparent resistivity curves will be now carried out for functions of $\rho(z)$ given graphically.

Example I

Fig. 8 shows the $\rho(z)$ curve used in this example. On the basis of this curve the values of $\rho(z)$ and $\frac{d}{dz}\rho(z)$ were determined for the depth

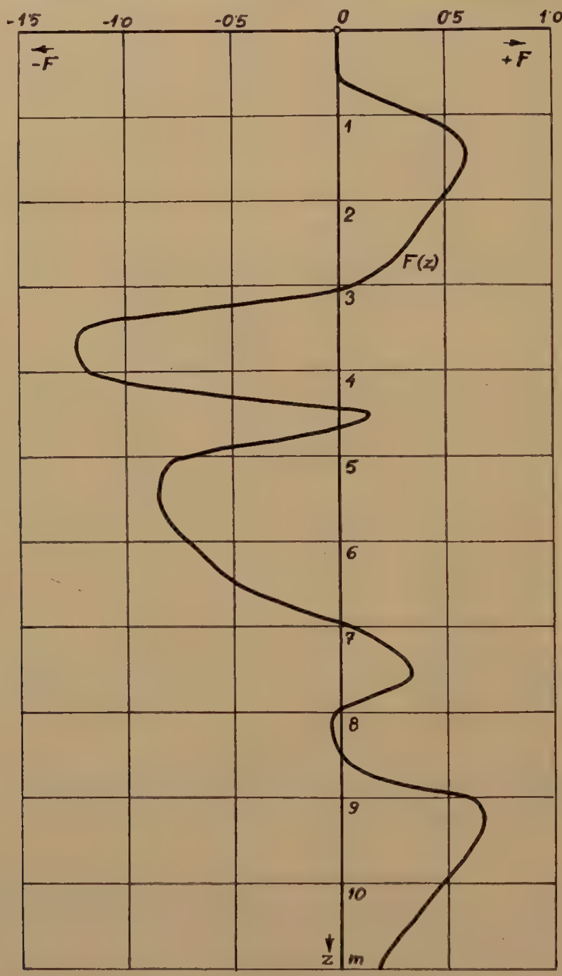

FIG. 9.—EXAMPLE I, GRAPH OF $F(z)$.

values $z = nh$ $(n = 1,2, \ldots)$. The values obtained thus for $h = \frac{1}{2}m$ are given in the columns 3 and 4 of Table 1. These values we use in order to compute by means of formula 16 the specific reflection coefficient $\kappa(z)$.

For $n = 3$ (i.e., for $z = 1.5m$), we find, for example, in this way

$$\kappa(3h) = \kappa(1.5m) = \frac{1}{2\rho(z)} \times \frac{d}{dz}\rho(z) = \frac{1}{2 \times 15} \times 10 = 0.3333m^{-1}$$

All values of $\kappa(z)$ obtained for $z = nh$ are listed in section 5 of Table 1.

Using now formula 35, we calculate the values of $\Theta(z)$, getting, for example, for $n = 3$ (i.e., for $z = 1.5m$):

$$\Theta(3h) = \Theta(1.5m) = 2\kappa(z)\sqrt{\frac{\rho(z)}{\rho_0}} = 2 \times 0.3333 \times \sqrt{\frac{15}{10}} = 0.8164m^{-1}$$

All the $\Theta(z)$ values determined are given in Table 1 in column 6.

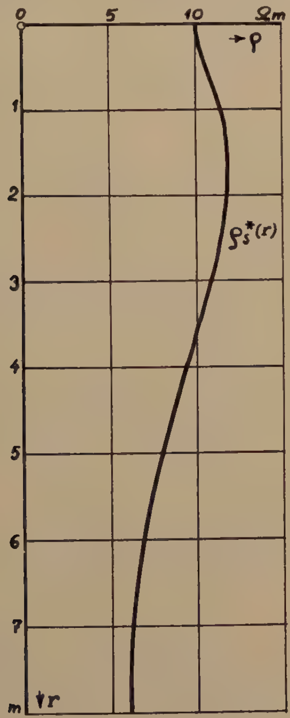

FIG. 10.—EXAMPLE I, DIAGRAM OF $\rho_s{}^*(r)$.

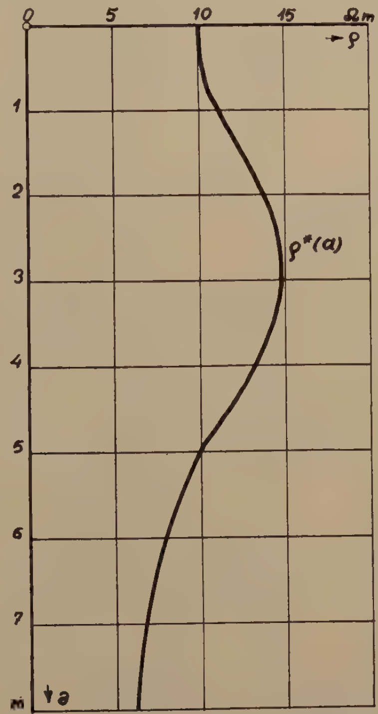

FIG. 11.—EXAMPLE I, APPARENT RESISTIVITY BY WENNER'S METHOD.

Further, with the aid of approximative equation 44 we calculate by turns the values of $\vartheta(nh,[2m + n]h)$. The obtained values are tabulated in section 7 of Table 1.

For illustration, we will show here the way of computing the ϑ value for $n = 5$, $m = 4$; i.e., the value $\vartheta(5h,[2 \times 4 + 5]h) = \vartheta(2.5m,6.5m)$.*

Equation 44 gives in this case:

$$\frac{1}{\kappa(5h)} \cdot \vartheta(5h,[2 \times 4 + 5]h) = \frac{1}{2}\Theta(4h) - \frac{1}{2}\Theta(9h) + h\begin{bmatrix} \vartheta(\ h,[2 \times 4 + 1]h) \\ +\vartheta(2h,[2 \times 4 + 2]h) \\ +\vartheta(3h,[2 \times 4 + 3]h) \\ +\vartheta(4h,[2 \times 4 + 4]h) \end{bmatrix}$$

$$+ h\begin{bmatrix} \vartheta(4h,[2 \times 0 + 4]h) \\ +\vartheta(3h,[2 \times 1 + 3]h) \\ +\vartheta(2h,[2 \times 2 + 2]h) \\ +\vartheta(1h,[2 \times 3 + 1]h) \end{bmatrix} - h\begin{bmatrix} \vartheta(9h,[2 \times 0 + 9]h) \\ +\vartheta(8h,[2 \times 1 + 8]h) \\ +\vartheta(7h,[2 \times 2 + 7]h) \\ +\vartheta(6h,[2 \times 3 + 6]h) \end{bmatrix}$$

* This value is placed in the double-framed cell of Table 1.

TABLE 1[a]

1	2	3	4	5	6	7													
n	z	ρ	$\frac{d\rho}{dz}$	$\kappa(z)$	$\Theta(z)$	$2 \times \vartheta(n.h,[2m+n]h)$[b]													
	m	Ωm	Ω	m^{-1}	m^{-1}	$m = 0$	$m = 1$	$m = 2$	$m = 3$	$m = 4$	$m = 5$	$m = 6$	$m = 7$	$m = 8$	$m = 9$	$m = 10$	$m = 11$	$m = 12$	$m = 13$
0	0	10.00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0.5	10.00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1.0	10.95	5.05	+0.2305	+0.4822	−0.1111	−0.1547	−0.0150	−0.0680	+0.0101	+0.4001	+0.3157	−0.3508	−0.1115	+0.2528	−0.0060	−0.0920	−0.1947	−0.2116
3	1.5	15.00	10.00	+0.3333	+0.8164	−0.2907	−0.2237	+0.0481	+0.2360	+0.7532	+0.7438	−0.0470	−0.2774	−0.1084	+0.3829	−0.1262	−0.4107	−0.4932	−0.2157
4	2.0	20.00	10.00	+0.2500	+0.7072	−0.2270	−0.1326	+0.1495	+0.7217	+0.7299	+0.1608	+0.2068	−0.1766	−0.1141	+0.2359	−0.3331	−0.5055	−0.3150	−0.1782
5	2.5	24.40	6.00	+0.1229	+0.3840	−0.0858	−0.0132	+0.3482	+0.4151	+0.1257	+0.2141	+0.1429	−0.1026	−0.1117	+0.0069	−0.2510	−0.2039	−0.1594	−0.2121
6	3.0	26.00	−2.47	−0.0475	−0.1532	+0.0097	−0.0953	−0.1374	−0.0518	−0.0947	−0.0916	−0.0516	+0.0571	+0.0888	+0.0226	+0.0662	+0.0714	+0.1031	+0.0828
7	3.5	18.00	−24.00	−0.6667	−1.7888	−0.9575	−0.9464	−0.1994	−0.9859	−1.0367	−0.8958	−0.4159	+1.0532	+1.2025	−0.1495	+0.5675	+1.1476	+1.0407	+0.7543
8	4.0	7.85	−13.00	−0.8280	−1.4664	−0.5259	+0.6482	−0.4335	−0.6370	−0.5947	−0.4956	−0.0979	+0.9126	+0.7381	−0.4294	+0.6478	+0.6809	+0.5797	+0.3845
9	4.5	6.00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

[a] In the table are given only the ϑ values for $m = 1 \ldots 13$. Although there were computed ϑ values for all values of m up to $m = 45$.
[b] Regarding formula 53, it appears to be convenient for numerical computations to tabulate values of $2 \times \vartheta$ instead of ϑ.

TABLE 2[a]

n	0	1	2	3	4	5	6	7	8	9	10
z	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
$F(z)$	0	0	+0.4266	+0.5937	+0.4744	+0.3328	+.00428	−1.2036	−1.1690	+0.1428	−0.7410
11	12	13	14	15	16	17	18	19	20	21	22
5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	10.5	11.0
−0.8273	−0.6909	−0.4742	+0.0562	+0.3382	−0.0288	+0.0017	+0.6187	+0.6194	+0.4664	+0.3014	+0.1789

[a] The values of $F(z)$ have been computed in the same way as the ϑ values of Table 1 up to $n = 45$.

Hence it is

$$\frac{1}{0.1229} \times 2 \times \vartheta(5h,[2 \times 4 + 5]h) = +0.7072 - 0 + \frac{1}{2}\begin{bmatrix} 0 \\ +0.0101 \\ +0.7532 \\ +0.7299 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} -0.2270 \\ -0.2237 \\ -0.0150 \\ +0 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 0 \\ +0.6482 \\ -0.1994 \\ -0.0518 \end{bmatrix} = +1.0228$$

or

$$2 \times \vartheta(5h,[2 \times 4 + 5]h) = 0.1229 \times 1.0228 = +0.1257m^{-2}$$

All values used in this computation are indicated in Table 1 by means of cells framed by heavy lines.

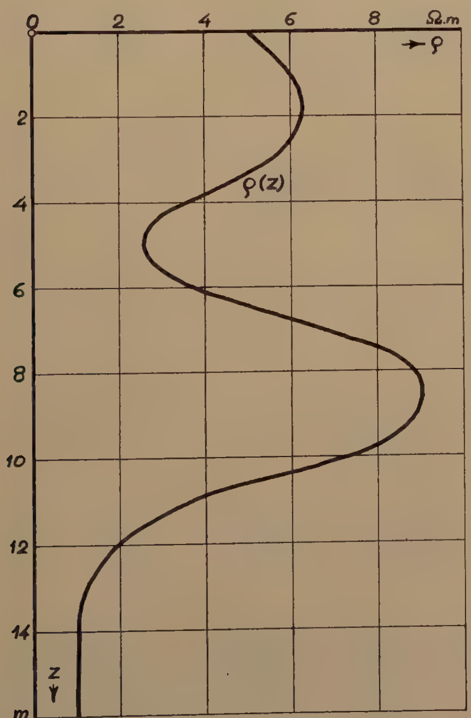

FIG. 12. EXAMPLE II, GIVEN $\rho(z)$ FUNCTION.

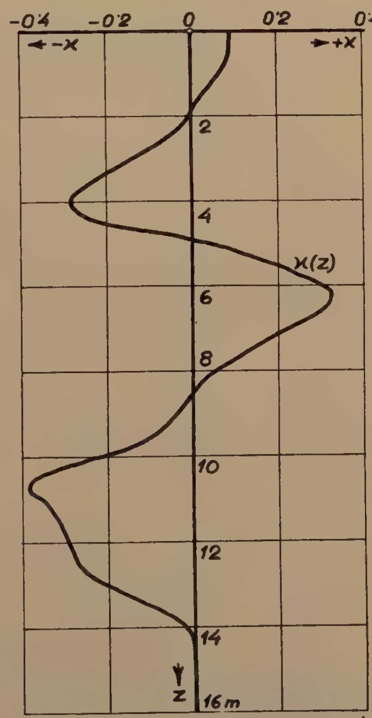

FIG. 13.—EXAMPLE II, GRAPH OF $x(z)$.

Applying now formula 53, we find the values of $F(z)$ for $z = nh$. The value of $F(z)$ for $z = 4h$ we determine by means of eq. 53, for example:

$$F(4h) = F(2m) = \Theta(4h) + 2h \begin{bmatrix} \vartheta(4h,[2 \times 0 + 4]h) \\ +\vartheta(3h,[2 \times 1 + 3]h) \\ +\vartheta(2h,[2 \times 2 + 2]h) \\ +\vartheta(1h,[2 \times 3 + 1]h) \end{bmatrix}$$

$$= +0.7072 + \tfrac{1}{2} \begin{bmatrix} 0 \\ +0.0101 \\ +0.7532 \\ +0.7299 \end{bmatrix} = 0.4744m^{-1}$$

The obtained numerical values of function $F(z)$ are given in Table 2, and a graph of this function is shown on Fig. 9.

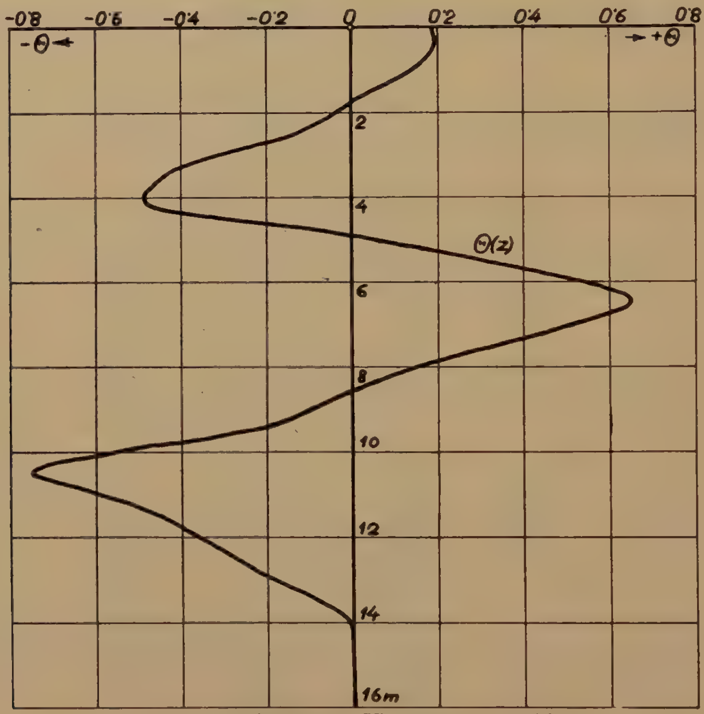

FIG. 14.—EXAMPLE II, GRAPH OF $\Theta(z)$.

At last, formula 57 enables us to find the function $\rho_s^*(r)$ of the apparent resistivity, corresponding to a single electrode arrangement. The values thus obtained are tabulated in Table 3, and the diagram of the function is shown on Fig. 10.

Formula 59 further enables the possibility of computing and draughting the curve of $\rho^*(a)$, which would be obtained in the case examined by measurements carried out with the aid of Wenner's method. The results of this computation are given in Table 4 and plotted graphically on Fig. 11.

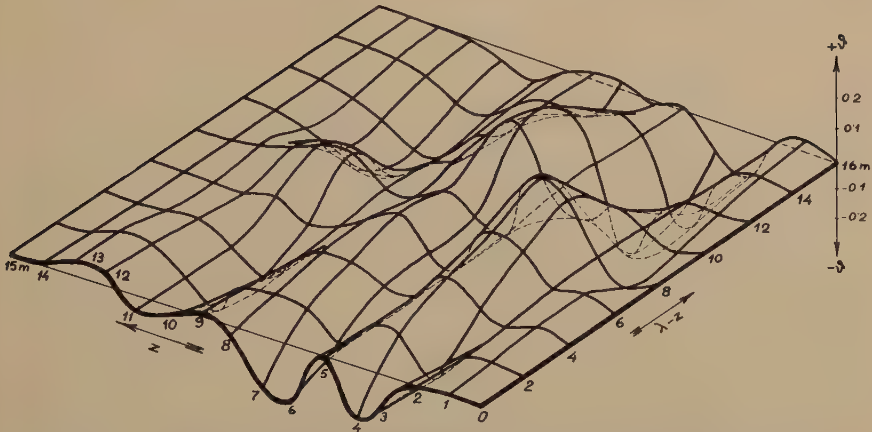

FIG. 15.—EXAMPLE II, THREE-DIMENSIONAL GRAPH OF FUNCTION $\vartheta(z, \lambda)$.

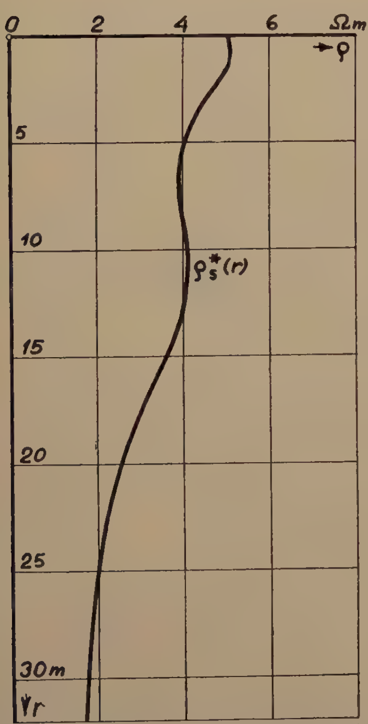

FIG. 16.—EXAMPLE II, GRAPH OF $\rho_s{}^*(r)$.

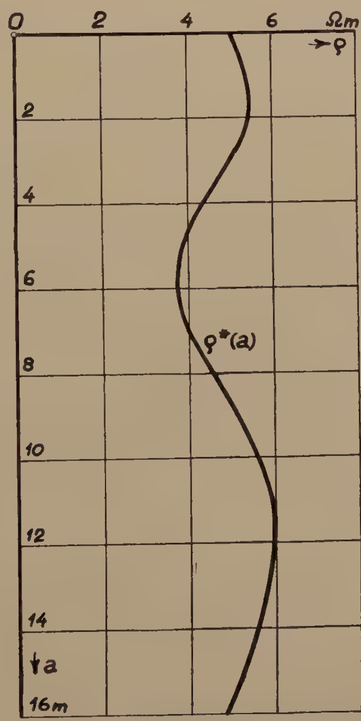

FIG. 17.—EXAMPLE II, APPARENT RESISTIVITY BY WENNER'S METHOD.

Example II

The $\rho(z)$ curve of this example is given diagrammatically on Fig. 12. The results of the computations for this example are given only in the form of diagrams (Figs. 13 to 17). Fig. 13 shows the graph of $\kappa(z)$, obtained by formula 16; Fig. 14 illustrates the curve of $\Theta(z)$ calculated by means of eq. 35; Fig. 15 shows a three-dimensional graph ("relief") of the function $\vartheta(z,\lambda)$ which was obtained with the aid of eq. 44; and the Figs. 16 and 17 show the functions of $\rho_s{}^*(r)$ and $\rho^*(a)$ of the apparent resistivity for the single-electrode method and for Wenner's arrangement.

TABLE 3

r	m	0	1	2	3	4	5	6	7	8
$\rho_s{}^*(r)$	Ωm	10.00	11.53	11.74	10.86	9.51	7.96	6.93	6.38	6.16

TABLE 4

a	m	0	1	2	3	4	5	6	7	8
$\rho^*(a)$	Ωm	10.00	11.32	13.97	14.79	12.86	9.90	7.86	6.76	6.32

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Application of Rapid Current Surges to Electric Transient Prospecting

By Gifford White*

(New York Meeting, February, 1940)

Considerable attention has been directed in recent years to methods of electric prospecting other than the conventional direct-current techniques. It has been extensively recognized that electrical data of a more general nature than that possible by direct-current methods might be useful in geophysical studies. To this end, a number of different types of electric currents have been employed, such as single-frequency alternating currents, square-topped periodic current waves,[4] periodic surges of a less specific nature,[3] and suddenly applied direct current.[1,2] Yet, to the author's knowledge, little analysis of these different types of driving currents has been made from the standpoint of network theory to determine just what type of currents would give the most useful data, and, most important of all, to determine the relation of the data taken by one method to those of another.

Without touching on the actual theoretical problem of how currents flow in the earth under specified boundary conditions, some very useful information can be obtained from the study of the fundamental integral equations of electrical networks, throwing light on the problem not ordinarily appreciated by the field worker. Some manipulation of the standard network equations will be made, with the sole purpose in mind of developing expressions leading to practical results.

Suppose that four points in the earth have been located and it is wished to obtain the most useful type of electrical data by inserting current into two of them and measuring the potential developed between the other two. The first problem that arises is that of which pair of the four shall be chosen as current grounds. In the direct-current case, where only steady potentials are measured, the selection is of little consequence, but as soon as currents varying with time are used at the current grounds, a new effect arises. The fact that a varying current is flowing in the current-carrying conductor between the current grounds means that magnetic and electric fields varying with time will be produced

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[4] References are at the end of the paper.

about the conductor. These fields induce voltages into the potential circuit, which will be added to the earth-conductivity voltages. If it be conceded that the object is to emphasize as strongly as possible the earth-conductivity terms, the wire spread must be set up in such a manner that these direct induction effects will be minimized. As an example, consider the case of using the Wenner spread[10] in which the potential probes are internal to the current grounds, and the wires of the two circuits parallel each other. Then, the varying fields of the current wire will induce fairly large voltages into the potential circuit. One worker reports a series of experiments with this spread,[3] using suddenly applied direct current and measuring the transient thus produced in the potential circuit. He correctly reached the conclusion that the transient voltages thus measured gave little usable information concerning the earth.

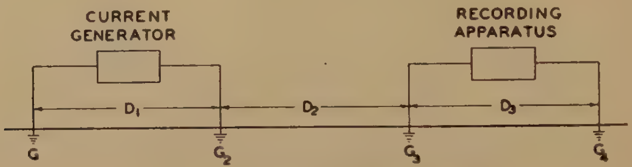

FIG. 1.—DIAGRAM OF FIELD SETUP.

The other logical mode of connecting four such grounds is illustrated in Fig. 1. Here the potential probes are entirely external to the current circuit, and the magnitude of the direct induction terms has been considerably reduced in comparison to the earth-conductivity terms in the measured voltage. Such a spread was used in all of the field work that is to be described later.

Having decided upon and set up a spread at a location, the problem now becomes that of deciding just what data to take to obtain the maximum electrical data about the earth. The direct-current resistivity can be measured, the response to single-frequency alternating currents found, and so on. The number of experiments that could be suggested is infinite. The question that poses itself then becomes: Is there a single piece of electrical data that would contain all other possible data taken in the given setup? The answer is simple if the network approach is used. The symbols and their meanings are as follows:

$A(t)$ is the transient voltage measured at the voltage probes G_3G_4 of Fig. 1, due to one ampere of direct current suddenly applied between G_1G_2. $A(t)$ is commonly called the unit function, or step function, response voltage.

$A'(t)$ is the derivative of $A(t)$ with respect to time.

$I(t)$ is any current wave that may be introduced into G_1G_2 as a driving force.

$E(t)$ is the voltage measured at G_3G_4 when $I(t)$ is applied.

All of these functions are just the oscillograms of current or voltage that would be obtained by experiment during the making of a set of measurements.

The time t will be measured in seconds, and in the integrals the variable t will be replaced by λ to distinguish it from the t of the upper limit. Where Q appears, it will be the charge in coulombs.

From the superposition theorem, all these variables are related by[5,6]

$$E(t) = \frac{d}{dt}\int_0^t I(t - \lambda)A(\lambda)\, d\lambda \qquad [1]$$

This equation displays the fact that $A(t)$ allows the computation of the voltage produced by any current wave. By taking the indicated derivative of the integral, another form is derived:

$$E(t) = I(t)\, A(0) + \int_0^t I(t - \lambda)A'(\lambda)\, d\lambda \qquad [2]$$

$A(0)$ is the value of $A(t)$ computed at $t = 0$, the time of application of $I(t)$. For the normal earth, $A(0) = 0$, since the currents are not propagated in a manner that permits discontinuities of voltage.

The function $A'(t)$ will need to be examined more closely than is possible in equation 2, to see its significance. As a previous paper has pointed out,[7] consider the result of making $I(t)$ a unidirectional surge of current, lasting a very short time, but carrying appreciable charge. If the charge injected into the earth by this unidirectional surge is Q, and the surge lasts a length of time negligible in comparison with the variation of the other time functions, it can be shown that eq. 2 reduces to

$$E(t) = QA'(t) \qquad [3]$$

Solving, this becomes

$$A'(t) = \frac{1}{Q}E(t) \qquad [4]$$

This equation says that if a rapid current surge is applied to the earth, the voltage resulting at a pair of measuring points divided by the total charge Q gives the derivative of the response to one ampere of suddenly applied direct current. The remarkable property of this result is that the wave form of the current surge does not enter into the problem at all; the sole stipulation is that the surge must be of short duration to give a good approximation to the function $A'(t)$. Just how short such a current surge must be will be taken up later.

Consideration of the two basic equations involving $A(t)$ and $A'(t)$ shows that either of these functions allows the computation of the voltage due to any other type of driving current, however arbitrary. Hence, either of these functions contains all the information about electrical

conductivity that it is possible to extract from the earth in the given electrode setup. This answers the question of what single piece of data would be equivalent to all other possible data. If either of these functions has been measured satisfactorily, any further data taken without moving or altering the electrode spread will be redundant, and any less information is not enough.

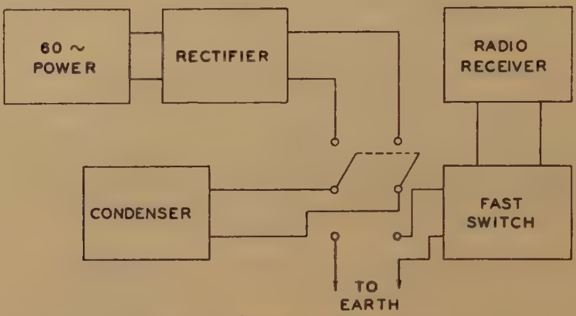

FIG. 2.—SURGE GENERATOR.

The basis for the electrical method known as the Eltran is based on the function $A(t)$. The experimental procedure is to insert a step function of current into the ground by the closing of a switch in a battery circuit, and then to make oscillograms (Fig. 8) or other measurements on the transient produced.[1,2] This particular record was taken during the course of some experimental work with large spreads, and the magnitude of the suddenly applied current was about 120 amperes. The various types of computations that may be made with such oscillograms constitute the Eltran data. There are, however, some engineering limitations to the size of spreads in which battery transient oscillograms can be taken, so attention was directed to the other network function, $A'(t)$.

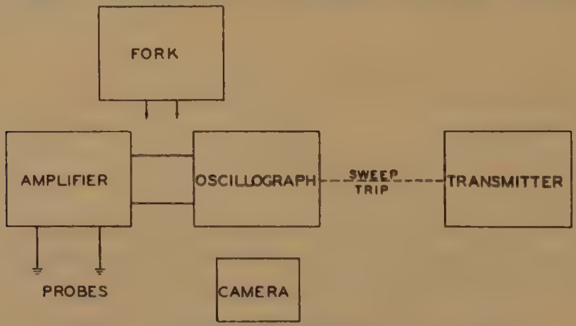

FIG. 3.—TRANSIENT RECORDING APPARATUS.

A surge generator was built up according to the block diagram of Fig. 2. A source of 60-cycle power, which may be a motor-generator set, supplies a rectifier capable of delivering about 3000 volts on open circuit. This rectifier charges up a bank of condensers connected in parallel.

After being charged, the condensers may be reconnected in series to increase the effective surge amplitude and thus reduce the discharge time. A manual switch changes the condensers from charge to shoot position, and the actual discharge is made by the closing of a high-speed mechanical switch. The action of such a switch is important because it must completely close the circuit in a fraction of a millisecond after its points become near enough together to allow the air gap to break down. The closing of this fast switch is controlled by radio, so that the synchronization of the surge with the recording apparatus is possible. The discharge of the condensers is fed to the grounds through heavily insulated cable.

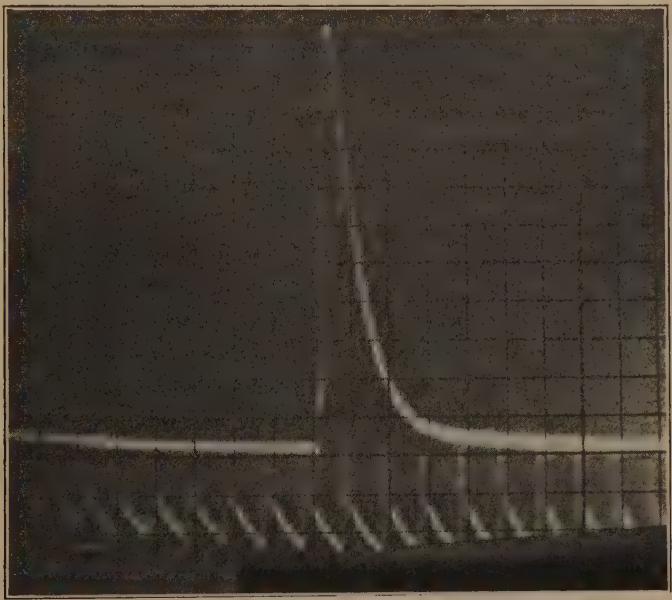

FIG. 4.—SURGE GENERATOR VOLTAGE TRANSIENT.

The recording apparatus (Fig. 3) consists of a high-fidelity amplifier connected to the potential grounds, and a cathode-ray oscillograph with its associated camera. A radio transmitter is used to transmit the synchronizing signals to the surge apparatus. The transmitter and the linear sweep circuit of the oscillograph are interconnected in such a manner that the signal that trips the high-speed surge-generator switch also initiates one sweep of the cathode-ray trace across the screen. Timing is arranged to make the transient fall in the middle of the screen. The camera shutter is opened just before the surge and a record of the oscillograph trace is made. Then, the wave of a 1000-cycle tuning fork is superimposed upon the transient photograph to provide a time axis. The vertical sensitivity of the recording apparatus must also be measured to provide a voltage axis.

In Fig. 4 is shown an oscillogram of the voltage transient produced by a surge from this apparatus. A transparent ruled screen was placed in front of the cathode-ray tube to provide reference axes for measurements on the oscillogram.

The purpose of taking such oscillograms is to obtain the function $A'(t)$, since the response equations 1 and 2 have shown it to be at least as useful as the step function $A(t)$. The question that remains is whether or not the surge apparatus actually gives $A'(t)$. This can be checked by computations, which can be made easily.

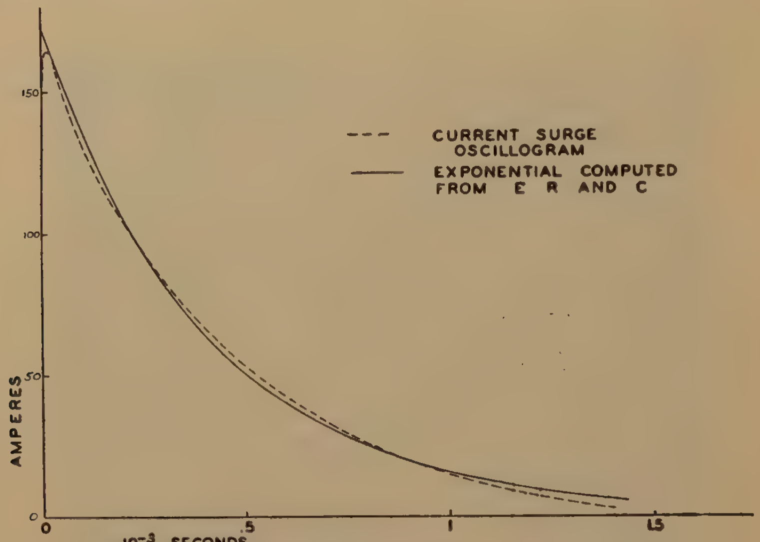

FIG. 5.—CURRENT SURGE COMPARED TO EXPONENTIAL.

Some oscillograms of the current surges were taken for a study of their shape, and it was found that they closely approximate an exponential in form. If the resistance between the current grounds is measured by some simple means and the charge voltage and the condenser capacity known, the current surge approximates the simple discharge of a condenser through a resistance. In Fig. 5 the measurements from a current oscillogram are compared to the theoretical curve of a condenser discharging through a resistance equal to that of the ground. The front of the actual current surge is rounded off, but otherwise the curves have a remarkable coincidence.

The fact that the current wave approximates an exponential in form is a most fortunate circumstance from the standpoint of making computations on the observed voltage transient. With a given ground resistance R, condenser capacity C, and charging voltage E, the current can be approximated as

$$I(t) = \frac{E}{R}\exp(-t/RC) \quad [5]$$

Apply this to eq. 2 and obtain*

$$E(t) = \int_0^t A'(\lambda)\frac{E}{R}\exp\left(-\frac{t-\lambda}{RC}\right) d\lambda \quad [6]$$

Take the derivative of this equation with respect to time to get

$$E'(t) = -\frac{E}{R}\frac{1}{RC}\int_0^t A'(\lambda)\exp\left(-\frac{t-\lambda}{RC}\right) d\lambda + \frac{E}{R}A'(t) \quad [7]$$

The integral in this equation is the same as eq. 6; make a substitution of 6 in 7 and obtain

$$E'(t) = -\frac{1}{RC}E(t) + \frac{E}{R}A'(t) \quad [8]$$

Solve for $A'(t)$ and obtain

$$A'(t) = \frac{R}{E}E'(t) + \frac{1}{CE}E(t) \quad [9]$$

It will be remembered that E/R is the maximum current in the surge discharge of the condenser, and CE is the quantity of charge on the condenser. This remarkable result shows that the voltage transient produced by a condenser surge in the earth allows the computation of the exact function $A'(t)$ by a simple calculation from the experimental oscillogram. The operation is one of pure arithmetic.

The problem of how nearly the condenser surge apparatus approximates the function $A'(t)$ can now be answered by means of eq. 9. Fig. 6 shows a surge transient oscillogram plotted after correction for scale factors. With it is the exact function $A'(t)$ computed by the correction equation. The correction terms have shifted the ordinates slightly to earlier times, but the amount of the correction is small. The surge on this station was from a 14-microfarad condenser charged to 5000 volts and discharged through an electrode circuit of 22 ohms resistance. The record picked for this example is an average one taken during the early experimental setups when insulation resistance of the current-carrying cable was a limiting factor in the type of surges that could be used.

This example shows that it is entirely feasible to approximate the function $A'(t)$ by using a practical surge generator as a driving force, and that a check on performance can be made by the application of a simple correction equation. The method has some advantages over others that make it attractive. The source of power is particularly simple, and can be built at a reasonable cost. A most important consideration is the light weight of the generator compared to other equivalent sources of power.

* The basis for this proof was suggested to the author by W. T. White.

The maximum size of the electrode spreads that can be used with such a surge generator is considerably greater than with any other method because of the extremely high instantaneous power available. For instance, the set of records of which Fig. 4 and Fig. 6 are typical was

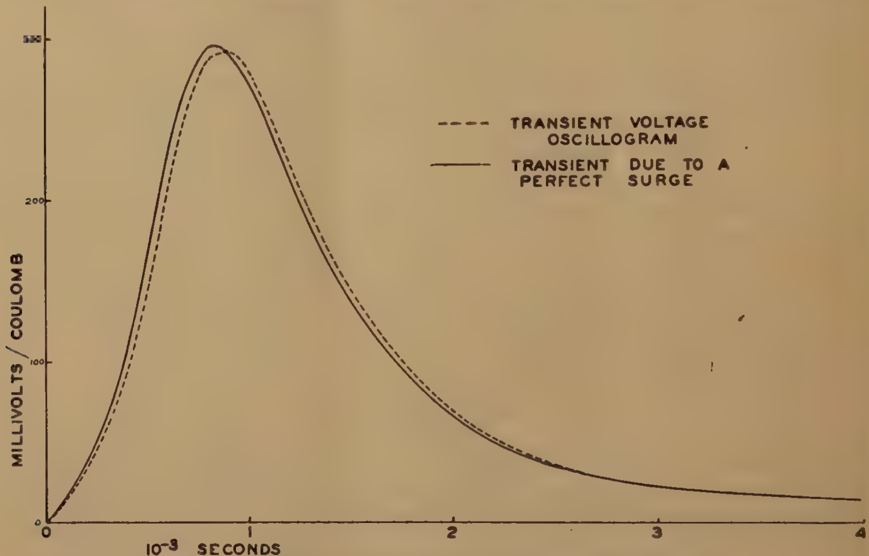

FIG. 6.—SURGE TRANSIENT AND ITS CORRECTED FORM.

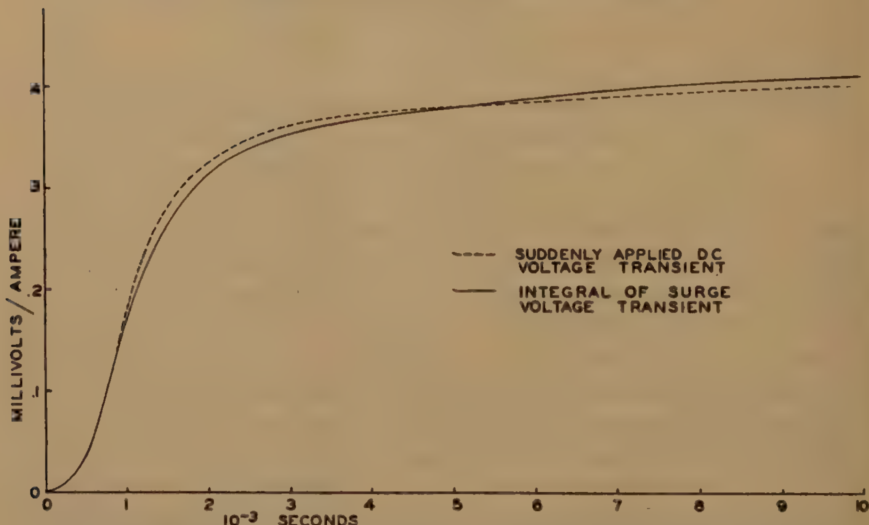

FIG. 7.—COMPARISON OF DIRECT-CURRENT TRANSIENT TO INTEGRAL OF SURGE.

obtained with a spread in which the potential electrodes were entirely external to the current electrodes by 8000 feet.

Aside from the practical possibility of taking $A'(t)$ directly as field data, considerable interest lies in its value for computations. As men-

tioned before, either $A(t)$ or $A'(t)$ allows the computation of the oscillogram that would have been obtained from the application of any other type of driving currents. Eq. 2 involving $A'(t)$ is the easiest to apply because it requires integration only. As examples, the surge of Fig. 6 was applied to a set of calculations, some of them done on the Massachusetts Institute of Technology Cinema Integraph.[9] The integrations may also be carried out graphically with little trouble.

The first obvious computation to make with the surge data is to find the transient due to one ampere of suddenly applied direct current, thus obtaining the function $A(t)$. The integral of the surge in Fig. 6 is plotted

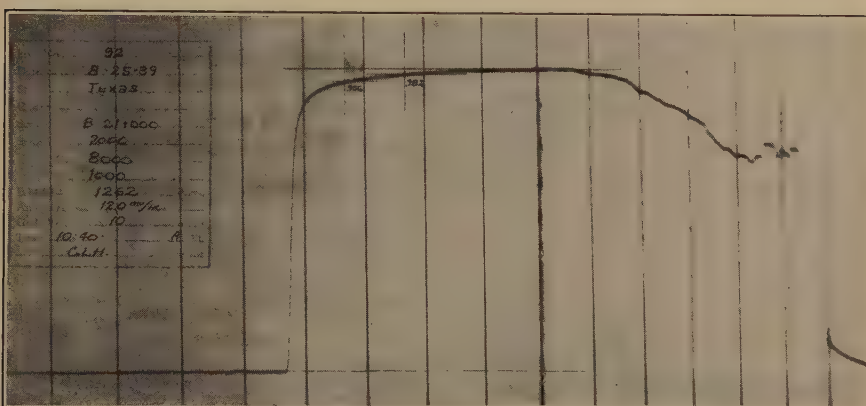

FIG. 8.—SUDDENLY APPLIED DIRECT-CURRENT TRANSIENT.

in Fig. 7, which shows that it rises to a constant steady-state value of about 0.4 millivolt per ampere. This means that for every ampere of direct current in the current grounds at this setup, 0.4 mv. will appear across the potential electrodes. This final height is the quantity usually measured in direct-current prospecting.

An actual oscillogram (Fig. 8) of the transient voltage due to suddenly applied direct current at this same station is also available, taken at a different time with different apparatus. This is the transient voltage due to 120 amp. of direct current suddenly applied to electrodes at the identical location in which the surge of Fig. 6 was taken. The vertical timing lines of Fig. 8 are 0.01 sec. apart. The plot of this experimental oscillogram in Fig. 7, with the integrated curve of the surge transient for comparison, shows that the two curves agree closely. The deviation is explained by the nature of the experimental difficulties in taking the step-function oscillogram of Fig. 8. Clearly, then, this reinforces the conclusion obtained from the response equations that the surge response and the step-function response contain the same data, since a knowledge of one allows the computation of the other.

Another calculation is shown in Fig. 9, where the voltage due to direct-current pulses of one ampere amplitude, 0.005 sec. length and 0.005 sec.

separation, is plotted. This current shape might be called a 100-cycle square-topped wave. The plot shows the transient build-up to steady state during the first cycle, and a complete cycle of the steady-state voltage that would have been measured at the given station. This computed wave should match the experimental wave at least as accurately as the step-function transient computed in the foregoing paragraphs matches the actual measured step-function transient.

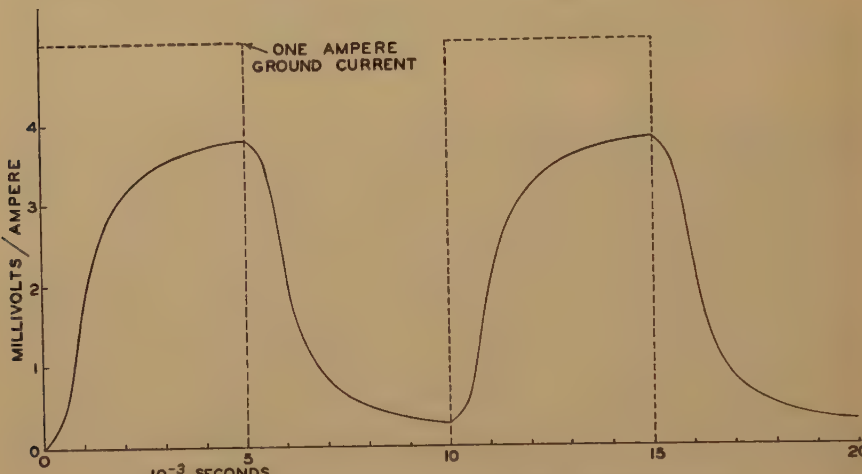

FIG. 9.—VOLTAGE DUE TO SQUARE-TOPPED CURRENT WAVE.

Suppose next that eq. 9 is manipulated by taking the integral of both sides from $t = 0$ to $t = \infty$

$$\int_0^\infty A'(t)\,dt = \frac{R}{E}\int_0^\infty E'(t)\,dt + \frac{1}{CE}\int_0^\infty E(t)\,dt \qquad [10]$$

The left-hand side is the value of the steady-state response to direct current, $A(\infty)$. The first integral on the right-hand side is zero, since the final value of the surge oscillogram is zero. Then we have

$$\begin{aligned} A(\infty) &= \frac{1}{CE}\int_0^\infty E(t)\,dt \\ &= \frac{1}{Q}\int_0^\infty E(t)\,dt \qquad [11] \end{aligned}$$

This means that the area of the oscillogram, just as it would be measured by a planimeter or other means, when divided by the total charge of the surge, is exactly the ordinary direct-current resistivity, and the slowness or rapidity of the surge is of no consequence. Eq. 11 offers an important possibility for taking direct-current resistivity data from the surge-voltage transients without making any oscillograms. With a surge generator as a source of power, and a ballistic type of meter to

integrate the amplified voltage transient, readings could be taken rapidly and accurately without the bother of using calibrated potentiometers and nonpolarizing electrodes, or making any photographic recordings. As mentioned before, the way to using large as well as small wire spreads would be opened by the superior power possibilities of the surge generator.

An interesting quantity can be measured directly from the surge oscillogram by taking the maximum height of the transient in millivolts and dividing by the total charge of the current surge. On the assumption that the surge was sufficiently rapid, the number thus arrived at is the maximum time derivative of the step-function transient, as a moment of consideration of the relation of $A(t)$ to $A'(t)$ will show. Such a set of

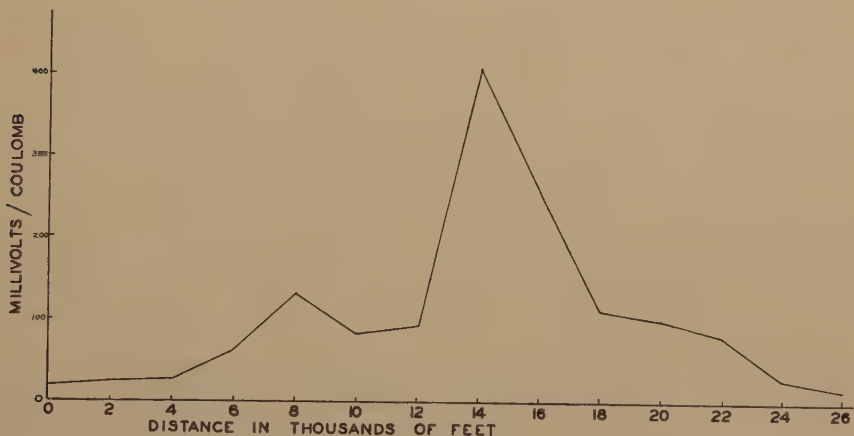

FIG. 10.—TRANSIENT VOLTAGE MAXIMA ACROSS STRUCTURE.

computations were made from the surge-transient oscillograms taken over a large prospect, using spread dimensions that, when labeled as in Fig. 1, were D_1 = 2000 ft., D_2 = 8000 ft., and D_3 = 1000 ft. In Fig. 10 are the results of a profile of such readings made across a known geophysical prospect. The stations are 2000 ft. apart, and the profile totals about 5 miles long. It might be mentioned that although an individual spread covers 2 miles, the center point was selected as the place to write the data on the map, according to the usual convention with electrical methods. A very large increase in the value of the transient maxima will be noticed in the center of the profile at about the axis of the geological structure.

A contour map of the variations in these maximum values is shown in Fig. 11. The stations cover the above-mentioned geophysical prospect, and sufficient stations were taken so that there is no doubt about the choice of contours. The striking feature of the prospect shown is that an area of extremely high maxima coincides with the known geological high, and that an area of much lower maxima, presumably normal, completely

surrounds the high. All of the stations were taken with the same electrode spread dimensions, and most of the stations were repeated on different dates with substantially duplicate results.

A number of workers in the electrical field have considered the possibility that a phenomenon analogous to the reflection seismograph effect might hold for electrical forces in the earth as well; that is, an electrical disturbance might give rise to wave trains of some sort that could be

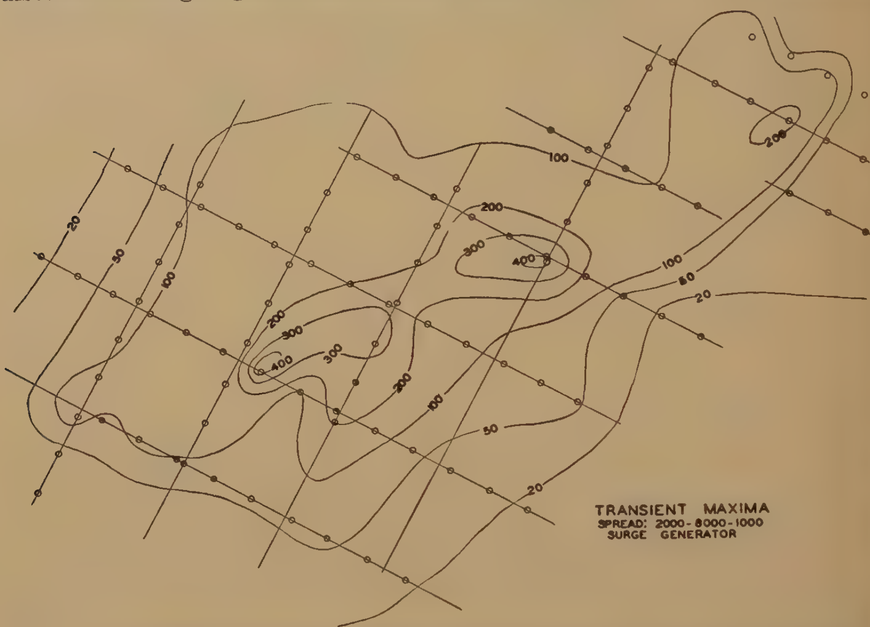

FIG. 11.—MAXIMUM HEIGHT OF VOLTAGE TRANSIENTS AT STATIONS ON GEOPHYSICAL PROSPECT.

reflected from electrical discontinuities and thus form irregularities on the oscillogram, which might be identified and used for correlation. If such an effect exists, the surge transients should show it, since the surge generator provides the sharpest electrical disturbance that can be produced. Several hundred records taken over widely different areas are available, and none so far has shown any reliable indication of such an effect.

ACKNOWLEDGMENTS

The author wishes to express great thanks to E. F. Neuenschwander and to the Eltran party of Humble Oil and Refining Co. for their earnest collaboration in the accumulation and development of the data for this paper, and to W. T. White, of Massachusetts Institute of Technology, who gave his assistance in making the computations. Most of all, the author is indebted to Dr. L. W. Blau, for making the work possible by his encouragement and assistance.

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DISCUSSION

(*L. W. Blau presiding*)

M. K. Hubbert,* New York, N. Y.—In addition to his very able analysis of a difficult problem, I am impressed by the fact that Mr. White has adopted the operational point of view and adhered rigidly to his factual data. While I have no objection to theoretical approaches to the distribution of electric current in an inhomogeneous and anisotropic medium such as the earth, most of the existing approaches along this line have either given unwieldy results or else have been based upon assumptions known not to be correct. In so doing, the fact has frequently been overlooked that in problems as little understood as this one still is our most important results are likely to arise as pure, empirical discoveries, unpredicted by existing theory. It seems to me, therefore, that Mr. White's refusal to speculate upon the current flow in the ground and his adherence to his observational data give a commendable as well as an impregnable position.

S. Pirson,† State College, Pa.—That no reflections of the electrical surge impulses traveling into the ground have been observed by the author may be due to the fact that the time duration of the impulses used is too great (10^{-3} sec.) instead of an impulse of a duration of the order of a few microseconds or less. If the reflected impulses are received when the energizing surge is at its peak value, it is very difficult to observe them. Therefore it is necessary, in order to observe the reflected impulses, to use a surge that will have practically dissipated completely before the reflected electrical impulses are received at the pickup circuit. It is possible that in order to observe reflections, amplification of the reflected wave trains should be resorted to in a manner similar to the present practice of the reflection seismograph.

F. W. Lee,‡ Washington, D. C.—The author's statement that the ratio of the maximum potential value in the potential transient when divided by the maximum

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† Head, Department of Petroleum and Natural Gas Engineering, The Pennsylvania State College.

‡ Chief, Geophysical Prospecting Section, U. S. Geological Survey.

value of the current transient gives the same values secured by direct-current measurements of the same ground configuration of electrodes is interesting. It would not be possible to predict this relation except by field measurements.

The theoretical distribution of current and potential in the ground, even with direct current, is infinitely complex and at the present time not amenable to theoretical evaluations except under the most simple assumptions, and then only in a few singular cases. With alternating currents and impulse currents the theoretical relations become even more complex. Consequently, nothing other than a more or less empirical analysis can be expected at this time. It must, however, be borne in mind that the more rigorous relationships often owe their origin to empirical observations, initially not so well understood but later coordinated into the relatively more exact scientific relations. These experimental observations and relations at the present time are far in advance of the rigorous mathematical theory, and it will probably require the creation or invention of a separate branch of mathematics for properly correlating the observations into a coordinated system of understanding.

G. E. White (author's reply).—It was not intended to comment on the potential theory aspects of the electrical conductivity problem in this paper. However, it is relevant in connection with the discussion on the search for reflections to refer to some theoretical work. Consideration of Maxwell's equations will show that in the case of a medium with the earth's physical constants, electric current flow takes place in a manner analogous to diffusion, and no wave-propagation phenomena can possibly occur except at the high radio frequencies. This means that reflections showing up as sharp wave fronts will not result except at such high frequencies. All available evidence from high-frequency experiments seems to be against any adequate penetration at such radio frequencies.[11]

A larger number of surge oscillograms are now available, and none shows any evidence of reflections in the audiofrequency band. Shorter surges have been tried, but no important changes have resulted in the voltage oscillograms. There appears to be no evidence, either experimental or theoretical, to justify the anticipations of electric reflections. All the phenomena so far discovered are of a diffuse nature.

In regard to Dr. Lee's comments on obtaining direct-current resistivity data from the surge transient data, a statement that appears in the paper should be emphasized. The direct-current data are obtained by taking the ratio of the area of the potential transient to the area of the current surge. These areas can be found by the use of ballistic meters directly or by taking the areas of field oscillograms. These areas have no relation to the maximum values of the voltages and currents, or to the particular manner in which these voltages and currents varied with time. This relation is true for any four-terminal linear electrical network, and the proof that is given arises from accepted network theory.

C. H. Dix,* New York, N. Y.—The use of equations 1 and 2 in Mr. White's paper is not quite the usual one. $E(t)$ is a voltage and $I(t)$ is a current, so that $A(t)$ is an impedance function and not the ordinary indicial admittance. If this is a correct use of the "superposition theorem," it appears to require justification either by a reference or a reconsideration of the "theorem."

From an electrical viewpoint, it appears that the present use of eqs. 1 and 2 is incorrect, or rather vacuous. Is it possible to "suddenly apply one ampere of direct *current* between G_1G_2"? This possibility implies a special character to the "load"

[11] A. S. Eve: *Trans.* A.I.M.E. (1932) **97**, 162.

* Socony-Vacuum Oil Co.

which we have no right to assume. In fact, it appears that no actual case exists, so that the theory has no application.

This exchange of voltage and current is a quite natural procedure for an electrical prospector. In direct-current resistivity measurements one considers an input current rather than an input voltage. The reason for this is that consideration of the input voltage requires a knowledge of the electrode-to-ground resistivities, whereas, when the total current is measured, the effect of electrode variations is automatically taken care of. Such a procedure, while quite correct for steady-state low-frequency currents, may be open to question for transient pulses.

One may suggest that the difficulty pointed out above may contribute as a cause of the apparent extreme shallowness of results of electric transient prospecting[12] along with the well-known "skin effect."

This would suggest that the electric transient method is unlikely to be of much economic importance. We arrive at this conclusion whether shallow effects are important or not. Shallow resistivity measurements are less expensive and appear to serve the same purpose.

G. E. White (author's reply).—As far as the mathematical theory is concerned, a current source capable of producing the desired step and surge current functions can be postulated and the equations derived in the forms given in the paper. The real criticism comes in the question of whether or not the results have any application to practice.

Actual experimental oscillograms of the current resulting from suddenly applied direct-current voltages have been made, and it has been found that on the time scales employed in Eltran recording the current entering the current wire is essentially a step. This indicates that the input impedance of the ground is nearly a pure resistance up to some frequency beyond the resolution of the time scale used. Frequency analyses have been made of the voltage transients appearing at the potential probes, and it has been found that the transmission of the conducted earth currents drops off rapidly with frequency, and that the transmission of the earth is by far a more important limiting factor on the high-frequency content than are departures of the driving current from a perfect step. With these stipulations, it appears admissible to apply the derived equations to the case of the earth transients obtained by the experimental technique sketched in the paper.

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[12] See E. E. Blondeau: *Geophysics*, **4**, 271–278.

Phase Measurements in Electrical Prospecting

By Helmer Hedstrom,* Member A.I.M.E.

(New York Meeting, February, 1937)

The purpose of this paper is to direct attention to the importance and the usefulness of phase measurements in electrical prospecting for ore, a subject about which virtually nothing has been published. The paper describes a new field method for ore prospecting, the "Turam" method, which makes use of phase measurements, and which has been used with great success for five years.

Of the principal geophysical prospecting methods, the gravimetric and magnetic methods determine the direction and strength of the static fields of force of the earth. By seismic and electrical methods, fields of force are *applied* to the block of ground under investigation. By these methods a third parameter can be measured, besides the direction and strength of the field; namely, the time of arrival of the applied field at the different observation points. This time of arrival has been practically the only parameter measured in seismic work, while in electrical prospecting it has generally not been considered, the measurements having been limited to determination of direction and strength of the applied field.

Phase Measurements

In the most widely used of the electrical prospecting methods, those using alternating current, the time of arrival of the "electrical vibration" at the different observation points can be carried out as *phase measurements.* The phase angle measured then expresses, in part of one "vibration," or one cycle, how much the electrical vibrations at the observation point are out of step with those close to the source of the field. A phase lag of 45°, for instance, means that the alternating field in the observation point is one-eighth of one complete "vibration," or cycle, out of step behind the field at the source. If the frequency is, say, 500 cycles per second, a frequency often used in electrical prospecting, a phase lag of 45° then means that the time required for the field of force to travel from the source to the observation point is $\frac{1}{8} \cdot \frac{1}{500} = \frac{1}{4000}$ sec. Since the phase angle can be measured with an accuracy of a fraction, say one-tenth, of a degree, and the frequency can be considerably higher than 500, it is readily seen that the time of arrival can in this way be measured

Manuscript received at the office of the Institute Feb. 20, 1937. Issued as T.P. 827, August, 1937.

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with an accuracy equal to a fraction of one-millionth part of a second. (Incidentally, this has been made use of in the last few years for accurate recording of the speed of propagation of mechanical vibrations in the ground, for investigation of building sites and the like. In that case the mechanical vibrations have been "translated," by means of suitable seismographs, into electrical vibrations, the phase angles of which have then been recorded).

Phase measurements in alternating electromagnetic fields have been carried out by the Electrical Prospecting Co., of Sweden, since 1925, mostly in connection with that company's method for structure mapping in oil and coal fields. This so-called *compensator method* measures amplitude and phase of the electromagnetic fields with reference to the primary current in the source, the "layout," which forms a long straight line or a big loop of insulated cable laid out on the ground. In this application of electrical prospecting, the ground generally has a comparatively high electrical conductivity, which causes a normal phase lag of the electromagnetic field of as much as 45° in a distance of a few hundred feet from the layout and also large attenuation or damping of the field.

The compensator method requires direct connection between the observation point and the layout through an insulated two-conductor cable, and the method therefore becomes somewhat cumbersome and slow when used in prospecting for ore, where it is desired to cover large areas in a short time, often in difficult country. Since further ore prospecting generally is carried out where the country rock is non-conducting (crystalline), so that normally there is no appreciable phase shift or attenuation in the electromagnetic field, it did not in general appear practical or necessary to use the compensator method for this purpose. A method used on a large scale for ore prospecting was the *two-frame method*, which measured only the direction and the strength of the electromagnetic field, through comparison of the fields at successive observation points, say 30 ft. apart, along profiles across the supposed strike of mineralization. The equipment was light and simple: it consisted of two induction coils connected in series by a two-conductor insulated cable, across which was connected an amplifier with a telephone. By tilting the one coil, at one observation point, a certain angle from the horizontal, and holding the other coil horizontal at the next observation point, a balance between the induced currents in the two coils could be obtained. From the tilting angle of the one coil could then be calculated the ratio between the vertical electromagnetic fields at the two points. A proposed modification of this method, the so-called three-frame method, was intended to make possible also the measurement of the difference in phase between the electromagnetic fields at the two observation points, but it never reached any appreciable application, owing to certain prac-

tical difficulties. Another suggestion for a modification of the two-frame method was later made by the Geophysical Experimental Survey of Australia, which proposed the use of its Ratiometer to measure the amplitude ratios and phase differences of the electromotive forces in the two induction coils. However, practical difficulties seem to have prevented the application of the Ratiometer for this purpose, and no results of surveys with the proposed arrangement are known to have been published. A more successful method was worked out by the author and adopted by the Electrical Prospecting Co. about five years ago as its standard field procedure. This so-called "Turam" method, which measures the phase difference as well as the amplitude ratio between fields at adjacent observation points, has since been widely used and has proved very successful in the field.

The Turam Method

The apparatus used is shown in Fig. 1. It consists of two induction coils, each with 1200 turns of insulated copper wire wound on a circular

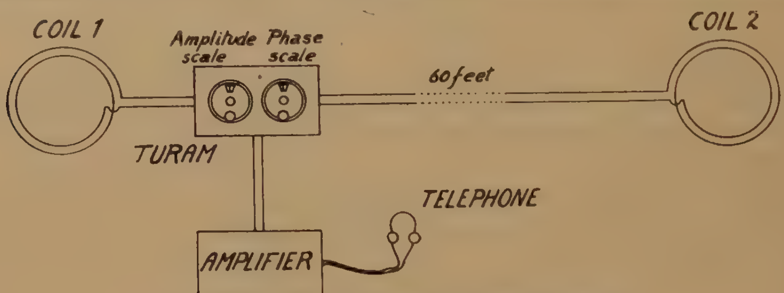

Fig. 1.—Apparatus used in Turam method.

aluminum frame; by means of insulated two-conductor cables these induction coils are connected to the Turam, a simple bridge arrangement, made up of variable resistances. The Turam, through a three-stage amplifier of special design, is connected to a telephone, which serves as a null instrument for adjustment of the Turam. For the taking of a reading the two coils are placed, for instance, 60 ft. apart and held horizontal, while the observer adjusts the two dials on the Turam until no sound is heard in the telephone. The right-hand dial on the Turam then shows directly the phase difference between the fields affecting the two coils, while the reading on the left-hand dial gives the ratio between the strength, or amplitude of the two fields. The instrument can, without change in the position of the two coils, take in phase differences from $+25°$ to $-25°$ and amplitude ratios from 0.6 up to about 10.00.

Fig. 2 shows the field procedure. The Turam, a small box weighing about 4 lb., is fixed on top of the amplifier, which weighs about 20 lb. The whole apparatus is carried on the back of an assistant, who notes

down the readings called out by the observer, while two other assistants handle two induction coils. For reconnaissance work the induction coils are generally kept horizontal, to take in the vertical field component, and at a distance from each other of from 30 to 150 ft. Sometimes the coils are kept vertical and at right angles to (and sometimes parallel with) the profile, in order to take in the horizontal component, and for some purposes one coil may be held vertical and the other horizontal. The reading is very easy to take, since the minima on the two dials are always sharp; the time required to take one reading usually is only about 15 sec. For this reason, the Turam method does not require any skill or any particular training of the observer, like the older electrical prospecting

Fig. 2.—Field procedure with Turam apparatus.

methods—for instance, the equipotential-line and the two-frame methods. It is therefore possible to use as observers workmen employed locally in the district where the prospecting is to be carried out.

Field Procedure

In the prospecting campaign carried out regularly by the Swedish Geological Survey in its search for sulphidic ore bodies, the electrical work is nowadays organized as follows.

Each prospecting party is led by two experts, one of whom, with a young assistant, takes care of all the office work in the camp, while the other one is in charge of all the field work. The latter has 14 workmen, employed locally, as assistants—one motorman, three men for the cutting and staking of base lines and two groups of five men each for the actual surveying.

Base lines are cut and staked out with 200-meter intervals, in a direction parallel to the main strike of the ore-bearing formation; they are marked out with small numbered pegs at every 20 m. A thin, insulated cable 2 or 3 miles long is laid out along one of these base lines and grounded at both ends by means of some 20 to 30 small iron pegs. A small alternating-current generator, driven by a light-weight two-cylinder gasoline engine, is carried up to and connected to this cable at some convenient spot; it is generally able to supply 1 to 1.5 amp. of current to the cable. The two surveying groups now start out at different points along the cable, walking out at right angles to it, crossing the base lines one after the other, all the while taking readings on the Turam every 20 m. and using the two-conductor cable connecting the two induction coils as a tape to measure off 20 m. at a time. One man with a compass walks ahead of the four men that are taking the readings, so that the true direction is kept and no time is lost in looking for the right peg on the next base line. When a surveying group reaches the second base line, 400 m. from the cable, which generally takes less than 15 min., they measure out 40 m. more and then move over 60 m. sideways and turn back on the next profile towards the cable. Where an electrical disturbance, or "indication," is met with, which immediately registers on the very sensitive phase dial, and also on the ratio dial, the men put their next profile only 20 m. away, in order to get a closer investigation of the disturbed area. In disturbed areas readings are also taken along one or two of the base lines, in order to tie all the different profiles together, so that the phase angles obtained along the profiles by addition of all the phase-differences readings can all be referred to the same zero phase. By surveying closed polygons in this manner, a valuable check is made on the accuracy of the phase-difference readings. The phase difference, incidentally, generally is found to be between 0.1° and 0.2°.

In this way the two groups search through during a day a strip of ground 400 m. wide and 1500 to 2500 m. long (an area of 150 to 250 acres) and early next morning the cable is moved up 400 m., so that next day another strip of the same width can be added.

The man carrying the Turam apparatus on his back makes the field notes on a rough map, on which are also put in topographic and geologic features according to instructions given by the supervising expert, and with the help of the compass man who picks out rock outcrops and other notable features. Incidentally, the supervising expert has much more time for geologic observations than if he had to take the instrument readings himself, which is particularly advantageous in new and little known areas.

The efficiency of this system has been outstanding. Taking the bad days with the good ones, and the areas with many indications, which retard the work, with the barren ones, it has been found that a field party as

described above covers, *on an average*, a profile length per day of a little over 6 miles, with observations every 60 ft. (On some days twice this profile length can be surveyed.) This corresponds to *an average*, taken over several months, of 0.6 sq. km., or 150 acres, per day, at a cost amounting to a little less than 70¢ an acre.

Calculation Work

The field notes are brought to camp in the evening where they are laid together to form a rough map, on which it is possible to see where electric disturbances occur, and mostly also how they line up between profiles. In planning the work for the next day, it is possible, therefore, to take immediately the day's results into consideration, which is a great advantage.

From the field notes a map is made up on a scale of 1:2000, with the base lines and the surveyed profiles, and with the readings plotted along the profiles. "Indications," which are characterized by phase differences and abnormal ratios, are lined up between the profiles so that the exact configuration of the existing conductors can be drawn. Where the apparent strike of the indications tends to turn away too much from the direction parallel to the base lines, the phase differences along the profiles are added, so that equiphase curves can be drawn, which generally give a clear picture of the actual strike. Fig. 3 gives an example of this use of the phase measurements. The conducting body indicated by the phase shift in this case proved, after so-called detailed electrical investigation and subsequent pitting, to be a large, valuable, copper-zinc-lead-silver ore.

Another example is given by Fig. 4. As shown on this figure the north-south profiles, at intervals of 60 m., and the east-west profiles at intervals of 100 m., had given indications shown by the short, heavy lines across the profiles. The picture was puzzling, to say the least, and it was not clear how the indications could be lined up between the profiles. When the equiphase lines were drawn the strike showed up, and the picture became quite clear, as shown in the figure, when the phase change across the equiphase lines was plotted as equiphase-change curves. The largest phase differences here were 35° per 10 m. The detailed investigation, followed by some trenching, showed that these indications were caused by strongly folded carbonaceous slates, with heavy mineralization of pyrite.

In areas where indications are obtained, the phase differences and the deviations of the amplitude ratios from the normal are plotted along the profiles, in accordance with the left part of Fig. 5. The dotted line to the left in the figure shows the phase anomaly, obtained simply by plotting the field readings directly. The full line to the left shows the ratio anomaly, obtained by dividing the measured ratios by the corresponding

FIG. 3.—EQUIPHASE CURVES IN ELECTROMAGNETIC FIELD OVER ORE BODY DISCOVERED ELECTRICALLY.

"normal" ratios, which, when a long, straight cable is used as primary layout, are equal to the inverse ratio of the distances from the two induction coils to the cable. The scale of the two curves is chosen so that 0.2° phase difference is equal to 1 per cent in the ratio curve, a relation which just about corresponds to the relative accuracy of the two measurements.

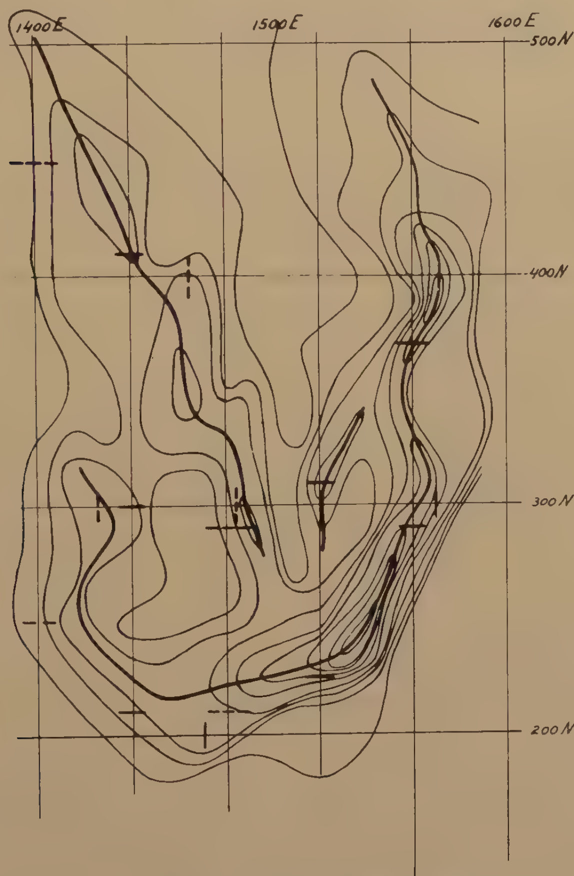

FIG. 4.—CURVES OF EQUAL PHASE DIFFERENCE.

The plotting of these curves makes possible the accurate locating of the indications, and also, as will be shown below, a grading of indications according to the electrical conductivity of the corresponding conducting bodies. A phase difference, without a ratio anomaly, or with a very small ratio anomaly, corresponds to a conductor of comparatively "low" conductivity, a large phase difference with a small ratio anomaly means a conducting body of "medium" conductivity, a medium phase difference with a large ratio anomaly, a "good" conductor, and a small phase

difference with a large ratio anomaly, a "very good" conductor. The example shown in Fig. 5 corresponds to a conductor of the "medium" variety. The maximum phase difference, it may be noted, is about 15° per 10 m., the maximum ratio anomaly 25 per cent above normal, i.e., a "reduced" ratio of 1.25.

The two curves to the right in Fig. 6 show the in-phase ("real") and the out-of-phase ("imaginary") vertical field components, calculated from the phase angles, obtained by summing up the phase differ-

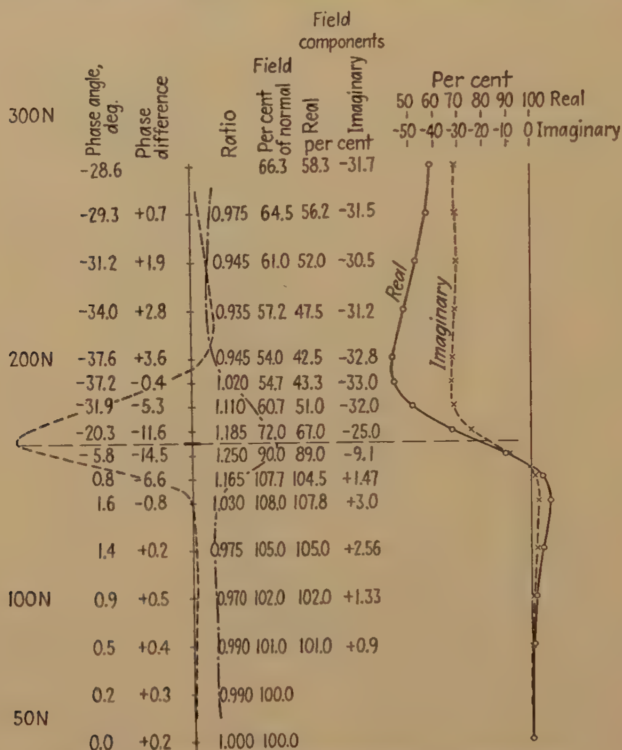

FIG. 5.—PLOT WHEN INDICATIONS ARE OBTAINED.

ences, and from the total field amplitudes, expressed in the "normal" field strengths in the same observation points as unit. These field amplitudes are calculated simply by putting the field equal to 100 per cent at an undisturbed point near the cable (the first observation point at the bottom of Fig. 5), and then dividing consecutively with the reduced ratios. It is easy to show that the figures so obtained will express the field amplitudes in percentage of the normal field strengths in the respective observation points as unit. These figures multiplied by the cosine and the sine of the phase angles will then give, respectively, the "real" and the "imaginary" field components, expressed in the normal field as

unit. The inflection points on these two curves, of course, indicate the exact location of the axis of the secondary current under the profile. It may be noted in Fig. 5 that these inflection points coincide closely with the tops, or maxima, of the curves for phase difference and ratio anomaly. For practical purposes, therefore, it is usually sufficient to plot these two curves in order to indicate the position of the curve of ratio anomaly, which means very little calculation work.

Fig. 6 shows the phase-difference and ratio-anomaly curves over an indication of the "low" conductivity variety, with two current axes.

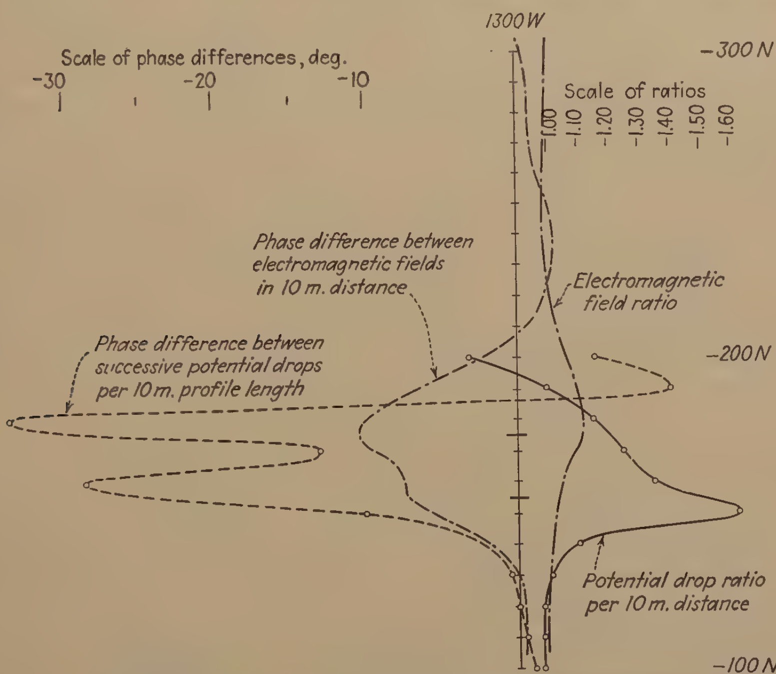

Fig. 6.—Double electrical indication showing phase measurements in electromagnetic and electric field.

The bottom one of these two current concentrations shows up only in the phase-difference curve, although with a pronounced anomaly That this phase anomaly in the electromagnetic field is due to the presence of a conductor under the profile is confirmed by the *potential drop ratios* and the *potential drop phase differences* measured along the same profile, using the Turam not as usual with induction coils but with a special double transformer device, which makes it possible to measure with the Turam potential drop ratios and phase differences between potential drops in the ground. For detailed investigations over conducting bodies of very low conductivity (also over bodies with a conductivity lower than in the surrounding country rock) this system gives a

sharp determination of the current axis, especially through the phase-difference measurement, which in this case has an enormous sensitivity. Ordinarily, however, the phase differences in the electromagnetic field are a sufficiently sensitive indicator, as in the case shown in Fig. 6, to locate everything in the way of conductors, even of the very lowest conductivity. Besides, Turam measurements of potential drops in the ground cannot be used on perfectly dry ground or on frozen ground, as can Turam work in the electromagnetic field, and the latter is therefore preferred and used as the standard method.

Indications that appear to be caused by ore bodies are investigated in detail; that is, they are surveyed from one layout on each side of the indication, and the vertical as well as the horizontal field components are measured. The field components are then calculated as just described in discussing Fig. 6, and the exact location of the current axes is determined from the inflection points on the vertical-component curves, and the maximum points on the horizontal-component curves. As the current axis in each case will be close to that side of the conducting body from which the survey is carried out, the distance between the two current concentrations will then indicate the width of the sub-outcrop of the conducting body. For this purpose, as will be shown below, the out-of-phase components, as measured by the Turam method, give more accurate results than the in-phase components.

Advantages with the Turam Method

The Turam method has been used for five years in more than a dozen countries in Europe, in Africa and in Australia. Because of its many advantages, it has now entirely supplanted the older reconnaissance methods used by the Electrical Prospecting Co.; viz., the equipotential-line and the two-frame methods.

One of the main advantages of the new method, apart from its high efficiency and low cost per acre, is that the field procedure is so simple to learn that it does not require trained observers. Mining companies wishing to start out on a prospecting campaign with the Turam method can therefore use their own employees for the field work, and do not have to rely on a staff of outsiders. Since sharp minimas are obtained under all conditions, the work does not tire the observer as the older methods did, and there is therefore less risk of errors. Besides, an error in a reading is easily discovered during the calculation work, because the ratio reading and the phase reading check each other, and it is therefore not possible for a negligent observer to fake observations.

In comparison with that of the equipotential-line method, the Turam work is easier and faster, and gives much more information. It is significant that the Geological Survey of Sweden, which formerly used the equipotential-line method for reconnaissance work, is now using the

Turam method exclusively, although a license fee is required for the latter while the equipotential method is free from patent protection. There are several reasons for this: One is that the Turam method has proved to be on an average about 50 per cent faster than equipotential-line surveying, using the same number of men, so that the cost per acre is approximately the same in both cases. Also, the Turam work gives so much more detail, owing to the extreme sensitivity of the phase measurements, that it has been possible thereby to *map the structure* of the ore-bearing formation, even when it contains only traces of mineralization. This had not been possible previously, either with the equipotential-line or the two-frame methods. Further, in areas where the ore-bearing formation includes such terrific conductors as are formed by long bands of graphitic slate, the equipotential lines become quite unmanageable, while the Turam method works right up to and across these conducting banks, working out the details of the distribution of electrical conductivity in a way that has never before been possible, not even with the two-frame method. This is a great advantage in an area where ore bodies may be expected right up against the graphitic slates. Also, equipotential-line work cannot, under all conditions, be considered quite safe to pick up all existing conductors, since it does not indicate ore bodies that lack outcrop under the overburden. It has happened, in a couple of notable instances in north Sweden, that ore has been drilled into under a roof of barren and insulating leptite, which prevented any indication of the mineralization from showing up in an equipotential-line survey on the surface of the ground, while very strong indications were obtained in the electromagnetic field—i.e., by Turam work. Further, Turam work can be used with the same efficiency, winter and summer, while equipotential work is impossible when the ground is frozen. For a prospecting program that anticipates also winter work—for instance, because the area concerned takes in also bogs and lakes, which are most easily accessible in the winter—the Turam method is most useful.

In comparison with the two-frame method, the Turam work, besides being faster and easier on the observer, is considerably more sensitive. As the two-frame method measures the ratio between the in-phase parts only of the fields at the two reference points, it misses entirely indications that are caused by conductors of comparatively high resistivity, which, therefore, cause practically only out-of-phase field components. In the experience with the Turam method so far, instances are known where previous two-frame work had given no indication whatever (for example, over known zinc-lead mineralizations, because of their high resistivity) but where later Turam surveys, using exactly the same layout and frequency, gave indications of as much as 8° phase difference per 20-m. profile length. This is a very pronounced indication, if one considers that the mean error of the phase determination generally is between

0.1° and 0.2°, and that indications of 1° per 20 m. are used successfully for the above-mentioned structure mapping of key beds in ore-bearing formations.

Another advantage of the Turam work over the two-frame method is that the phase differences measured are so easily summed up to form phase maps, which show the strike of the conducting bodies, regardless of what striking direction has been assumed by the planning of the survey and the placing of the primary layouts. In very hilly country, the primary field at the different observation points may be difficult to calculate and to correct for, as is necessary in two-frame work. It is then very convenient to have recourse to the measurement of the phase differences—which, of course, are a purely "secondary" phenomenon and therefore need no correction for irregularities in the primary field.

A special advantage of the Turam method over previous methods is the added information given by the measuring of the out-of-phase component. It was described earlier in this paper how this is used for the grading of indications obtained according to the conductivity, or, which amounts to the same thing, the resistivity of the corresponding conducting bodies. It may seem strange that it should be possible to tell something about the *resistivity* of the conductors; one would think that a thin body with low resistivity would give the same electrical indication as a thicker body with higher resistivity, as long as the ratio between cross-section area and resistivity remained the same. This was also the general experience with the older methods of electrical prospecting. That this is no longer true when using phase measurements is shown by Figs. 7 and 8, which represent the results from laboratory measurements over two ore models of the same shape and size, and of the same ratio between cross-section area and resistivity. In the scale 1:1000 these models represent good-sized ore bodies of the resistivities 23 ohm-cm. (the lead model) and 1.75 ohm-cm. (the copper model), corresponding to the "good" and the "very good" variety of conductors referred to above that one may find during field work.

Because lead has about 13 times higher resistivity than copper, the lead model was made 13 times thicker than the copper model. As might be expected, the strength of the total secondary fields is also the same in both cases, in the horizontal as well as in the vertical components, so that the electrical indications would have been the same in both cases had not phase measurements been employed. However, the *out-of-phase components* of the vertical and horizontal fields are much larger over the model with the higher resistivity. The maximum phase lag in the vertical component is 18° over the lead model (observation point 8) but only 8° over the copper model (observation point 7), as can be measured in the vector diagrams to the right in the two figures. In these diagrams, the long vertical line (in-phase axis) represents the amplitude and the phase

of the primary field, which is generated by an alternating current of 440 cycles frequency flowing in a long, straight wire parallel to and 350 mm. to the left in the figure from the ore model. The field components measured at each observation point are expressed in the primary field at the same point (measured with the model removed) as unit, and plotted in the scale given by the said vertical line and also shown to the left in the picture, with positive values counted upwards and to the right in the vector diagram. This diagram shows how, when the ore model is approached

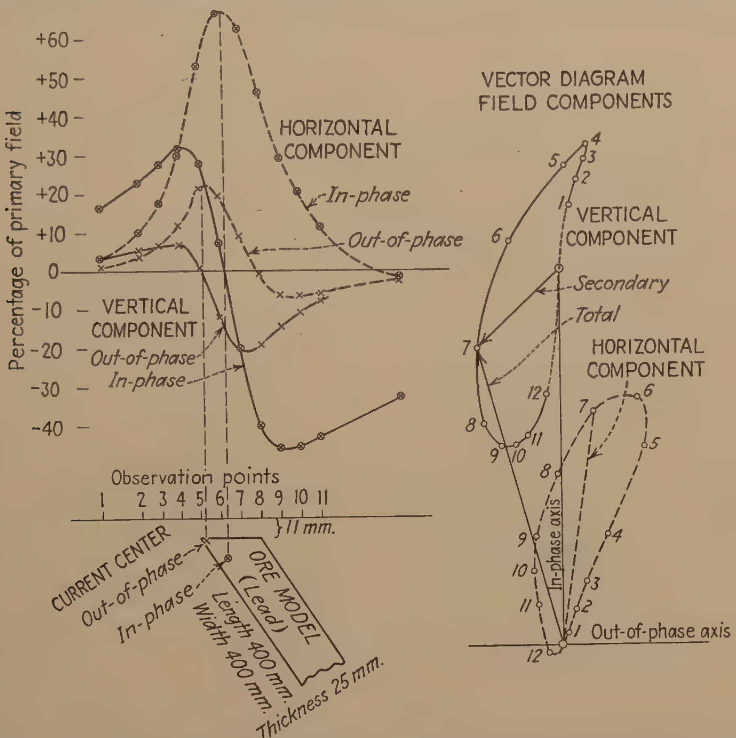

FIG. 7.—ELECTROMAGNETIC SECONDARY FIELD COMPONENTS OVER ORE MODEL.

(observation points 1, 2, 3, 4), the vertical component increases above its normal value, while at the same time its phase swings over to the positive side (leading phase). When passing over the model (observation points 5–9) one experiences a quick phase shift to the negative (lagging phase) side, while at the same time the amplitude quickly decreases below its normal value. On the far side of the model (observation points 10 and up) the vertical field component then gradually comes back to normal strength and phase. In the same way as with the vertical component, the phase of the horizontal component quickly swings over to the left (to lagging phase) over the ore model, showing that the electromagnetic field is being *retarded* by the ore model. (In comparing the phase

angles of the vectors of the vertical and horizontal components in these diagrams, it should be noted that the horizontal-component vectors in the figures should rightly be turned around 180°; this, however, has only theoretical interest, since for practical purposes 0° phase and 180° phase are identical.)

These vector diagrams (Figs. 7 and 8) show plainly the difference in the reaction obtained from the "good" and the "very good" conductor even though the strength of the indication (amplitude of the secondary fields) is the same in both cases. The lower resistivity of the "very good" conductor (Fig. 8) is shown by smaller phase differences and by the fact that the secondary fields, horizontal (observation points 5, 6) as well as vertical (observation points 8, 9) have a phase angle much nearer to the 180° phase of an "ideal" conductor than they have with the "good" conductor. In an "ideal" conductor, of course, the only resistance offered to the secondary current will be caused by self-induction, and the secondary current and consequently the secondary fields, will therefore have a phase angle 90° after the induced electromotive force, which in turn is lagging 90° of phase after the primary field. In a conductor of high resistivity, on the other hand, the secondary current, and consequently the secondary fields, will be only slightly phase-displaced after

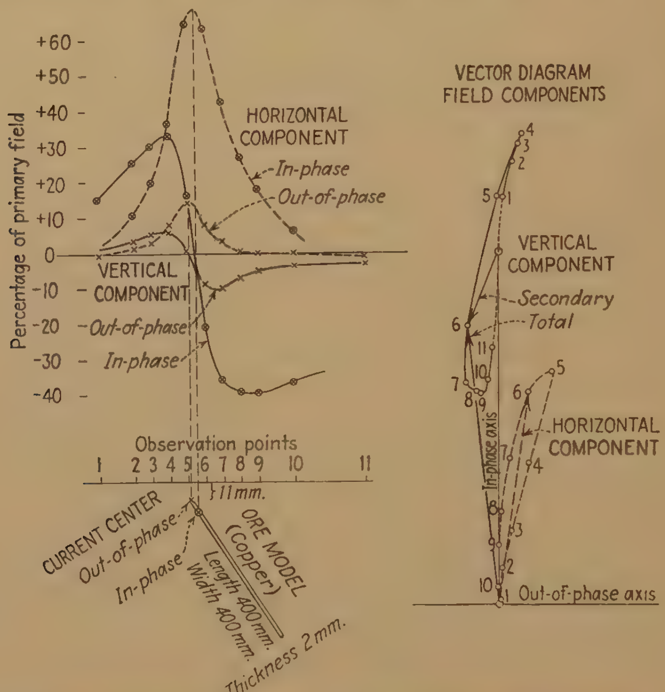

FIG. 8.—ELECTROMAGNETIC SECONDARY FIELD COMPONENTS OVER ORE MODEL.

the induced electromotive force, and the phase angle of the secondary fields will therefore be only a little more than 90° after the primary field. In that case, then, vector diagrams like those in Figs. 7 and 8 will show secondary field almost at right angles to the in-phase axis, meaning that no perceptible damping of the primary field will occur, so that measurement of the vertical field strength only will not reveal the presence of the conductor, except possibly by broader minima than usual in the taking of readings.

Such vector diagrams have proved very useful for the identification and following up of different current axes from one profile to another during detailed work over disturbed areas.

Figs. 7 and 8 show also how the current axis corresponding to the out-of-phase field components falls at the very edge of the conductors, while the in-phase current axis is found a considerable distance from the edge. The current axes are here located below the inflection points of the vertical-component curves and below the maxima on the horizontal-component curves, while the depth to a current axis has been taken equal to the distance from the maximum on the respective vertical-component curve to the inflection point above the current axis, and also equal to the distance from the maximum on the horizontal-component curve to that point in front of the maximum where the curve has risen to one-half of its maximum value. It is easy to show, through simple mathematical deductions, that these rules hold for linear currents.

This demonstrates, of course, as has also been experienced in the field, and as has been already mentioned above, that the use of phase measurements will also give a more accurate determination of the outlines of conducting bodies than has been possible with previous methods.

DISCUSSION

(J. C. Karcher presiding)

H. Lundberg,* Montreal, Que.—In Canada we have reached the same conclusion as Mr. Hedstrom—that the phase analysis is important. We have even got results that at first were startling, in that with an electromagnetic method and phase analysis it has been possible to trace and to outline quartz veins under considerable overburden.

Mr. Hedstrom mentioned a figure of 70¢ per acre. How close do the lines run and what are the distances between readings?

H. Hedstrom.—The lines are 60 m. apart and the readings 20 m. apart.

J. C. Karcher,† Dallas, Texas.—What frequency is used with this instrument?

H. Hedstrom.—Usually 500; for those laboratory experiments, 440 cycles was used.

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* Consulting Engineer.

† Geophysical Services, Inc.

J. C. Karcher.—At about what depth is the ore body shown near the center of Figs. 2 or 3?

H. Hedstrom.—The first one is about 20 ft. only.

D. A. Keys,* Montreal, Que.—There is one point in connection with any bridge method or compensation method—that is, very often in the use of the telephone it is difficult to get an absolute minimum. A magic eye instead of the telephone—an instrument used on radio instruments for getting the tuning on a station—works remarkably well for getting the minimum.

H. Hedstrom.—This apparatus is designed to be as simple and foolproof as possible. I think the telephone is very good for that purpose.

J. C. Karcher.—The difficulty in the use of a telephone usually arises from harmonics in the waves, and if there is a good sine wave in the generator, the telephone probably gives good results.

H. Hedstrom.—It is one of the main advantages of this method that we always get sharp minima, which are not obtained with the other methods of determining direction or amplitude only. That is why unskilled observers can be employed.

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* Professor of Physics, McGill University.

Radioactivity Tests of Rock Samples for the Correlation of Sedimentary Horizons

By H. Landsberg,* Member A.I.M.E. and M. R. Klepper†

(New York Meeting, February, 1939)

Many of the sedimentary rocks contain small amounts of radioactive constituents. These vary in quantity in different layers. Some recent deposits show rather high activity as; for example, the deep sea oozes and some lake deposits.[1,2] Measurements made in boreholes have also shown considerable changes in activity from horizon to horizon. The first to publish data on this fact was Ambronn,[3] and recently Russian geophysicists have shown remarkable variations of activity from one layer to the next in various wells. Spak[4] felt even that he was able to line up horizons from data measured in two neighboring wells.‡

The measuring of radioactive radiation is fairly difficult. Large errors are possible by the circulation of gaseous emanations in the well and porous strata. It is, therefore, never certain whether the activity of the layer where the instrumental device or collector is located will be measured, or that of some adjacent layer. For this reason it seemed to be advantageous to take samples of rocks into the laboratory and test them there for their contents of radioactive material. Such a method has been followed in the past to distinguish radioactive ore from other rocks. Radioactivity of rocks, especially of the igneous type, was also determined in the laboratory by numerous authors for absolute age investigations.[6] Corresponding data from studies of sedimentary formations are scarce. Specimens taken from a core of the Bradford oil sandstone were tested by Landsberg and Ingham,[7] with the results illustrated in Fig. 1, which shows that within relatively small vertical distances the differences of relative activity were considerable.

If one assumes that the distribution of radioactive material is fairly uniform throughout a layer, measurements of radioactive radiation in various horizons would offer additional information for stratigraphic

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Manuscript received at the office of the Institute Feb. 15; revised July 10, 1939. Issued as T.P. 1103, September, 1939.

* Assistant Professor of Geophysics, School of Mineral Industries, The Pennsylvania State College, State College, Pa.

† Graduate Scholar in Geology, School of Mineral Industries, The Pennsylvania State College.

[1] References are at the end of the paper.

‡ Similar results for cased wells were published by Howell and Frosch[5] in the interval between presentation, in February 1939, and publication of this paper.

correlation. It is difficult to decide to what extent such an assumption is justified. A layer deposited in the same sedimentation area and deriving its material from the same erosion area should with similar mineral content also show the postulated uniformity of radioactive substances. In this connection it should be understood that the terms "layer" and "horizon" designate a persistent lithologic unit and not a thin band or lamination.

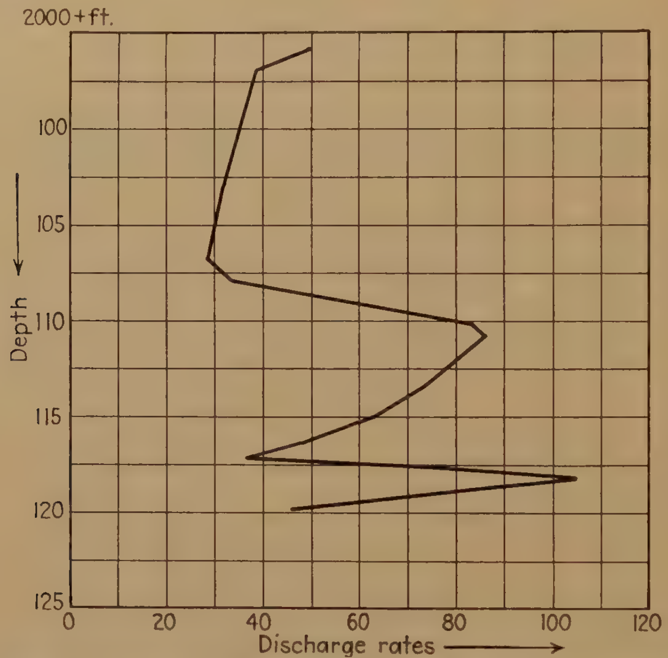

FIG. 1.—RELATIVE RADIOACTIVITY OBSERVED IN CORE FROM PRODUCTIVE HORIZONS OF BRADFORD OIL SANDSTONE.

Depth of core samples is plotted versus rates of discharge observed on an electrometer.

An attempt was made to test the assumption outlined in the preceding paragraph by taking rock samples in corresponding horizons in two localities from the Silurian sediments in Central Pennsylvania and measuring their radioactivity.

THE SAMPLES

The samples for the test were collected from the same five formations in two profiles, separated from each other by a horizontal distance of approximately five miles. The locations where the samples were taken are shown on the geological map (Fig. 2), showing a section southeast of the Tussey Mountain.* The horizons from which the samples came

* Masseyburg, in the center of the area in question, is on latitude 40°40′ N. and longitude 77°55′ W.

are known from the geological evidence to correspond in age. Care was taken to collect fresh rock specimens; to all outward appearances they had not been altered by weathering, which might have changed the contents of radioactive material by leaching. Table 1 indicates the horizons from which the samples were taken, together with a few characteristics of the rocks. It should be noted that in the numbering used sample No. 1 corresponds stratigraphically approximately to sample

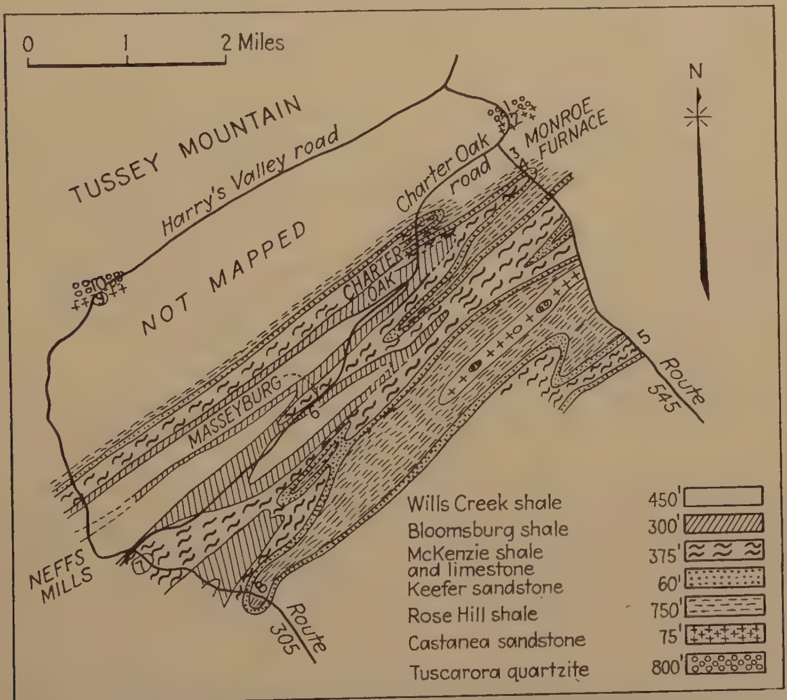

FIG. 2.—POSITIONS OF PLACES AT WHICH ROCK SAMPLES WERE COLLECTED
Geology taken from a preliminary survey by C. A. Bonine.

No. 10, No. 2 to No. 9, No. 3 to No. 8, No. 4 to No. 7, and No. 5 to No. 6. The samples were ground in a mortar to a uniform size of minus 100 mesh and 70 grams of each sample was used for the tests.

PROCEDURE OF MEASUREMENTS

Prior to 1936 practically all of the geophysical investigations on radioactivity were carried on with electrometers. The data on which Fig. 1 is based were gained by means of a Lindeman electrometer. These measurements are cumbersome and require considerable experimental skill. Electrometer methods, especially in the tests conducted for determining the age of rocks, also necessitate fusing the material to drive all the emanations out of it.

TABLE 1.—*Samples Used for Tests*

Profile	Sample No.	Name	Formation	Character
First........	1	Tuscarora quartzite	Basal Silurian	Thick bedded, hard, white, fine grained, firmly cemented.
	2	Castanea sandstone	Clinton	Hard, dark reddish brown quartzite. Grains coated and cemented with hematite.
	3	Rose Hill shale	Clinton	Greenish brown, thin bedded, argillaceous.
	4	Keefer sandstone	Clinton	Rusty colored, medium bedded.
	5	Bloomsburg shale	Basal Cayugan	Light reddish brown, lumpy, argillaceous.
Second......	6	Bloomsburg shale	Basal Cayugan	Reddish, otherwise like No. 5.
	7	Keefer sandstone	Clinton	Like No. 4.
	8	Rose Hill shale	Clinton	Like No. 3.
	9	Castanea sandstone	Clinton	Like No. 2.
	10	Tuscarora quartzite	Basal Silurian	Like No. 1.

Lately electrometers have been replaced by the Geiger-Müller counter. This method was applied for testing rocks almost simultaneously in France,[8] Russia,[9] and this country.[7,10] The ease of handling the counter is a big advantage only partly cancelled by the effects of cosmic radiation on the accuracy of the measurements. Generally γ-ray counters are used. Samples are tested by simply exposing as large a surface as possible to the counting tube. Some γ-radiation is lost by absorption in the sample, but since all samples are treated in the same way this should be proportional to the total radiation in each sample.

The counter and amplifier used in our experiments was a standard commercial set with a GLC 10 Geiger-Müller tube counter and a single-stage, low-current, vacuum-tube amplifier. The impulses could be counted directly by earphones and were recorded by a galvanometer. The tube itself was shielded by a lead mantle 6.5 cm. thick, in order to eliminate as much extraneous radiation as possible. For the same reason all measurements were made in the basement of a three-story building. This precaution resulted in a reduction of nearly 75 per cent of the outside radiation, which amounted to 15 impulses per minute on the average. Thus the basic zero count had the mean value of a little less than four impulses per minute. The simultaneous use of multiple counters, one for the samples, the others for the background radiation, as previously described by Evans and Mugele,[10] would permit the elimi-

nation of alternating blank and sample tests, but for our work only one counter was available.

In the beginning blank tests were alternated with tests using samples for periods of 9 min. each. It was soon found, however, that the variability of background and sample radiation was too large to get satisfactory results. A procedure used by Spak in his field tests was adopted thereafter. In this procedure the time interval for counting a fixed number of impulses is measured. This number is determined by observation of the variability. In using a large enough number of impulses the mean value of impulses per minute will become more and more reliable. It was found that the variability of the mean could be reduced to 2 per cent by using 90 impulses as a basis for the average values. A smaller number of impulses would show large deviations of various tests among each other, so that all possible relations would be obscured and any small systematic deviations be covered up. In the measurements reported here the time required for testing one sample in this fashion was about 20 min. While this procedure results in a smaller percentage deviation of the average number of impulses per minute, it does not guarantee strict reproduction of data between two series. This is mainly due to the background or zero count, which is subject to all the vagaries of cosmic radiation. In order to eliminate the extraneous variability and to get a good picture of the relative radioactivity of the samples, averages of various series have to be used.

The counting equipment at our disposal was not very elaborate and in some cases did not permit the distinction of double impulses. Even with galvanometric recording this difficulty could not be quite avoided. In doubtful cases, therefore, such especially intense impulses were rated at one and one-half the weight of ordinary impulses.

General Experimental Data

For reliability, it was very important to obtain, as outlined above, the basic number of impulses to be used for comparison. Fig. 3 shows the percentage deviation from the mean value of an observation in relation to the number of impulses counted. Beginning with a value of 20 per cent for 15 impulses, the curve asymptotically approaches zero; for 90 impulses the deviation is about 2 per cent. The differences between samples (see below) were as much as 20 per cent, hence for this exploratory investigation there was no reason to extend observations beyond 90 impulses. The values for the deviation obtained in these laboratory tests of rock samples agree closely with the findings of Spak[4] in his observations in boreholes. Fig. 3 also shows why the original procedure of observing a sample for 9 min. would yield poor results. For a 9-min. interval, 36 impulses were observed on an average. A deviation of about 7.5 per cent

can be expected, therefore, and thus smaller differences between samples would be obscured.

Another test of importance was to compare the results obtained by direct counting and by recording. The variations observed, regardless of the method used, were virtually identical. Using galvanometric recording, the deviations between various samples were found to be 24.1 per cent, whereas for direct counting they were 25.1 per cent of the total number of impulses counted.

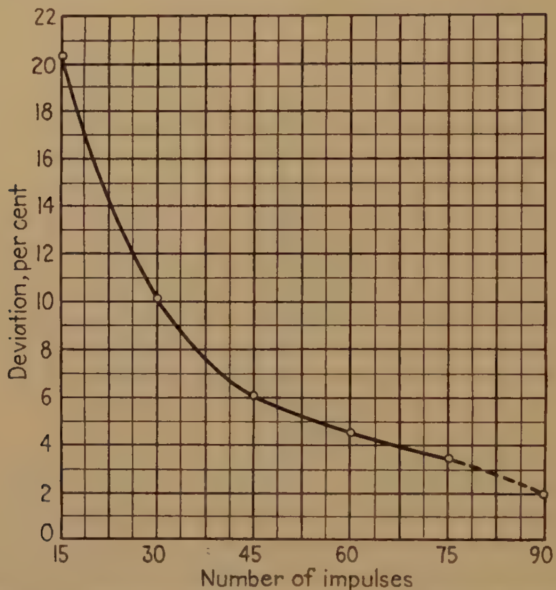

FIG. 3.—RELIABILITY OF MEASUREMENTS WITH GEIGER-MÜLLER COUNTER EXPRESSED AS PERCENTAGE DEVIATION FOR A GIVEN NUMBER OF IMPULSES.

CORRELATION DATA

Tests were run on different days, to eliminate background variations as far as possible. In every series each sample was tested for 50 to 75 min. The results obtained are plotted in Fig. 4. The lower part of the figure gives the variation of γ-radiation in the first profile (Table 1); the upper part represents the second profile. Maxima and minima of radiation coincide for corresponding horizons. Significantly, samples 3, 5, 6 and 8, all shales, seem to be completely barren of active material. All of the other samples, which are sandstones and quartzites, indicate the presence of radioactive material. But it can also be seen that the samples showing some radioactivity are lower in their absolute value in the second profile (samples 6 to 10) than in the first profile (samples 1 to 5). This might be explained by the fact that samples of the first profile were collected along a rather recent road cut, whereas those of the second profile came

from older cuts. Even though the samples did not appear to be weathered, the action of water may have removed some of the active material.

In testing the geological significance of radioactive measurements made at the surface, particularly for oil prospecting, Clark and Botset[11] found a close correlation between Radon measurements and the percentage of heavy minerals in the soil. One cannot expect to find this correlation everywhere by merely using percentages of heavy minerals, since it is obviously important what types of heavy minerals are present. A separation of the heavy minerals was made in bromoform (sp. gr. 2.8) and the results are shown in Table 2.

The figures for samples 2 and 9 have been placed in parentheses because these samples were boiled in acid to remove the hematite coating and therefore are not strictly comparable with the rest of the samples; boiling in acid destroys certain heavy minerals, notably magnetite. The percentages as such show neither a correlation between corresponding horizons nor the characteristic differences between the sandstones and the shales observed in the radioactive profile. If, however, the types of heavy minerals are singled out, better light is thrown on the situation. The last column of Table 2 gives the types of heavy minerals observed in the various samples.

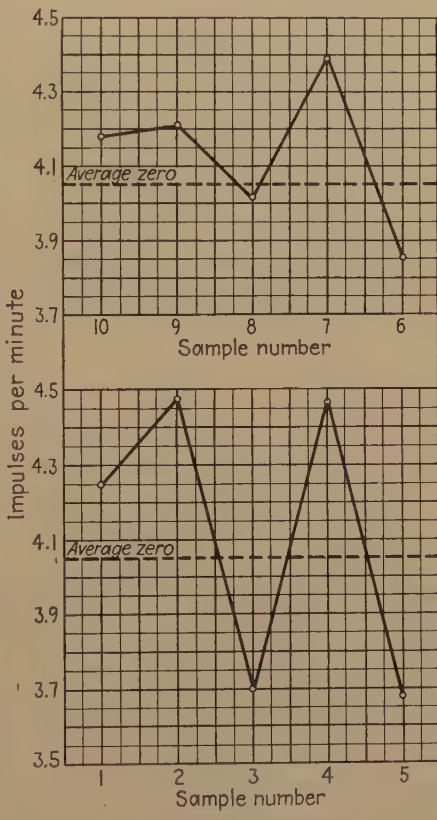

FIG. 4.—RADIOACTIVITY SHOWN BY ROCK SAMPLES.
Two vertical profiles with five horizons corresponding in both.

The table shows that in all samples that indicated radioactivity (Nos. 1, 10; 2, 9; 4, 7) there are abundant zircon grains (order of magnitude 10 to 15 per cent of the total heavy minerals) and also frequent occurrences of tourmaline. In the inactive samples (Nos. 5, 6; 3, 8) tourmaline is completely absent, and zircon is present only in traces of sample 6. Besides, the active samples show evidence of such minerals as monazite and xenotime, not found in the inactive samples. One is tempted to associate the activity with the presence of zircon. The following sentence shows that Clark and Botset clearly recognized the

TABLE 2.—*Heavy Minerals in Samples*

Profile	Sample No.	Percentages of Heavy Minerals	Types of Heavy Minerals[a]
First	1	0.49	Several varieties of zircon (with inclusions) abundant. Tourmaline frequent; red and yellow rutile; chlorite.
	2	(0.07)	Zircon abundant; tourmaline frequent; red and yellow rutile.
	3	0.035	Much pyrite.
	4	1.17	Zircon and tourmaline abundant; rutile, (monazite).
	5	0.37	Hematite, chlorite, apatite.
Second	10	0.31	Zircon (with inclusions) abundant; rutile, tourmaline, (xenotime).
	9	(0.09)	Zircon, tourmaline (xenotime).
	8	0.30	Fluorite, chlorite, pink garnet.
	7	0.51	Hematite, zircon, tourmaline, chlorite, muscovite, (monazite).
	6	0.78	Biotite, muscovite, hematite, barite, apatite, fluorite, yellow rutile and very small amount of zircon.

[a] Minerals in parentheses have not been identified with absolute certainty. In each of the samples there was also a large quantity of opaque material.

importance of this constituent of the heavy minerals: "Since these elements (uranium and thorium) are isomorphous with zirconium, it is only logical to look for a possible relationship between radon content of the soil and the amount of zircon and other rare-earth minerals in the soil."[11] The heavy-mineral analysis, therefore, tends to confirm the measurements of radioactivity. The tests for radioactivity, however, do not consume as much time as a complete analysis of the heavy minerals requires.

The amount of radioactive matter present in the active samples is very low. Even if the number of γ impulses counted is considered to be less than the total emitted by disintegrating atoms, the radium equivalent in these sediments as computed from our measurements does not exceed the order of magnitude of 5×10^{-14} g Ra per gram rock.* For Kimberley zircon, Gockel[12] reports an order of magnitude of 2×10^{-11} g Ra per gram mineral. There is about 10^{-3} gram of zircon per gram of rock in the active samples. If this zircon contains as much radioactive material as the Kimberley zircon, 2×10^{-14} g Ra per gram rock would be

* For the methods of calculation of the amount of radium (or radium equivalent of other radioactive substances) present the reader is referred to the chapter by A. F. Kovarik, Calculating the Age of Minerals from Radioactivity Data and Principles.[6]

present. This value and the actual value are of the same order of magnitude and strengthen the probability of the zircon being the bearer of the activity.

Outlook and Summary

As this work was undertaken in a field where previous data were few, some changes in procedure suggested themselves during the course of the experiments. It seems that core samples should be used for such tests in preference to outcrop material, in order to avoid the possibility of weathering. Furthermore, if samples are tested to about 1000 impulses, inaccuracies in activity data should be reduced to less than one per cent.

While these things had to be learned in the course of the present investigation, the following conclusions seem warranted:

Testing for radioactivity of rock samples by means of the Geiger-Müller counter affords an additional tool for stratigraphic work.

Rock samples taken from two vertical profiles were used for an example. In the vertical sequence three active and two barren horizons were indicated. The same trend of activity was evident from both profiles.

The heavy-mineral separation showed that the presence of zircon and associated minerals characterized the active horizons.

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DISCUSSION

(Sherwin F. Kelly presiding)

F. C. Farnham,* Rolla, Mo.—It appears to me that in the results reported in this paper we have another example of the fact that geophysical markers or horizons exist, and that such horizons do not necessarily coincide with geologic horizons. This fact is often lost sight of when there is attempted correlation between the results of geophysical measurements and the known geologic section.

S. H. Hamilton, Philadelphia, Pa.—Some practical commercial use was made some years ago, near Plum Tree, N. C., in tracing mica-rich pegmatite veins with the ordinary gold-leaf electroscope. Such tests with the electroscope at stations previously blocked out were expressed qualitatively and when plotted gave a picture in agreement with geologic probability. Mining for commercial sheet mica resulted successfully.

L. W. Blau,† Houston, Texas.—I desire to call attention to a paper by Dr. L. G. Howell on the subject of gamma-ray well logging.[13] It has been observed in our work that gamma-ray activity is higher in shales than in sands and seems to be inversely proportional to particle size. This relation suggests that the radioactive materials came down in finely divided form and were deposited with the shales.

L. F. Kindle,‡ Sudbury, Ont.—I would suggest that the explanation may be that in certain varieties of arenaceous strata there is less feldspathic mineral than in those of shale and clay, hence a greater probability of the presence in the latter of the "K_{49}" variety of potassium, which element is characteristically radioactive, giving rise on decomposition to a calcite. Barite, also, though less likely, may explain some radioactivity since barium was found as a by-product in the recently described experiments on the atomic disintegration of uranium.

M. Muskat,§ Pittsburgh, Pa.—How did the samples differ with respect to their properties, such as grain analysis, porosity, etc., other than that of their radioactivity?

H. Landsberg (author's reply).—Regarding the difference between the radioactivity of clays and sandstones suggested by Mr. Kindle, we believe that there is no general rule. Their relative activities probably depend entirely on the source of sedimentation; i.e., the original material that governs how much radioactive matter is deposited in a given stratum.

We cannot give any information in reply to Mr. Muskat's question, because no tests were made on the properties of the samples except for the radioactivity and the heavy mineral contents.

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* Assistant Professor of Physics, Missouri School of Mines and Metallurgy.

† Director Geophysical Research, Humble Oil and Refining Co.

[13] See reference 5.

‡ Consulting Mining Geologist.

§ Gulf Research and Development Corporation.

INDEX

(Note: In this index the names of authors of papers and discussions and of men referred to are printed in SMALL CAPITALS, and the titles of papers in *italics*.)

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